MATH 21200 Sample Final Exam (2020 version)

1. Integrate the left side of the equality $\int_0^{\pi/2} \sin^n(x)\cos^3(x)dx = 1/24$ and solve for a positive integer n; $n =$

- ☐ 3
- ☐ None of These
- ☐ 2
- ☐ 4
- ☐ 5

2. Let $g(x)$ be a differentiable even function [i.e., $g(-x) = g(x)$] such that $g(\pi/2) = 1/4$ and $\int_0^{\pi/2} g'(x)\sin(x)dx = 2/\pi^2$.

Evaluate $\int_{-\pi/2}^{\pi/2} g(x)\cos(x)dx$.

[Hint: $\cos(x)$ is an even function.]

- ☐ $\frac{1}{2}$
- ☐ $-\frac{4}{\pi^2}$
- ☐ $\frac{4}{\pi^2}$
- ☐ None of These
- ☐ $\frac{1}{2} - \frac{4}{\pi^2}$

3. Let $g(x)$ be a differentiable function such that $g(1) = 2$ and $g(2) = 4$.

Evaluate $\int_1^2 \frac{g'(x)}{g(x)} \frac{1}{[\ln(g(x))]^2} dx$.

[Use $r\ln a = \ln a^r$.]

- ☐ $\ln(2)$
- ☐ $\frac{1}{\ln(4)}$
- ☐ 1
- ☐ $\ln(4)$
- ☐ $\frac{1}{\ln(2)}$

4. $\int_0^{\pi} 4\sin^2(x)dx =$

- ☐ $\frac{\pi}{4}$
- ☐ $\frac{\pi}{2}$
- ☐ π
- ☐ 4π
- ☐ None of These

5. $\int_0^2 x + 4xe^x\,dx =$

- ☐ $4e^2+2$
- ☐ e^2-4
- ☐ $4e^2$
- ☐ None of These
- ☐ $4e^2+6$

6. $\int_8^{15} \frac{7}{x^2-7x}\,dx =$

- ☐ $\ln\left(\frac{15}{64}\right)$
- ☐ $\ln\left(\frac{64}{15}\right)$
- ☐ $\ln(960)$
- ☐ $\ln\left(\frac{15}{8}\right)$
- ☐ $\ln\left(\frac{8}{15}\right)$

7. Which of the following integral(s) diverge?

- ☐ None of These
- ☐ $\int_0^{\infty} \frac{2}{\sqrt{x}(x+5)}\,dx$
- ☐ $\int_1^{3} \frac{1}{(x-2)^2}\,dx$
- ☐ $\int_0^{1} \frac{1}{(1-x)^{1/4}}\,dx$
- ☐ $\int_1^{\infty} \frac{1}{\sqrt{x-3}}\,dx$

8. Let $f(x)$ be non-negative for all x. Which of the following statement(s) is/are correct?

- ☐ If $f(x) \geq e^{-x}$ for all x, then $\int_1^{\infty} f(x)dx$ is divergent.
- ☐ If $f(x) \geq c > 0$ for all x, where c is a constant, then $\int_1^{\infty} f(x)dx$ is divergent.
- ☐ None of These
- ☐ If $f(x) = \frac{1}{x}$ for $x > 0$, then $\int_1^{\infty} f(x)dx$ is convergent.
- ☐ If $f(x) \leq e^{-x}$ for all x, then $\int_1^{\infty} f(x)dx$ is convergent.

9. Which of the following series requires at least 2 tests to determine whether it is absolute convergent, conditional convergent or divergent?

- ☐ $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$
- ☐ $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2-n)}{n^2+n}$
- ☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+n}$
- ☐ $\sum_{n=1}^{\infty} \frac{2^n}{3^{n+1}}$
- ☐ $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

10. Which of the following series diverge?

- ☐ $\sum_{n=1}^{\infty} \frac{3}{n^2 \ln(n)}$
- ☐ None of These
- ☐ $\sum_{n=1}^{\infty} \frac{3^n}{n!}$
- ☐ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+2n}$
- ☐ $\sum_{n=1}^{\infty} \frac{\sin(n)}{3^n}$

11. Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x+a)^n}{\sqrt{n+1}}$, where a is a constant.

