Sample Final Exam
Math 346 – Prof. Santoro

Instructions: All questions are worth the same number of points. **Important: No books, calculators, or notes are allowed. Turn off cell phones, alarms, and anything else that makes noises!**

Answer 8 out of the 10 questions from this exam. Please indicate very clearly which questions you are choosing not to answer.

You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct).

You have 135 **minutes** to complete this exam. Good luck!

[1] Solve the system

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 9 \\ 14 \end{bmatrix}.$$

[2]

Consider the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & k \end{bmatrix}$. For which values of k will the system $Ax = \begin{bmatrix} 2 \\ 3 \\ 7 \end{bmatrix}$ have:

(a) A unique solution?

(b) An infinite number of solutions?

(c) For the value of k you picked for item b), find a **particular** solution to the system above.

(d) Still for the value of k you picked for item b), find the **complete** solution to the system above.

[3] Find bases for the four fundamental subspaces of A:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \end{bmatrix}$$

[4]

(a) Find the orthogonal projection of the vector $b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$ on the subspace spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$.

(b) If P is the plane spanned by the vectors $v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ in $\mathbb{R}^3$, find a basis for the orthogonal complement $P^\perp$.

[5] Let A be the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 3 \\ 1 & 1 & 4 & 8 \\ 1 & 1 & 1 & 5 \end{bmatrix},$$

Compute the determinant of the matrix $B = A^4(A^T)^3A^{-5}$, justifying all of your steps.

[6]

(a) Find the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

(b) Find $\lim_{k\to\infty} A^k$, justifying your answer.

[7] Find the line that best fits the points $(x, y) = (0, 2), (1, 3), (2, 6)$.

[8] Solve the linear system of differential equations

$$\begin{cases} x' = 2x + y \\ y' = x + 2y \\ x(0) = 0 \\ y(0) = 1 \end{cases}$$

[9] Suppose a 4×4 matrix A has eigenvalues $0, 0, 1$ and 2. What is the determinant of the matrix $B = (A^2 + I)^{-1}$? Justify your answer.

[10] Find the Singular Value Decomposition of the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$$