Answer ALL questions (10 points each). Show all work.

1. Let l_1 and l_2 be the lines

$$l_1: \quad \begin{aligned} x &= 2+t \\ y &= -3t \\ z &= -1+4t \end{aligned} \qquad\qquad l_2: \quad \begin{aligned} x &= 5-t \\ y &= 1+3t \\ z &= 1-4t \end{aligned}$$

(a) Are l_1 and l_2 parallel, perpendicular or neither?

(b) Find an equation for the plane through l_1 and l_2.

2. The air pressure at all points (x, y, z) in some region is $P(x, y, z) = 10 + \dfrac{25}{z^2+1} + \sin(2x^2 + y^3 + z)$.

(a) Find the rate at which the pressure is changing per unit distance at the point $(1, 0, -2)$ in the direction $\mathbf{v} = \langle 1, 2, 5 \rangle$.

(b) The position at time t of a fly in the region is $(2t-1, e^{2t-2}-1, t^3-t-2)$. Find the rate at which the pressure the fly experiences is changing per unit time when it is at position $(1, 0, -2)$.

3. Find all local maxima, local minima and saddle points of the graph of $f(x, y) = -3y^3 - 4x^2 + 8x + 9y$.

4. (a) Find the mass of a lamina that occupies the region bounded by $y = \sqrt{x}$, $x + y = 2$ and $y = 0$ and has density $\delta(x, y) = y$ at each point (x, y).

(b) Use differentials (linear approximation) to approximate $\dfrac{25}{3.1^2 + 3.8^2}$.

5. (a) Find the volume of the region bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$.

(b) Find an equation of the tangent plane to the graph of $xy^2 = 2 + \ln(2z - 1)$ at the point $(2, -1, 1)$.

6. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=1}^{\infty} \dfrac{(-1)^n 2^n}{2^n + 1}$ (b) $\sum_{n=1}^{\infty} \dfrac{(-1)^n 2^n (2n+1)}{n!}$ (c) $\sum_{n=1}^{\infty} \dfrac{(-1)^n \ln n}{n}$

7. (a) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \dfrac{(n+1)(x+2)^n}{(n+3)^2}$.

Remember to check the endpoints, if applicable.

(b) Find the limit or show it does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{x^2}{x^2 + y^2}$

8. (a) Find a unit vector in the direction of the tangent vector at the point $(1, 5, 2)$ to the curve with vector representation $\mathbf{r}(t) = \langle \sqrt{t + e^t}, 2t + 5, t^3 + 2) \rangle$.

(b) Graph the equation $x^2 - 4y^2 - 4z^2 - 6x + 5 = 0$, labelling the coordinates of the vertices.

9. Do part (a) **OR** (b). If you do both parts, only part (a) will be graded. Mark clearly, by crossing out any work you do want graded.

(a) Find the mass of the region in space bounded by $z = x^2 + y^2$, $x^2 + y^2 = 1$ and $z = 0$ and having density $\delta(x, y, z) = 2z(x^2 + y^2)$.

(b) Find the mass of the region above the xy-plane and inside both the cone $\phi = \pi/4$ and the sphere $\rho = 2$ which has density $\delta(x, y, z) = z$.

10. (a) Let $f(x) = e^{-x^4}$.

(i) Find the first four nonzero terms of the Maclaurin series (i.e., the series centered at 0) representation of $f(x)$.

(ii) Use the result in (i) to find $\int_0^{1/2} f(x)\, dx$ with an error less than or equal .001. Justify that your answer has the required accuracy.

(b) Find the surface area of the portion of the surface $z = y^2$ which is above the triangle in the xy-plane with vertices $(0,0,0)$, $(0,1,0)$ and $(1,1,0)$.