**1.** (a) For points $A = (1,0,-1)$, $B = (2,-1,0)$ and $C = (1,2,3)$, $\mathbf{AB} = \langle 1,-1,1\rangle$ and $\mathbf{AC} = \langle 0,2,4\rangle$. A normal to the plane through $A$, $B$ and $C$ is $\mathbf{AB}\times\mathbf{AC} = \langle -6,-4,2\rangle$, and an equation for this plane is $-6(x-1) - 4y + 2(z+1) = 0$. Any point on the plane and any normal to the plane could have been used to find an equation for the plane.

(b) The direction of the line through $D = (4,1,-3)$ and $E = (2,0,2)$ is $\mathbf{DE} = \langle -2,-1,5\rangle$. A set of parametric equations for the line through $(5,8,0)$ and parallel to the line through $D$ and $E$ is $x = 5-2t$, $y = 8-t$, $z = 5t$.

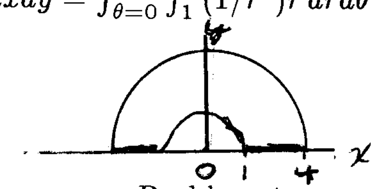
(c) $\mathbf{n} = \langle 1,2,-1\rangle$ is a normal to the plane $z = x+2y$. For $\mathbf{v} = \langle 2,0,2\rangle$, $\mathbf{n}\cdot\mathbf{v} = 0$, so $\mathbf{v}$ is perpendicular to $\mathbf{n}$, and thus $\mathbf{v}$ is parallel to the plane.

**2.** (a) For $h(x,y) = 4 + x^2 - \ln(y^2+1)$, $\nabla h = \langle 2x, \dfrac{-2y}{y^2+1}\rangle$, and $\nabla h(3,1) = \langle 6,-1\rangle$. A unit vector in the direction from $(3,1)$ to $(4,0)$ is $\mathbf{u} = \langle 1,-1\rangle/\sqrt{2}$. The directional derivative of $h$ at $(3,1)$ in the direction $\mathbf{u}$ is $\nabla h(3,1)\cdot\mathbf{u} = 7/\sqrt{2}$.

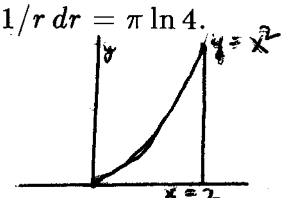
(b) The path $\mathbf{r}(t) = \langle 2t, \sin t\rangle$ has derivative $\dfrac{d\mathbf{r}}{dt} = \langle 2, \cos t\rangle$, so $\mathbf{r}(\pi/2) = \langle \pi, 1\rangle$ and $\dfrac{d\mathbf{r}}{dt}(\pi/2) = \langle 2,0\rangle$. Now $\nabla h(\pi,1) = \langle 2\pi,-1\rangle$ and $\dfrac{dh}{dt} = \dfrac{dh}{dx}\dfrac{dx}{dt} + \dfrac{dh}{dy}\dfrac{dy}{dt} = \nabla h\cdot\dfrac{d\mathbf{r}}{dt}$. At $t = \pi/2$ and $(x,y) = (\pi,1)$, $\dfrac{dh}{dt} = \langle 2\pi,-1\rangle\cdot\langle 2,0\rangle = 4\pi$.

**3.** For $f(x,y) = 2x^4 - x^2 + 3y^2$, the stationary points are the points satisfying these equations: $f_x = 8x^3 - 2x = 2x(4x^2-1) = 0$ and $f_y = 6y = 0$. Thus the critical points are $(0,0)$, $(1/2,0)$ and $(-1/2,0)$. The discriminant is $D = f_{xx}f_{yy} - f_{xy}^2 = (24x^2-2)(6) - 0 = 12(12x^2-1)$. Thus $D(0,0) = -12 < 0$, so $(0,0)$ is a saddle point; $D(\pm 1/2,0) = 24$ and $f_{yy}(\pm 1/2,0) = 6 > 0$ (or $f_{xx} = 4 > 0$), so $(\pm 1/2,0)$ are local minima.

**4.** The mass of the lamina $R$ between $x^2+y^2 = 1$ and $x^2+y^2 = 16$ and above the $x$-axis which has density $\delta(x,y) = 1/(x^2+y^2)$ is, using polar coordinates,
$\iint_R \delta(x,y)\,dxdy = \int_{\theta=0}^{\pi}\int_1^4 (1/r^2)r\,drd\theta = \int_0^{\pi} 1\,d\theta \int_1^4 1/r\,dr = \pi\ln 4.$



Problem 4



Problem 5(a)

**5.** (a) The region bounded by $x = 2$, $y = 0$, $y = x^2$, $z = 1$ and $z = x+2$ is the region between heights $z = 1$ and $z = x+2$ which is above the region in the $xy$-plane shown for Problem 5(a). Its volume is $\int_{x=0}^{2}\int_{y=0}^{x^2}(x+2) - 1\,dydx = \int_0^2 x^3 + x^2\,dx = 20/3$.

