Math 202 Final Exam Spring, 2009

Part I Do all parts of the following five problems.

(1) Compute the derivative $\frac{dy}{dx}$ for each of the following (15 points) :

$$\text{(a)} \quad y = x^x + x^2; \qquad \text{(b)} \quad y = 2^{\sin(x)} + e^2;$$

$$\text{(c)} \quad y = \arctan(e^{\sqrt{x}}).$$

(2) Compute each of the following integrals(30 points):

$$\text{(a)} \int \frac{2x+1}{x^2+5x+6}\, dx; \qquad \text{(b)} \int \frac{x^2}{\sqrt{9-x^2}}\, dx;$$

$$\text{(c)} \int e^x \sec^2(e^x)\, dx; \qquad \text{(d)} \int_1^2 \sqrt{x}\, \ln(x)\, dx;$$

$$\text{(e)} \int_0^{\pi/4} \sqrt{\sec(x)}\, \tan(x)\, dx.$$

(3) Compute each of the following limits (10 points):

$$\text{(a)} \quad Lim_{x\to 0} \frac{e^x - 1 - x}{1-\cos(x)}; \qquad \text{(b)} \quad Lim_{x\to 0} \frac{\sin(3x)}{1-e^{2x}}.$$

(4) Compute the area of the entire region between the two curves $y = x^3 - 3x^2 + 2x$ and $y = 3(x-1)$. While your answer should be numerical, you need not combine the numbers you get. (Hint: It is helpful to sketch the curves.) (8 points).

(5) Sketch the curve given by the equation $r = 1 - \sin(\theta)$ in polar coordinates, labeling the x and y intercepts, and compute the area it encloses. (7 points)

Part II Do all parts of three out of the following five problems (10 points each)

(6) The region R in first quadrant of the xy plane is bounded by the curves $y = \sqrt{x}$, $x = 1$ and $y = 0$. Set up two integrals (method of washers and method of shells) for the volume of the solid obtained by rotating R around y axis. Use one of the integrals to compute the area.

(7) (a) A population of bacteria begins with 10 bacteria and then grows at a rate proportional to its size. After 6 hours there are 100 bacteria. Find a mathematical formula for $P(t)$, the number of bacteria after t hours.

(b) A spring has natural length L feet. Find L given that the work done stretching the spring from length 2 feet to length 3 feet is twice the work done stretching it from 3 feet to 4 feet.

(8) (a) Compute the arclength of the portion of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 1$.

(b) A curve is given parametrically by $x(t) = t^3, y(t) = 1 - t$. Find an equation for the tangent line to the curve at the point $(8, -1)$.

(9) (a) Compute $Lim_{x \to 1^+} \ (t-1)^{\sqrt{t-1}}$.

(b) Evaluate the integral or show it is divergent:

$$\int_1^\infty \frac{2}{t^{6/5}} \, dt.$$

(10) (a) Identify the conic section whose equation is $(y+1)^2 + 9x^2 = 36$. Find the center and each focus and vertex.

(b) Compute the integral

$$\int \frac{1}{x^4 + 5x^2 + 4} \, dx.$$