1) Turn-off cell phones and put them and all notes out of sight.
2) CALCULATORS are allowed, NO scrap paper (use sheets provided)
3) Points will be deducted if a solution is given without written proof of your work
4) If you need additional space to answer a question, please use the facing side of each sheet.
5) Note: See page 2 for Normal Distribution tables.

SHOW ALL WORK

PART 1: [pages 3 to 8] Answer ALL questions in this part. (70 points) If you need additional space to answer a question, please use the reverse side of each sheet.

1) [10 points] Consider the differential equation $\frac{dy}{dx} = y(3x+1)$ subject to the initial condition $y(0) = 3$. Solve the equation explicitly and compute the exact value of $y(1)$.

2) [10 points] Consider the initial value problem $\frac{dy}{dx} = y(3x+1)$, where $y(0) = 3$. Use the Euler method with 4 steps to estimate the value of $y(1)$. (Compute your answer to 3 decimal places.)

3) [10 points] For the differential equation $\frac{dy}{dt} = y^2(7-y)$, find the steady state solutions and the inflection points. Sketch the solution curves $y(t)$ for the initial values $y(0) = -1$, $y(0) = 0$, $y(0) = 1$, $y(0) = 5$, and $y(0) = 8$.

4) [15 points] You invest \$20,000 in the bank with a 5% annual interest rate, compounded continuously and you withdraw \$1100 per year, spread uniformly throughout the year.

   a) Find the initial value problem which describes the amount of money, $P(t)$, that you have in the bank at time *t*.

   b) How long it would take to run out of money? (That is for what *t* is $P(t) = 0$)

   c) Repeat part a and b if, instead of \$1100 per year, you withdrew \$1200 per year instead.

5) [15 points] Given the bivariate data (Compute your answer to 3 decimal places.):

X	4	10	6	14	10	16	24
Y	3	6	15	9	3	0	6

a) Compute the mean and standard deviation for each of X and Y.

b) Compute the correlation coefficient and write down the equation of the regression line.

c) Use the regression line to estimate the value of Y when $X = 8$.

6) [10 points] An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\frac{dN_1}{dt} = 0.02N_1(1 - \frac{N_1}{60} - \frac{N_2}{90})$$
$$\frac{dN_2}{dt} = 0.04N_2(1 - \frac{N_2}{80} - \frac{N_1}{40})$$

a) Find all steady-state solutions of this system and determine which are stable (Hint: it is helpful to sketch the null-clines where $\frac{dN_1}{dt} = 0$ and where $\frac{dN_2}{dt} = 0$).

b) When the populations are $N_1 = 30$ and $N_2 = 40$, determine whether each population is increasing or decreasing, and at what rate. Explain

PART 2: [pages 9 to 12] Completely answer any 3 questions. Each question is worth 10 points.
If you answer more than 3 questions, cross out work you do not want graded or only the first 3 questions will be graded.

7) [10 points] Check to see whether the expression $y = \frac{5e^x + 1}{1 + x}$ is a solution of the differential equation $\frac{dy}{dx} = \frac{1 + xy}{1 + x}$ with initial condition $y(0) = 6$.

8) A bowl contains 6 red balls, 4 blue balls, 3 white balls and 1 green ball. You pick two balls without replacement.
[3 points] a) What is the probability that both balls are white? (Answer may be left as a fraction or as a three-place decimal.)

[4 points] b) What is the probability that both balls are the same color? (Answer may be left as a fraction or as a three-place decimal.)

[3 points] c) What is the probability that both balls have different colors? (Answer may be left as a fraction or as a three-place decimal.)

9) [10 points] 140 light bulbs were randomly selected from a large batch and placed through a simulation of everyday use until they burned out. The table below summarizes the distribution of their lifetime (=number of hours until burn-out). For example, 20 bulbs lasted between a little more than 900 hours and up to 1000 hours.

Hours until burnout	(800, 900]	(900, 1000]	(1000, 1100]	(1100, 1200]	(1200, 1300]
# of bulbs	15	20	45	35	25

a) Based on the data, prepare a <u>relative</u> frequency histogram for the time until burnout for bulbs from the sample.

b) Based on data, estimate as accurately as you can the <u>median</u> number of hours that a sample bulb lasted until burnout. (Compute your answer to 3 decimal places, if necessary.)

c) Based on the data estimate as accurately as you can the <u>average</u> number of hours that a sample bulb lasted until burnout. (Compute your answer to 3 decimal places, if necessary.)

10) [10 points] Assume that 7% of cars pulled over at a police check point are found to have a vehicular infraction.

a) If 20 cars are stopped at a check point what is the probability that 3 or more cars will be found to have some infraction? (Compute your answer to 3 decimal places.)

b) The local precinct is aiming to give out at least 16 tickets for vehicular infractions to cars stopped at the check point. If 200 cars pass through the check point, use a normal distribution to estimate the probability that the police will reach their target. (Compute your answer to 3 decimal places.)