(b) Write the equation $x^3 + y^2 - z = 2$ as $z = f(x,y) = x^3 + y^2 - 2$; now $f_x(1,2) = (3x^2)_{(1,2)} = 3$ and $f_y(1,2) = (2y)|_{(1,2)} = 4$, so an equation of the tangent plane is $z - 3 = 3(x-1) + 4(y-2)$. OR: Let $F(x,y,z) = x^3 + y^2 - z$; $\nabla F(1,2,3) = \langle 3,4,-1\rangle$, and an equation of the plane is $3(x-1) + 4(y-2) - (z-3) = 0$.

**6.** (a) Let $a_n = \dfrac{(-1)^n n}{3n+1}$. Then $\lim |a_n| = 1/3 \neq 0$. Therefore $\lim a_n \neq 0$ and $\sum a_n$ is divergent by the test for divergence.

(b) Let $a_n = \dfrac{(-1)^n 5^n}{3^{2n}}$. Then $\dfrac{a_{n+1}}{a_n} = \dfrac{(-1)^{n+1}5^{n+1}}{3^{2n+2}} \dfrac{3^{2n}}{(-1)^n 5^{2n}} = -5/9$ for all $n$, so $\sum a_n$ is a geometric series with common ratio $-5/9$. A convergent geometric series is absolutely convergent by the ratio test.

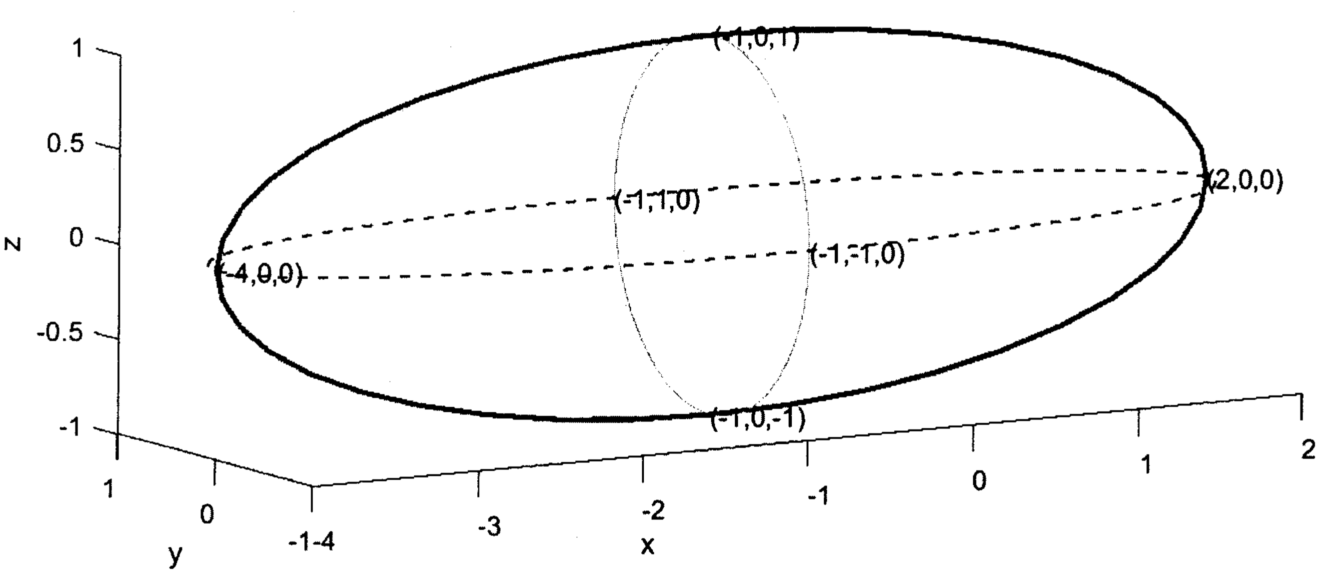
(c) Let $a_n = \dfrac{(-1)^n}{3n+1}$. Since $\lim |a_n| = 0$, $\lim a_n = 0$. Since, also, $|a_n|$ is steadily decreasing $\sum a_n$ is convergent by the alternating series test. By comparing $\sum |a_n|$ with $\sum 1/n$, the limit comparison test shows $\sum |a_n|$ is divergent. Hence, $\sum a_n$ is conditionally convergent.

