Department of Mathematics

Math 391 **Final Examination** Fall 2005

Part I. Answer ALL questions. Total 70 points.

1. [6 Points] Find the general solution to $y^{(4)} - 4y'' + 4y = 0$.

2. [10 Points] Solve $\left(e^{y^2} - 5y\sin(xy) + \dfrac{1}{\sqrt{x}}\right) dx + \left(2xye^{y^2} - 5x\sin(xy) - \dfrac{1}{\sqrt{y}}\right) dy = 0.$

3. [5 Points] Suppose $u(x,t)$ satisfies the partial differential equation:

$$tu_{xx} + 2xu_{xt} + xtu_x = 0.$$

Use the separation of variables method to replace the partial differential equation by two ordinary differential equations.

4. **(a)** [5 Points] Using only the definition, find the Laplace Transform $Y(s)$ of $y(t) = e^{a\,t}$ where a is a constant. For what values of s does the Laplace Transform $Y(s)$ exist?

(b) [10 Points] Solve, using Laplace Transforms, the initial value problem:

$$y'' - 4y' + 4y = 3, \qquad y(0) = 0, \quad y'(0) = 1.$$

(See table at end of exam.)

5. [6 Points] Find two linearly independent solutions of $2x^2y'' + 3xy' - y = 0$ for $x > 0$ and compute their Wronskian.

6. [10 Points] A spring has a spring constant of 8 lb/ft. A 4-lb weight is pulled down $\frac{1}{2}$ ft below equilibrium and then given an initial upward velocity of 5 ft per sec. The damping constant γ is equal to 2 lb-sec/ft. Assume the acceleration due to gravity is 32 ft/sec ^{2}.

(a) Solve for $u(t)$, the displacement of the weight from equilibrium at time t.

(b) Find the distance of the weight from equilibrium at $t = 1$ sec.

7. [10 Points] Consider the general power series solution $y(x) = \sum_{n=0}^{\infty} a_n x^n$ near $x = 0$ for

$$(x^2 - 4)y'' + 3xy' + y = 0.$$

Find the recurrence relation and compute the first four terms for the solution satisfying $y(0) = 4,\ y'(0) = 3$.

8. [8 Points] Solve $x^2y' - 2xy = 4x^3 - 9, \quad y(1) = 5$.

More Problems on the back.

Part II. Answer any THREE (3) COMPLETE questions, each 10 Points. Total: 30 points. Omit two question.

9. Use the method of variation of parameters to find the general solution to

$$y'' - 4y' + 4y = \frac{e^{2x}}{x}.$$

10. For the equation $3xy'' - y' + 5y = 0$, consider a series solution $y(x) = \sum_{n=0}^{\infty} a_n x^{r+n}$ valid for $x > 0$. Find the indicial equation, the recurrence relation and the first three non-zero terms of the solution corresponding to the larger root of the indicial equation.

11. Consider the function $f(x)$ defined by:

$$f(x) = \begin{cases} x, & \text{if } 0 \le x < 2 , \\ \\ -x, & \text{if } -2 \le x < 0; \end{cases} \qquad f(x+4) = f(x) \text{ for all } x.$$

(a) Find the Fourier series $F(x)$ for $f(x)$ (that is, give the entire series in summation form, not just the Fourier coefficients).

(b) Sketch the graph of the function to which the Fourier series converges for the interval $[0, 8]$.

12. Solve: $y'' + 3y' - 4y = 10e^t + 16t, \quad y(0) = y'(0) = 0.$

13. (a) Find the general solution to $y^{(6)} + 16y'' = 0$.

(b) Given that the equation $L[y] = y^{(4)} - 3y^{(3)} + y'' + 5y' = 0$ has the general solution

$$y_c(t) = c_1 + c_2 e^{-t} + c_3 e^{2t}\cos(t) + c_4 e^{2t}\sin(t),$$

write the form of $y_p(t)$ for the equation $L[y] = 2te^{-t} + 4e^{2t}\cos(t)$. You need not evaluate the coefficients of $y_p(t)$.

End of Exam Questions

Table of Laplace Tranforms

$f(t)$	1	t^n	e^{at}	$\cos at$	$\sin at$
$\mathcal{L}\{f(t)\}$	$\frac{1}{s}$	$\frac{n!}{s^{n+1}}$	$\frac{1}{s-a}$	$\frac{s}{s^2+a^2}$	$\frac{a}{s^2+a^2}$