Name ______________________________

Do all problems.

1. (24 points) Evaluate the following integrals
   (a) $\int_1^e x^4 \ln x \mathrm{d}x$.
   (b) $\int \cos^3(2t)\sin^5(2t)\mathrm{d}t$.
   (c) $\int_0^{\frac{\sqrt{2}}{4}} \frac{2\mathrm{d}t}{\sqrt{1-4t^2}}$.
   (d) $\int \frac{e^t}{e^{2t}+3e^t+2}\mathrm{d}t$.

2. (15 points) State for each series whether it converges absolutely, converges conditionally, or diverges. Name a test which supports your conclusion and justify why it applies, by showing a calculation or giving an explanation.
   (a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n+1}$.
   (b) $\sum_{n=1}^{\infty} \frac{\ln(n^2)}{n}$.
   (c) $\sum_{n=1}^{\infty} \frac{n!}{n^2 10^n}$

3. (8 points) Let a curve C be given parametrically by $x = t^3,\ y = \frac{3t^2}{2},\ 0 \le t \le \sqrt{3}$.
   (a) Find the length of C.
   (b) Find an equation for the line tangent to C at $t = \frac{1}{2}$.

4. (a) (3 points) Using a known series, find the Taylor series centered at 0 of $f(x) = \frac{1}{1+x^2}$. Express your answer using summation notation.
   (b) (3 points) Use your answer to part (a) to obtain the Maclaurin series for $g(x) = \tan^{-1}(x)$.
   (c) (2 points) Use part (b) to estimate $\tan^{-1}(0.1)$ to the nearest thousandth. Describe your reasoning to guarantee your answer has the required accuracy.

5. (a) (4 points) Find the interval of convergence (including possible endpoints) for the power series $f(x) = \sum_{n=1}^{\infty} \frac{(2x+1)^n}{n+1}$.
   (b) (3 points) Using $f(x)$ from part (a) find $f'(-\frac{1}{2})$.

6. (8 points) Let $f(x) = \frac{1}{(x-1)^{\frac{2}{3}}}$.
   (a) Make a rough sketch of the graph of $f(x)$ between $x = 1$ and $x = 3$.
   (b) Is the area under the graph of $f(x)$ and above the x-axis between $x = 1$ and $x = 3$ finite? Justify your answer. If so, evaluate $\int_1^3 \frac{\mathrm{d}x}{(x-1)^{\frac{2}{3}}}$.

7. (6 points) Find the points in which the line $x = 1 + 2t,\ y = -1 - t,\ z = 3t$ intersects the coordinate planes.

8. (8 points) (a) Is the line $x = 1-2t,\ y = 2+5t,\ z = -3t$ parallel to the plane $2x+y-z = 8$? Give reasons for your answer.
   (b) Find a vector parallel to the line of intersection of the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

9. (7 points) Find the point on the plane $x + 2y + 2z = 13$ closest to the point $(2, -3, 4)$.

10. For the given functions $w = x^2 - y^2 + z, x = \sinh t, y = \cosh t, z = e^{2t}$
    (a) (3 points) express $\frac{\mathrm{d}w}{\mathrm{d}t}$ as a function of t using the chain rule.
    (b) (3 points) express $\frac{\mathrm{d}w}{\mathrm{d}t}$ as a function of t by expressing w in terms of t and differentiating directly with respect to t.
    (c) (2 points) Use either part (a) or part (b) to evaluate $\frac{\mathrm{d}w}{\mathrm{d}t}$ at $t = 1$.