**7.** (a) The power series $\sum_{n=0}^{\infty} \dfrac{(x-2)^n}{(n+2)3^n}$ has coefficients $c_n = \dfrac{1}{(n+2)3^n}$ and center 2. Since $\lim |c_{n+1}/c_n| = \lim \dfrac{(n+2)3^n}{(n+3)3^{n+1}} = 1/3$, the radius of convergence of the series is $1/(1/3) = 3$ and the series converges at least in the interval $(2-3, 273) = (-1, 5)$. At $x = -1$ the series is $\sum \dfrac{(-1)^n}{n+2}$, which is convergent. At $x = 5$ the series is $\sum \dfrac{1}{n+2}$, which is divergent. The interval of convergence is $[-1, 5)$.

(b) The limit $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^4}{x^4+y^2}$ does not exist since, along the paths $y = 0$, $\lim_{x\to 0} \dfrac{x^2}{x^4} = \infty$ and, along the paths $x = 0$, $\lim_{y\to 0} \dfrac{y^4}{y^2} = 0$.

**8.** (a) The curve $\mathbf{r}(t) = \langle 2t+7, e^{2t+2}, t^3+t^2 \rangle$ is at $(5,1,0)$ when $2t+7=5$, i.e., when $t=-1$. Now $\dfrac{d\mathbf{r}}{dt} = \langle 2, 2e^{2t+2}, 3t^2+2t \rangle$, so $\dfrac{d\mathbf{r}}{dt}(-1) = \langle 2,2,1 \rangle$ is a tangent vector and $\langle 2,2,1\rangle/3$ is a unit tangent vector.

(b) The second degree equation $x^2+2x+9y^2+9z^2 = 8$ can be written in standard form by completing the square $x^2+2x = (x+1)^2-1$, transposing the constants to the right and dividing both resulting sides by the constant: $\dfrac{(x+1)^2}{9} + \dfrac{y^2}{1} + \dfrac{z^2}{1} = 1$, which is the equation of an ellipse centered at $(-1,0,0)$ with semiaxes 3, 1 and 1 in the $x$, $y$ and $z$ directions, respectively. Taking any two of the six vertices, say $(-3-1,0,0)$ and $(3-1,0,0)$, the graph is as below.



**9.** (a) An iterated integral for $\iiint_E f\,dV$, where $E$ is the hemisphere $\{(x,y,z) : x^2+y^2+z^2 \leq 4; z \geq 0\}$, in spherical coordinates is $\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/2}\int_{\rho=0}^{2} f\,\rho^2 \sin\phi d\rho d\phi d\theta$ and cylindrical coordinates $\int_{\theta=0}^{2\pi}\int_{r=0}^{2}\int_{z=0}^{\sqrt{4-r^2}} fr\,dzdrd\theta$.

(b) Let $f(x,y) = x/\sqrt{y}$. For $(x,y) = (10.1, 3.8)$ and $(x_0, y_0) = (10,4)$, $x - x_0 = 1/10$ and $y - y_0 = -1/5$; $f_x = 1/\sqrt{y}$ and $f_x(10,4) = 1/2$; $f_y = \dfrac{-x}{2y^{3/2}}$ and $f_y(10,4) = -5/8$. The linear approximation is $f(10.1, 3.8) =$

$$f(10,4) + f_x(10,4)(x-x_0) + f_y(10,4)(y-y_0) = 5 + (\frac{1}{2})(\frac{1}{10}) - (\frac{5}{8})(-\frac{1}{5}) = 207/40.$$

**10.** (a) (i) The Maclaurin series for $f(x) = 1/(1+2x) = 1/(1-(-2x))$ is

$$1 + (-2x) + (-2x)^2 + (-2x)^3 \pm ... = 1 - 2x + 4x^2 - 8x^3 \pm ....$$

(ii) $f(1/100) = 1 - 2^1/10^2 + 2^2/10^4 - 2^3/10^6 \pm ....$ Since the term $2^2/10^4 = 1/2500 < 1/1000$, we use the error estimate for alternating series to obtain $f(1/100) \approx 1 - 2/100 = .98$

(b) The surface area $S$ of the portion of the surface $z = x^2 + y^2$ inside the cylinder $x^2 + y^2 = 1$ is $\iint_R \sqrt{1 + z_x^2 + z_y^2}\,dA$, where $R$ is the unit circle in the $xy$-plane. Using polar coordinates and the substitution theorem with $u = 1 + 4r^2$, we have

$$S = \int_{\theta=0}^{2\pi}\int_{r=0}^{1}\sqrt{1+4r^2}\,r\,drd\theta = \int_0^{2\pi} 1\,d\theta \int_1^5 u^{1/2}\frac{1}{8}\,du = (2\pi)\frac{1}{8}\frac{2}{3}u^{3/2}|_1^5 = \frac{\pi}{6}(5^{3/2} - 1).$$