- ☐ $(-a-1, -a+1)$
- ☐ $[-a-1, -a+1)$
- ☐ $[-a-1, -a+1]$
- ☐ None of These
- ☐ $(-a-1, -a+1]$

12. For $f(x) = x\sin(x)$, an infinite series for $\int_0^{1/2} f(x)\,dx$ is

- ☐ $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+3)2^{2n+3}} = \frac{1}{(3)(2^3)} - \frac{1}{(5)(2^5)} + \frac{1}{(7)(2^7)} \pm \cdots$
- ☐ $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(2n+3)2^n} = \frac{1}{3} - \frac{1}{(3!)(5)(2)} + \frac{1}{(5!)(7)(2^3)} \pm \cdots$
- ☐ $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!(2n+3)2^{2n+3}} = \frac{1}{(3)(2^3)} - \frac{1}{(3!)(5)(2^5)} + \frac{1}{(5!)(7)(2^7)} \pm \cdots$
- ☐ $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!2^{2n+3}} = \frac{1}{2^3} - \frac{1}{(3!)(2^5)} + \frac{1}{(5!)(2^7)} \pm \cdots$
- ☐ None of These

13. The graph of the parametric equations $x = \sqrt{1-t^2}$, $y = 2 + (1-t^2)^{3/2}$, $-1 \le t \le 0$, is the same as the graph of

- ☐ $y = 2 + x^3, 0 \le x \le 1.$
- ☐ $y = 2 + x^3, -1 \le x \le 1.$
- ☐ $y = (x-2)^3, -1 \le x \le 0.$
- ☐ None of These
- ☐ $y = 2 + x^3, -1 \le x \le 0.$

14. Let **u** and **v** be orthogonal unit vectors. Which of the following is/are **NOT** true?

- ☐ $\mathbf{u} \times \mathbf{v} = \mathbf{0}$
- ☐ None of These
- ☐ $3\mathbf{u} - 4\mathbf{v}$ and $4\mathbf{u} + 3\mathbf{v}$ are orthogonal
- ☐ $\mathbf{u} \cdot \mathbf{v} = 0$
- ☐ $3\mathbf{u} - 4\mathbf{v}$ and $3\mathbf{u} + 4\mathbf{v}$ are orthogonal

15. Find an equation for the plane which is perpendicular to both of the planes $z = 2x + y$ and $z = 3x$ and which passes through the point $(0,5,-4)$.

- ☐ $-x - (y-5) - 3(z+4) = 0$
- ☐ $5(y-1) - 4(z+3) = 0$
- ☐ $-x + y - 3z = 0$
- ☐ None of These
- ☐ $-x + y - 5 - 3(z-4) = 0$

16. Lines l_1 and l_2 have parametric equations

l_1: $x = 1 + 2t$, $y = 4$, $z = 6 - t$

l_2: $x = 3 + 2s$, $y = 4$, $z = 5 + s$

Which of the following statement(s) is/are true?

- ☐ l_1 and l_2 do not intersect and are parallel.
- ☐ l_1 and l_2 intersect and are both parallel to the xz-plane.
- ☐ l_1 and l_2 intersect and are perpendicular.
- ☐ None of These
- ☐ l_1 and l_2 do not intersect and are both in the xz-plane.

The graph of $4x^2 - 9y^2 - 36z^2 = -36$ is a

- ☐ hyperboloid of one sheet, and the x-axis does not intersect the graph.
- ☐ hyperboloid of two sheets, and the x-axis does not intersect the vertices
- ☐ hyperboloid of one sheet, and the x-axis intersects the graph.
- ☐ hyperboloid of two sheets, and the x-axis intersects the vertices.
- ☐ None of These

$\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2 - \sin^2 y}{x^2 + y^2} =$

- ☐ 1
- ☐ The limit does not exist.
- ☐ 0
- ☐ -1
- ☐ 2

19. If $g(t)$ has a continuous second derivative and $f(x,y) = xg(y) + \tan^{-1}(xy)$, then $f_{xx}(2,1) =$

- ☐ Not enough information is given to find the numerical value of $f_{xx}(2,1)$.
- ☐ $-\dfrac{1}{25}$
- ☐ $\dfrac{1}{25}$
- ☐ $-\dfrac{4}{25}$
- ☐ $\dfrac{4}{25}$

20. If $f(x,y) = \sqrt{xy}$, then $f_x f_y + f_{xy} - f_{yx} =$

- ☐ $\dfrac{1}{4xy}$
- ☐ $\dfrac{1}{4} + \dfrac{1}{2\sqrt{xy}}$
- ☐ $\dfrac{1}{4} - \dfrac{1}{2\sqrt{xy}}$
- ☐ $\dfrac{1}{4\sqrt{xy}}$
- ☐ $\dfrac{1}{4}$