Math 19500 PreCalculus   Final. NO ELECTRONICS

1. (5 points) Solve $\frac{2x}{x+1} = \frac{2x-1}{x}$ for x.

2. (5 points) Find the center and the radius of the circle with equation $x^2 + y^2 + 6x - 2y + 6 = 0$. Sketch its graph.

3. (5 points) Simplify $\frac{\left(8s^3t^3\right)^{\frac{2}{3}}}{\left(s^4t^{-8}\right)^{\frac{1}{4}}}$ completely, writing your answer with only positive exponents.

4. (5 points) Find an equation of the line with x-intercept 6 and y-intercept 4.

5. (5 points) Solve the inequality $x^3 + x^2 > 2x$. Write your answer in interval notation.

6. (5 points) Given the function $r(t) = 6 - \frac{24}{t}$ and the values $t = 6, t = 12$.
   (a) Determine the net change between the given values.
   (b) Determine the average rate of change between the values.

7. (5 points) (a) Evaluate $\log_3\left(\frac{1}{27}\right)$.
   (b) Solve $\log x + \log(x - 3) = 1$ for x.

8. (5 points) If $f(x) = \frac{1-x}{x+2}$. Find a formula for the inverse function $f^{-1}(x)$.

9. (5 points) Given $f(x) = 1 - (x - 2)^2$. Sketch the graph of f. Label the intercepts on your graph. Find the maximum value of f.

10. (5 points) Sketch the graph of the piecewise defined function

$$f(x) = \begin{cases} 1 - x & \text{if } x \le 1, \\ 1 & \text{if } x > 1 \end{cases}$$

   and evaluate $f(f(-3))$.

11. (5 points) Let $f(x) = 1 - x^2$. Find and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$.

12. (5 points) A bacteria culture starts with 600 bacteria. After 1 hour there are 1000 bacteria. Assuming the size of the culture grows exponentially, find the time required for the population size to double. (You may leave ln, log, and (or) e in your answer).

13. (4 points) Sketch the graph of $y = \log_3(x-1) - 2$ not by plotting points, but by starting from the graph of $y = \log_3(x)$ and applying transformations. State the domain and range. Find all the intercepts and asymptotes and label them clearly on your graph.

14. (4 points) The sophomore class at Southland High School raised \$860 from the sale of tickets to a concert. Tickets sold for \$2.50 if purchased in advance and \$4.00 if purchased at the door. If a total of 275 tickets were sold, how many tickets were sold at the door?

15. (4 points) Find the quotient and remainder $\frac{x^2-3x+6}{x-1}$.

16. (4 points) Sketch the graph of one complete period of the function $y = 2\sin\left(3x + \frac{\pi}{2}\right)$. Label all intercepts, maximums, and minimums.

17. (4 points) Find the exact value of
   (a) $\tan\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$.
   (b) $\cos\frac{-7\pi}{6}$.

18. (4 points) Verify the identity $\cos\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{3}\right) = 0$.

19. (4 points) Perform the division $\frac{x^2-x-42}{x^2+6x} \div \frac{x^2-6x-7}{x^3+x^2}$ and simplify completely.

20. (4 points) Use an appropriate Half-Angle formula to evaluate $\sin\left(67.5°\right).$

21. (4 points) Find $\tan t$ if $\sin t = -\frac{4}{5}$ and $\cos t > 0.$

22. (4 points) Solve $6\sin t + 3 = 0$ for t when $-\pi \le t \le \pi.$