PART I: Answer ALL questions in this part (10 points each)
Show all work and simplify all answers. **NO CALCULATORS!!!**

1. Find all solutions: $(x\cos y + 1)y' + \sin y - e^x$

2. (a) Solve explicitly: $xy^3y' = 2x - 3$, $x > 0$, and $y(1) = -1$
(b) Replace the partial differential equation, in the function $u(x,t)$,

$$xtu_{xx} + 2u_{tt} + x^2tu = 0$$

with two ordinary differential equations. Do **NOT** solve the equations.

3. (a) Find the general solution: $ty' + (t-1)y = t^3$, $t > 0$
(b) Find, to within a multiplicative constant, the Wronskian of two solutions to the equation $2ty'' + 4y' + (t\sin t)y = 0$

4. Find the general solution: $y'' + 6y' - 7y = 1 + 2e^t$.

5. (a) For the function

$$f(x) = \begin{cases} -1, & -1 < x < 0; \\ 1, & 0 \le x < 1. \end{cases}$$

find the constant coefficient and the coefficient of $\sin 5\pi x$ of the Fourier series representation of the function of period 2 which coincides at all points of continuity with $f(x)$. You DO NOT have to compute the general coefficient.
(b) Find the value to which the Fourier series converges at $x = 0$.

6. (a) Find the terms through x^6 of the power series expansion centered at 0 for the general solution to $y'' - xy' + 2y = 0$.
(b) Find the coefficient of x^{38} in this series.

7. Use Laplace transforms to solve the initial value problem $y'' + y = 4$, $y(0) = 0$ and $y'(0) = 2$. No credit will be given for any other method.

CONTINUED ON BACK

Part II: Answer any THREE COMPLETE questions (10 points each).
Write the numbers of the TWO OMITTED questions on the cover of your answer booklet.

8. A 10 gallon tank is filled to the top with pure water. Water which has 20 mg of impurity per gallon pours into the tank at the rate of 1 gallon per minute and water (which is well mixed in the tank) flows out of the tank at the rate of 2 gallons per minute. Find the function $Q(t)$, the number of milligrams of impurities in the tank at time t.

9. A 4 lb weight in its equilibrium position stretches the spring from which it is suspended by 1/2 foot. Beginning in its equilibrium position, the weight is given an initial velocity of 4 ft/sec downwards. Air resistance which is proportional to velocity is 8 lbs when the weight has a velocity of 8 feet/sec. Find the function $u(t)$ which describes the displacement in feet of the weight t seconds after it is released. [Assume the acceleration due to gravity is 32 feet/sec^2.]

10. (a) For the equation $x^2y'' - (x+2)y = 0$, find and solve the indicial equation.
(b) For the larger solution r to the indicial equation, find the recursion relation for a solution to the differential equation of the form $y = x^r \sum_{n=0}^{\infty} a_n x^n$ with $a_0 \neq 0$.
(c) Find the terms of the series described in (b) through the x^4 term.

11. (a) Find the general solution: $t^2y'' - 4ty' + 6y = 0$
(b) Use the answer to (a) above to find the general solution to $t^2y'' - 4ty' + 6y = t^4e^t$

12. (a) Use the fact that the equation is homogeneous to find, in explicit form, the general solution of $y' = \dfrac{xy - y^2}{x^2}$, $x > 0$.
(b) What is the largest interval containing the point $x_0 = 5$ on which the equation $(x^3 - 4x^2)y'' + (x+1)y' - xy = 0$ is guaranteed to have a continuous solution? Find all singular points for this equation and state which of these, if any, are regular.