Department of Mathematics **The City College of New York**
Math 39200 **Final Examination** **Spring 2006**

SHOW ALL WORK. Answer 5 questions from Part I and 2 from Part II

PART I. Answer 5 complete questions from this part. (14 points each)

1. **a)** Let A be the matrix $\begin{pmatrix} 0 & 1 & -2 & -2 \\ 0 & 1 & -2 & 1 \\ 1 & 2 & -1 & 0 \\ 3 & 2 & 1 & 1 \end{pmatrix}$.

Find the determinants of the following matrices: **i)** A , **ii)** A^{-1} and **iii)** $2A^3$

b) Let $B = \begin{pmatrix} 1 & 2 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ Find **i)** BC and **ii)** $B^T C^T$.

2. Use Gaussian elimination to solve each of the following systems of equations. In part a), write the solution as a linear combination of vectors.

a) $\begin{cases} 3x - y + z = 0 \\ 2x + 4y - 2w = 2 \\ -7y + z + 3w = -3 \end{cases}$ **b)** $\begin{cases} x - y + 3z = 0 \\ 2x - y + 2z = 0 \\ x - z = 1 \end{cases}$

3. a) Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 8 \\ 1 & 2 & 4 \end{pmatrix}$.

b) Use the matrix A^{-1} that you found in (a) to solve the system $\begin{cases} x + 2y + 3z = 1 \\ 4y + 8z = -1 \\ x + 2y + 4z = 4 \end{cases}$

4. a) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

b) Let S be the surface parametrized by $x = u + 3;\ y = v + 2;\ z = uv^2$. Show that the point P with (*x,y,z*) co-ordinates (4,4,4) lies on S and find an equation of the tangent plane to S at point P.

5. a) Find the eigenvalues and eigenvectors of the matrix $A = \begin{pmatrix} 6 & 1 \\ -3 & 2 \end{pmatrix}$.

b) Use your answer to part a) to solve the following simultaneous differential equations for $y_1(t)$ and $y_2(t)$ subject to initial conditions $y_1(0) = 1$ and $y_2(0) = 2$:

$$\begin{cases} y_1' = 6y_1 + y_2 \\ y_2' = -3y_1 + 2y_2 \end{cases}$$

Please turn the page for the continuation of Part I and for Part II

Part I, continued

6. Let $\vec{F}(x,y,z) = < \dfrac{\ln y}{2\sqrt{x}} + yz, \dfrac{\sqrt{x}}{y} + xz, xy >$.

a) Find a potential function $f(x,y,z)$ for $\vec{F}$ so that $\nabla f = \vec{F}$ and

b) Evaluate $\displaystyle\int_C \vec{F}\cdot d\vec{r}$ where C is the straight line segment from $P(1,e,1)$ to $Q(4,e^2,2)$.

7. a) Let S be the surface described by $x^2 + y^2 = 1; 0 \le z \le 3$. Evaluate the integral $\displaystyle\iint_S z^2 dS$.

b) Find the length of the part of the parametrized curve $\vec{r}(t) = < \dfrac{t^2}{2}, \dfrac{2\sqrt{2}}{3}t^{3/2}, t >$ between the points P(0,0,0) and Q$(\dfrac{1}{2,}\dfrac{2\sqrt{2}}{3},1)$.

End of Part I. Make sure you answered five complete questions from this part.

PART II: Answer 2 complete questions from this part (15 points each).

8. Let R be the region in the *x,y*-plane bounded by the curves $x = y^2$ and $y = x - 2$.

Find $\displaystyle\int_C -ydx + xdy$ (where C is the boundary of R, oriented clockwise)

a) directly, as a line integral **AND**
b) as a double integral, by using Green's Theorem.

9. Let S be the surface described by $z = x^2 + y^2$; $z \le 4$; $y \ge 0$ and let C be the boundary curve of S with the orientation of your choice. Let $\vec{F}(x,y,z) = < y,z,x >$. Find $\displaystyle\int_C \vec{F}\cdot d\vec{r}$

a) directly as a line integral **AND** **b)** as a double integral, by using Stokes' Theorem.

10. Let T be the solid bounded below by $z = 0$, bounded above by $y + z = 1$, and bounded on the side by $x^2 + y^2 = 1$. Let S be the boundary surface of T. Let $\vec{F}(x,y,z) = < x,y,z >$. Use the outward pointing normal vector to evaluate $\displaystyle\iint_S \vec{F}\cdot d\vec{S}$

a) directly as a surface integral **AND**
b) as a triple integral, by using the Divergence Theorem.

END OF EXAM. Please check that you answered five complete questions from Part I and two complete questions from Part II.