Instructions: Show your work. If you use a theorem or a test to help solve a problem, state the name of the theorem or test.

Question 1 Compute the limit $\lim_{(x,y)\to(0,0)} \frac{x^3y^2+2y^3}{x^3+y^3}$ or prove that it does not exist.

Question 2 For parts a), b) and c) let $f(x,y) = x^2 - 3xy$.
a) At the point with $x = 1$ and $y = -1$ compute the unit vector pointing in the direction of greatest increase of the function $f(x,y)$ and compute the rate of increase in that direction.
b) Compute an equation for the plane tangent to the surface given by the equation $z = f(x,y)$ at the point in space with $x = 1$ and $y = -1$.
c) Find the rate at which $f(x,y)$ is changing at $(1,-1)$ in the direction toward the point $(5,2)$.

Question 3 Let E be the solid bounded by $y = x^2$, $y = x$, $x = z$, and $z = 0$ whose mass density is given by $\rho(x,y,z) = x$. Sketch E and find its mass.

Question 4 Find and classify the absolute extrema of the function $f(x,y) = x^2 - y^2$ over the region $x^2 + y^2 \leq 1$.

Question 5 Compute $\iiint_H z\,dV$, where H is the solid region bounded above by the xy-plane and below by the sphere of radius 4 centered at the origin.

Question 6 Let $f(x,y) = e^{3x-y}\cos(x-1)$. Estimate $f(.98, 3.01)$ using differentials (linear approximation).

Question 7 Change the following triple integral to cylindrical coordinates and then to spherical coordinates:

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx.$$

Now use one of the three integrals to compute the common value.

Question 8 Evaluate

$$\oint_C \arctan(x)\,dx + (3x-4-5y)\,dy,$$

where C is the circle of radius 4 centered at $(2,5)$ parameterized counterclockwise.

Question 9 The fluid flow in a region is given by a vector field $\vec{F} = (x-2y)\hat{\imath} + (2x+6y)\hat{\jmath}$. Compute the total outward flux of the fluid passing through a rectangular box, with opposite corners at the origin and at (4,2). Is there more flow into or out of the box?

Question 10 Consider the following curves:

$$A = \begin{cases} x = 3 + \cos t \\ y = 3 + \sin t \\ t = [0, 2\pi] \end{cases}, \quad B = \begin{cases} x = 3\sin t \\ y = 3\cos t \\ t = [0, 2\pi] \end{cases}, \quad C = \begin{cases} x = 12\cos t \\ y = 9\sin t \\ t = [0, 2\pi] \end{cases}.$$

Suppose we have a vector field $\vec{F}$ defined on all of the plane except the points (3,3) and (0,0). Also suppose that we know that $\nabla \cdot \vec{F} = 0$ everywhere on the plane except those two points.

If $\oint_A \vec{F} \cdot \mathrm{n}\, ds = 2\pi$ and $\oint_C \vec{F} \cdot \mathrm{n}\, ds = -1$, then what is $\oint_B \vec{F} \cdot \mathrm{n}\, ds$?