Instructions: Show your work. If you use a theorem or a test to help solve a problem, state the name of the theorem or test.

Question 1 Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2\cos(x+y)}{x^2+y^2}.$$

Question 2 Fill in the blanks with real numbers to make the following statement as precise as possible:

If at some point (x_0,y_0), $\frac{\partial f}{\partial x}(x_0,y_0)=3$ and $\frac{\partial f}{\partial y}(x_0,y_0)=-4$ then

$$\_\_\_\_\_\_\_\_\_\_\_\_ \leq D_{\vec{u}}f(x_0,y_0) \leq \_\_\_\_\_\_\_\_\_\_\_\_$$

for any unit vector $\vec{u}$. $D_{\vec{u}}f$ is the directional derivative of f in the direction of $\vec{u}$.

Question 3 The cylindrical solid enclosed by $x^2+y^2=4, z=0$ and $z=5$ has mass density given by $\sigma(x,y,z)=8-\arctan(z^2)$. Set up integrals to find the mass and z-coordinate of the center of mass. You do not need to actually do any of the integrals.

Question 4 Find and classify the critical points of the function
$f(x,y)=x^3-12x+3y^2-6y$.

Question 5 Evaluate $\int\int_T (2x+6y)\ dA$, where T is the triangular region in the xy-plane with corners at the origin, $(0,4)$ and $(2,0)$.

Question 6 Find the volume of the solid between the xy-plane and the surface
$z=12x^2y+4$ enclosed by the planes $x=-2$, $x=1$, $y=2$ and $y=4$.

Question 7 Let $\vec{F}=\left(2ye^{2xy}+2\right)\hat{\imath}+\left(2xe^{2xy}+4y\right)\hat{\jmath}$. Find the work done along the path $\vec{r}(t)=\left(t^2+t^3\right)\hat{\imath}+\left(t^2+t^7\right)\hat{\jmath}$, with t in $[0,1]$. It may help to notice that $\vec{F}$ is conservative.

Question 8 Let S be the sphere of radius 5 centered at the point (11,9,7). Think of the sphere as a globe with the north pole at point with the largest z-value. Let C be the curve which is the equator of S, travelled clockwise when viewed from above.
a) Parameterize the curve C.
b) For $\vec{F}=\cos(y^2)\hat{\imath}+\ln(x)\hat{\jmath}+(z-3)\hat{k}$, set up the following integral:

$$\oint_C \vec{F}\cdot \mathrm{T}\,ds.$$

Leave your integral for part (b) in terms of your parameter from part (a). Do not evaluate your integral.

Question 9 Sketch the region of integration, change the order of integration, and evaluate:

$$\int_0^9 \int_{x/3}^{\sqrt{x}} 6x \; dy \; dx.$$

Question 10

a) Evaluate $\displaystyle\int_C y\,dx$ directly as a line integral, where C is an ellipse parameterized by $x = 3\cos(t), y = 2\sin(t)$, with $0 \leq t \leq 2\pi$.

b) Apply Green's theorem to find the area enclosed by the curve C of part (a).