

**The City College Department of Mathematics
Fall 2008**

MATH 20900 Final Exam

- 1) Turn-off cell phones and put them and all notes out of sight.
- 2) CALCULATORS are allowed, NO scrap paper (use sheets provided)
- 3) Points will be deducted if a solution is given without written proof of your work

SHOW ALL WORK

PART 1: Answer ALL questions in this part. (60 points) If you need additional space to answer a question, please use the facing side of each sheet.

- 1) [10 points] Solve the differential equation $\frac{dy}{dx} = 6\sqrt{x}e^{2y}$ subject to the initial condition $y(1) = 3$.
- 2) [10 points] Consider the initial value problem $\frac{dy}{dx} = \sqrt{2y+2} + \frac{3x}{y}$, where $y(2) = 1$. Use the modified Euler method of estimation technique to approximate $y(2.5)$. (Calculations rounded off to 3 decimal places)
- 3) [10 points] Use geometric analysis to analyze the differential equation $\frac{dy}{dt} = (5-y)(y+3)(y-1)$. Your answer should include:
 - a) a graph of $g(y)$ vs. y , where $g(y)$ is the derivative of y as a function of y .
 - b) sketches of the solution curves $y(t)$ for the initial value problem $y(0) = 3$ and all steady state solutions.
 - c) show concavity and show the y value of all inflection points for solution curves.
 - d) stability analysis for all steady state solutions.
- 4) [8 points] a) Consider the five points $(1,2)$, $(4,1)$, $(5,-4)$, $(2,2)$, and $(0,8)$. Find the quantities s_x , s_y and r and use these to find the regression line. (Calculations rounded off to 3 decimal places)

[2 points] b) What estimate would you give for the value of y when $x = 3$? (Calculations rounded off to 3 decimal places)

- 5) [10 points] A milk company unconditionally guarantees that an unopened, properly refrigerated container of its milk will be fresh for 5 days after the date stamped on the carton. The company's data shows that unopened containers remain fresh on average for 6 days, with a standard deviation of 0.65 days. Assume that the length of time milk stays fresh follows a normal distribution. What percent of the milk containers will spoil in greater than 7 days after the date stamped? (Calculations rounded off to 3 decimal places)
- 6) A bowl contains 6 red balls, 4 blue balls and 1 white balls. You pick two balls without replacement.
- [3 points] What is the probability that both balls are blue? (Answer may be left as a fraction or as a three-place decimal.)
 - [4 points] What is the probability that both balls are the same color? (Answer may be left as a fraction or as a three-place decimal.)
 - [3 points] What is the probability that both balls have different colors? (Answer may be left as a fraction or as a three-place decimal.)

PART 2: Answer any 4 complete questions. Each question is worth 10 points.

If you answer more than 4 questions, cross out work you do not want graded or only the first 4 questions will be graded. If you need additional space to answer a question, please use the facing side of each sheet.

- 7) [10 points] An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\frac{dN_1}{dt} = 0.15N_1 \left(1 - \frac{N_1}{75} - \frac{N_2}{75}\right)$$

$$\frac{dN_2}{dt} = 0.05N_2 \left(1 - \frac{N_2}{50} - \frac{N_1}{25}\right)$$

- Find all steady-state solutions of this system.
- Based on the above model, would you characterize the species as competitive? In the long-term to which of the possible steady state solutions will the populations tend? Explain

8) [10 points] Your daughter needs to be able to draw \$50,000 a year from her college savings fund (you started from birth) to pay for college expenses to obtain a medical degree (assume that she will take four years for Bachelor's degree, four more years at medical school, then additional two years as a resident and yearly spending will be consistent from year to year). At start of her college career, she intends to invest her savings in government securities that should return 5.5% a year compounded continuously.

- Obtain the equation for $\frac{dP}{dt}$ and then find the general solution of $P(t)$ with, as yet undetermined, initial value P_0 .
- How large must your daughter's initial college savings be so that she can continue drawing her \$50,000 income until she becomes a doctor?

9) [10 points] 150 light bulbs were randomly selected from a large batch and placed through a simulation of everyday use until they burned out. The table below summarizes the distribution of their lifetime (=number of hours until burn out). For example, 25 bulbs lasted between a little more than 850 hours and up to 950 hours.

Hours until burnout	(750, 850]	(850, 950]	(950, 1050]	(1050, 1150]	(1150, 1250]
# of bulbs	10	25	50	40	25

- Based on the data, prepare a relative frequency histogram for the time until burnout for bulbs from the sample.
- Based on data, estimate as accurately as you can the median number of hours that a sample bulb lasted until burnout.
- Based on the data, estimate as accurately as you can the average number of hours that a sample bulb lasted until burnout.

10) [10 points] On a certain chemistry class, a professor has observed that, on her past final examinations, 72% of the students pass the final examination. Assume that students in the current class perform similarly to students in the past.

- If the professor currently has a large lecture class of 125 students. What is the estimate probability that 75 to 100 students will pass the final examination? (Compute your answer to 4 decimal places.)
- In addition to the large lecture, if the professor has an evening class of 30 students, what is the actual probability that 18 to 22 students will pass the final examination? (Compute your answer to 4 decimal places.)

- 11) [10 points] Answer parts a) and b) below based on the following information: Blood types O, A, B and AB occur in the population with frequencies 0.43, 0.41, 0.11, and 0.05 respectively. A person may receive blood from another person with the same Rh factor and with the same type or type O. Within all the blood types, the Rh factors, (plus and minus), occur with frequencies 0.86 and 0.14 respectively.
- a) To two decimal places, what is the probability that a randomly selected individual can be a donor for a person of type O, Rh+?
- b) A hospital needs three units of blood to give to a type B, Rh- recipient. An individual can donate only one unit. If 10 people arrive at the blood donation center, what is the probability that the hospital will be able to obtain the blood that it needs? Be careful to consider all possibilities that will satisfy the hospital's needs. Show details of the calculation using appropriate probability formulas.