Math 185 PR — **FINAL** — December 17, 2009

Answer ANY 10 COMPLETE questions OUT OF 12. Show your work/computations. Only non-computer/non-cell phone calculators allowed. Good luck! Total: 100 pts.

1. (a) If $Q = \{x \mid x$ is a rational number$\}$, $I = \{x \mid x$ is an irrational number$\}$, and $R = \{x \mid x$ is a real number$\}$, find each of the following sets:
   (i) $Q \cup I$, (ii) $Q \cap I$, (iii) $Q \cap R$.

   (b) List the first ten counting numbers in **BASE THREE**.

2. (a) If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 2, 4, 5\}$ and $B = \{2, 3, 4\}$, find:
   (i) $A \cap B$, (ii) $A \cup B$, (iii) $\overline{A}$, (iv) $A - B$, (v) $n(A \times B)$
   (where $\overline{A}$ denotes the complement of A, and n denotes cardinal number).

   (b) Convert the repeating decimal $0.2\overline{7}$ to a ratio of integers, *reduced to lowest terms*.

3. (a) Suppose the set C is a proper subset of the set D, and that $n(C) = 3$ and $n(D) = 5$. What is $n(\overline{C} \cap D)$?

   (b) Do the following subtraction directly **IN BASE FIVE**, *showing any borrowing or carrying*:
   $$\begin{array}{r} 1222_{\text{five}} \\ -333_{\text{five}} \\ \hline \end{array}$$

4. (a) Assume that giving birth to a boy or a girl are equally likely events. Find the probability that a couple having three children will have all boys.

   (b) Find the prime factorization of 160. **Show your work!**

5. (a) **True or false:** The greatest common factor of two different prime numbers can be equal to one of the two prime numbers.? **Explain your answer.**

   (b) Convert 26_{seven} to base ten.

6. (a) If there are 3 red marbles and 5 blue marbles in a bag and you draw them out at random, *without replacement*, find the probability that the second marble you take out is red, if the first marble you drew was blue.

   (b) Convert the base ten number 99 to base four.

Exam continues on reverse.

7. (a) Find a **rational** number that lies strictly between $\frac{1}{4}$ and $\frac{1}{3}$.

   (b) How many different ways can 5 people be stood in order in a line?

8. (a) Suppose you roll a single, fair, six-sided die. What is the probability of getting:

     i. an even number?
     ii. a number less than 5?
     iii. a 5?
     iv. a 7?
     v. a number less than 10?

   (b) Using the tests for divisibility (and **not** long division or your calculator), explain whether 5438021490 is divisible by (i) 9, or by (ii) 6.

9. (a) Find the greatest common factor (GCF) [also called greatest common divisor (GCD)] of 252 and 120.

   (b) (i) What is the nth term a_n in the arithmetic sequence $5, 8, 11, \ldots$?
   (ii) Use the method of Gauss to find the sum of the first 100 terms of this sequence. (You may check your answer by using your calculator to repeatedly do additions, but you must show the Gauss method to get credit.)

10. (a) Find the least common multiple (LCM) of 252 and 120.

    (b) (i) What is the nth term a_n in the geometic sequence $5, 15, 45, \ldots$?
    (ii) Use the formula for a_n to find exactly the 100th term of this sequence (approximations given by a calculator are not acceptable).

11. (a) Find a non-repeating pattern for which you can write a formula for the nth term a_n in the (neither arithmetic nor geometric) sequence $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \frac{1}{25}, \ldots$, and write down the formula for a_n in terms of n.

    (b) Give (i) a specific example of an **irrational** number that is NOT in decimal form, and (ii) a specific example of an **irrational** number that IS in decimal form.

12. (a) Prove that for ANY two-digit number, if the digits are reversed and the numbers subtracted, the difference is a multiple of 9. (Hint: write the subtraction in expanded form, applying the general test for divisibility by 9 will not be helpful.)

    (b) Draw a Venn diagram with three overlapping circles and label them to represent sets A, B, and C, then shade the region corresponding to $(A \cup B) \cap \overline{C}$.