Math 19500 Final Exam No electronics. Do all problems. Show your work.

1. (5 points) Evaluate $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(-\frac{1}{2}\right)$.
2. (5 points) Solve $\frac{x+5}{x-2} = \frac{5}{x+2} + \frac{28}{x^2-4}$ for x.
3. (5 points) Simplify $\log_2\left(\frac{8}{\sqrt{32}}\right)$.
4. (5 points) A 20 gram sample of the radioactive element mathtestium takes 10 years to decrease to 15 grams. Find the half-life of mathtestium. (You may use $e, \ln$, or $\log$ to express your answer.)
5. (5 points) Verify the identity $\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 = 1 - \sin x$.
6. (5 points) Given $f(x) = -3 - x^2$. Find and simplify the quotient $\frac{f(a+h)-f(a-h)}{h}$ when $h \neq 0$.
7. (5 points) Solve the inequality $\frac{2x-3}{x+1} \leq 1$. Express the solution using interval notation.
8. (5 points) (a) Find the quotient and remainder of the rational expression $\frac{3x-3}{x+2}$.
   (b) Use transformations and part (a) to graph of $y = \frac{3x-3}{x+2}$. Label intercepts and asymptotes on your graph.
9. (5 points) Sketch the graph of $f = 8x - x^2$. Label all local maximums and minimums and intercepts on your graph. State the intervals on which f is increasing and on which f is decreasing.
10. (5 points) Find the center and radius of the circle with equation $x^2 + y^2 - \frac{1}{2}x + y = \frac{1}{2}$.
11. (5 points) (a) Sketch the graph
$$f(x) = \begin{cases} 1 - x & \text{if, } x \leq 1 \\ 2x + 1 & \text{if, } x > 1. \end{cases}$$
   (b) Evaluate $f(-2) - 2f(3)$.
12. (5 points) Sketch the graph of the polynomial $y = -x^4 - 10x^2$. Make sure that your graph shows all intercepts and exhibits the proper end behavior.
13. (5 points) Use transformations to graph the function $f(x) = -\log_2(x - 4)$. State the domain, range, and asymptote(s).
14. (5 points) Solve $2 - \log_2(x + 1) = \log_2(x + 4)$ for x.
15. (5 points) Sketch the graph of one complete period of the function $y = -\sin\left(\pi x + \frac{\pi}{2}\right)$.
16. (5 points) Given $f(x) = \frac{x^2-1}{1-x}$ and $g(x) = x + 2$. Find and simplify $f(g(2)) - g(f(2))$.
17. (4 points) Find
   (a) $\cos\left(-\frac{\pi}{3}\right)$
   (b) $\cot\frac{25\pi}{2}$.
18. (4 points) Solve $4\cos^2(\theta) - 3 = 0$ for all θ when $0 \leq \theta \leq 2\pi$.
19. (4 points) If $\sin\theta = \frac{5}{13}$ and $\cos\theta < 0$. Find $\tan\theta$.
20. (4 points) Evaluate $\cos^2 112.5°$.
21. (4 points) Let $f(x) = \sqrt{1 - x}$.
   (a) Find the inverse function f^{-1}.
   (b) Sketch the graph of f and f^{-1} on the same coordinate axes.