Math 202 Final Exam Fall,2008

Part I Do all parts of the following six problems.

(1) Compute the derivative $\frac{dy}{dx}$ for each of the following (18 points) :

(a) $y = \ln(\dfrac{1}{x+1})$. (b) $y = \arcsin(\sqrt{x})$;

(c) $y = \sin(x)^x + \sin(x)$;

(2) Compute each of the following integrals(24 points):

(a) $\displaystyle\int \frac{x^3+2}{x^3-x}\, dx$; (b) $\displaystyle\int \tan(x)\sec^4(x)\, dx$;

(c) $\displaystyle\int \sqrt{4-x^2}\, dx$; (d) $\displaystyle\int_0^1 \arctan(x)\, dx$.

(3) Compute each of the following limits (10 points):

(a) $Lim_{x\to\infty} \sqrt{x}\, e^{-x}$; (b) $Lim_{x\to 1^+} x^{1/(x-1)}$.

(4) The region $R$ in first quadrant of the $xy$ plane is bounded by the curves $y = \sin(\pi x)$ and $y = 2x$. Set up two integrals (method of washers and method of shells) for the volume of the solid obtained by rotating $R$ around the line $y = 2$. Do not compute the value of the integrals(10 points)

(5) Sketch the curve given by the equation $r = 3+\sin(\theta)$ in polar coordinates, labeling the $x$ and $y$ intercepts, and compute the area it encloses. (8 points)

Part II Do all parts of three out of the following four problems (10 points each)

(7) (a) A 16 pound pull extends a spring 6 inches (= one half of a foot). Compute the work done stretching the spring an additional foot.

(b) Evaluate the integral or show it is divergent: $\int_0^1 \ln(t)\ dt$.

(8) (a) Calculate the arc-length of the section of the curve $y = \ln(\sec(x))$ between $x = 0$ and $x = \pi/4$. Leave your answer in terms of logs, but you should evaluate any trig functions which appear.

(b) Find all values of $t$ such that the tangent line to the curve given parametrically by $x = t^2 + t, y = 3t + 3t^2 - t^3$ is parallel to the line given by $y = 2x + 3$.

(9) (a) A radioactive substance has a half-life of 12 years. Derive a formula for the amount left after $t$ days if you begin with 400 pounds of the substance (Show your work). After how many years will you have 100 pounds left?

(b) Compute $\int \sec^3(x)\ dx$ (Hint: Use integration by parts with $u = \sec(x)$).

(10) (a) Draw a labeled sketch of the conic whose equation is $y^2+2y = 9x^2+35$. Identify which sort of conic it is.

(b) Use the definitions of the hyperbolic trig functions to derive the identity: $\cosh^2(t) - \sinh^2(t) = 1$.