MATH 375, SPRING 2015, FINAL EXAM

PLEASE WRITE YOUR NAME HERE:

STANDARD NORMAL TABLE IS AT THE BACK OF THE EXAM

1 (10 points). Calling a particular 800 number results in success with probability .8. What is the probability that it would take more than 4 tries to reach this number?

2 (10 points). You have a well shuffled standard deck of 52 cards. You open one card at a time. What is the probability that $3\clubsuit$ comes before $7\heartsuit$ and both are opened before $Q\spadesuit$?

3 (10 points). You flip a fair coin and if it lands on heads, you roll a die twice and let X denote the sum of the two outcomes. If the coin lands on tails, you only roll a die once and let X be the outcome of that roll. What is the probability that X is at least 5?

4 (10 points). On a particular day, according to the weather forecast, the chance of rain and the chance of snow are 30% each. Moreover, if it is going to rain or snow on that day, the chance that it would start between 12 am and 1 am is 40% for rain and 20% for snow. If there is no rain or snow between 12 am and 1 am on that day, what is the probability that it snows later in the day?

5 (10 points). Suppose that $X \sim \mathrm{Exp}(\alpha)$ and the distribution of Y given $X = x$ is also $\mathrm{Exp}(\alpha)$. Find the expected value of $Y - X$.

6 (10 points). In a forest, the heights of trees are independent identically distributed random variables with mean $\mu = 10$ m and standard deviation $\sigma = 2$ m. Use the Central Limit Theorem to estimate the 90th percentile of the sum of the heights of 100 trees.

7 (10 points). An auto insurance company classifies its customers into two categories–low risk and high risk. According to this company, 70% of the customers are low risk and 30% are high risk. The probability that a low risk customer has an accident in any given year is .1 and for a high risk customer it is .3. If a customer has an accident in 2015, what are the chances that she will have an accident in 2016?

8 (10 points). If X is a random variable with mean $\mu = 5$ and standard deviation $\sigma = 2$, prove that

$$\mathrm{P}(X \geq 8) \leq \frac{4}{9}.$$

9 (10 points). Let $f_{X,Y}(x,y) = Cye^{-xy},\ x,y > 1$, and let $Z = X - Y$.

(a) (2 pts) Find the constant C.

(b) (2 pts) Find the joint pdf $f_{X,Z}(x,z)$.

(c) (2 pts) Write down the integral for the marginal pdf $f_Z(z)$, but do not evaluate it.

(d) (2 pts) Write down the formula for the conditional pdf $f_{X|Z}(x|z)$, but do not evaluate it.

(e) (2 pts) Write down the formula for the conditional expectation $E(X|Z = 1)$, but do not evaluate it.

10 (10 points). Let $X, Y \sim \text{Bernoulli}(p)$. Is $X + Y$ always $\text{Binomial}(2, p)$? Prove or give a counterexample. **Hint:** Flip a coin.