

Answer ALL questions (10 points each). Show all work.

1. (a) Find an equation of the plane containing the points $(1, 0, -1)$, $(2, -1, 0)$ and $(1, 2, 3)$.

(b) Find parametric equations for the line through $(5, 8, 0)$ and parallel to the line through $(4, 1, -3)$ and $(2, 0, 2)$.

(c) Is the vector $\mathbf{v} = \langle 2, 0, 2 \rangle$ parallel, perpendicular or neither to the plane $z = x + 2y$?

2. After drifting, the height h in inches of the snow at point (x, y) in a parking lot is $h(x, y) = 4 + x^2 - \ln(y^2 + 1)$.

(a) Find the rate the height of the snow at $(3, 1)$ changes per unit distance traveled in the direction towards $(4, 0)$.

(b) A person walking in the lot is at position $(x(t), y(t)) = (2t, \sin t)$ at time t . Find the rate at which the height of snow the person is walking in changes per unit time at $t = \pi/2$.

3. Find all local maxima, local minima and saddle points of the graph of $f(x, y) = 2x^4 - x^2 + 3y^2$.

4. Find the mass of a lamina that occupies the region above the x -axis and between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 16$ and has density $\delta(x, y) = \frac{1}{x^2 + y^2}$ at each point (x, y) .

5. (a) Find the volume bounded by $x = 2$, $y = 0$, $y = x^2$, $z = 1$ and $z = x + 2$.

(b) Find an equation of the tangent plane to the graph of $x^3 + y^2 - z = 2$ at the point $(1, 2, 3)$.

6. State, for each series, whether it converges absolutely, converges conditionally or diverges. Name a test which supports each conclusion and show the work to apply the test.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n n}{3n + 1}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n 5^n}{3^{2n}}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{3n + 1}$

7. (a) Find the interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(x - 2)^n}{(n + 2)3^n}$.

Remember to check the endpoints, if applicable.

(b) Find the limit or show it does not exist: $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^4}{x^4 + y^2}$

8. (a) Find a unit vector in the direction of the tangent vector at the point $(5, 1, 0)$ to the curve with vector representation $\mathbf{r}(t) = \langle 2t + 7, e^{2t+2}, t^3 + t^2 \rangle$.

(b) Graph the equation $x^2 + 2x + 9y^2 + 9z^2 = 8$, labelling the coordinates of the center and any one of the vertices.

9. (a) Write an iterated integral, using either spherical or cylindrical coordinates, to evaluate $\iiint_E f \, dV$, where E is the hemisphere $\{(x, y, z) : x^2 + y^2 + z^2 \leq 4, z \geq 0\}$.

Note: The function is not specified, so calculation of the iterated integral is not possible.

(b) Use differentials (linear approximation) to approximate $\frac{10.1}{\sqrt{3.8}}$.

10. (a) Let $f(x) = \frac{1}{1 + 2x}$.

(i) Find the first four terms of the Maclaurin series (i.e., the series centered at 0) representation of $f(x)$.

(ii) Use the result in (i) to find $f(.01)$ with an error less than or equal .001. Justify that your answer has the required accuracy.

(b) Find the surface area of the portion of the surface $z = x^2 + y^2$ which is inside the cylinder $x^2 + y^2 = 1$.