Department of Mathematics
Math 209 Final Examination Spring 2009

Instructions: You may use a calculator and the accompanying tables. Round off numerical answers to three decimal places, unless noted otherwise. The time for the exam is 2 hours, 15 minutes.

Part I (70 points) Answer questions 1 through 7 in the space provided. Each problem is worth 10 points.

1. Use separation of variables to solve the initial value problem

$$\frac{dy}{dx} = \frac{e^x}{y}, \; y(0) = -4.$$

2. Use Euler's method with 4 subintervals to estimate the solution at $x = 3$ of the initial value problem $\frac{dy}{dx} = x - y$, $y(1) = 2$. Make a table showing your calculations.

3. The deer population of a national park has a natural growth rate of 4% per year. Each year the authorities allow 300 deer to be eliminated from the herd through hunting.

a) Write a differential equation to model the deer population $D(t)$ in the national park.

b) If the initial number of deer in the national park is (i) 6000, or (ii) 8000, using qualitative methods, sketch on the same axes graphs of the solution $D(t)$ in each case, clearly indicating what will happen to the population of deer as time goes on.

4. A study of classroom size discovered that in Shepard Hall, 20 classrooms can seat 25 students, 10 classrooms can seat 30 students, 8 classrooms can seat 35 students, 15 classrooms can seat 40 students, and 7 classrooms can seat 60 students.

a) Find the mean of the number of students that a classroom can hold.

b) What is the median number of students that a classroom can hold? Explain how you arrived at your answer.

5. A study of weight (y) vs. height (x) of adult males gave a regression line of $y = -180 + 5.2x$, where x is measured in inches and y in pounds. The standard deviations for x and y were found to be $s_x = 2.5$ and $s_y = 18$.

a) Find the correlation coefficient r.
b) What is the average weight for a male whose height is 68 inches?
c) If a male who is more than two standard deviations above the mean weight for his height is considered obese, is a male with height 68 inches and weight 200 pounds considered obese? Explain the reason for your answer.

6. In a discount supermarket, it is found that 60% of the sales are for less than $40. Find the probability of the following events:
a) Three randomly selected sales are each for less than $40.
b) At least one of three randomly selected sales is for less than $40.
c) Nine or more of 11 randomly selected sales are for less than $40.

7. The EverGreen Salad company asserts that on average its packaged salad greens remain fresh for 4 days after the "use by" date on the package, with a standard deviation of 0.45 days. Assume that the length of time a package of greens stays fresh after the "use by" date is normally distributed.

a) What is the probability that a randomly selected package of greens will be fresh for more than 5 days after the "use by" date on the package?

b) The EverGreen Company has decided to offer a refund for packages that are not fresh *x* days after the "use by" date on the package. Find the largest value of *x* (a whole number of days) so that the company can expect to issue refunds for at most 1 of 250 containers. Justify your answer.

Part II: Do any <u>two</u> problems (15 points each)

8. Consider the experiment of tossing a pair of six sided dice. Let X = the <u>absolute value</u> of the difference between the tosses. For example, if you toss a 2 and a 5 then $X = 3$.

a) Determine the probability distribution for X and draw a probability histogram
b) Find μ_X. What is the statistical interpretation of this number?
c) If you repeat this experiment twice, what is the probability that you will get a value of $X \leq 1$ each time? Justify your answer.

9. The growth of a population is described by a logistic differential equation. The intrinsic or natural growth rate is 4% and the carrying capacity is K = 350.

a) Write a differential equation to model the population.

b) For each of the following initial values N_0 use the first and second derivatives of the population function $N(t)$ to sketch the graph of $N(t)$: Your graphs should clearly show any long term limiting behavior of $N(t)$ and any changes in concavity. Place all graphs on the same set of axes.

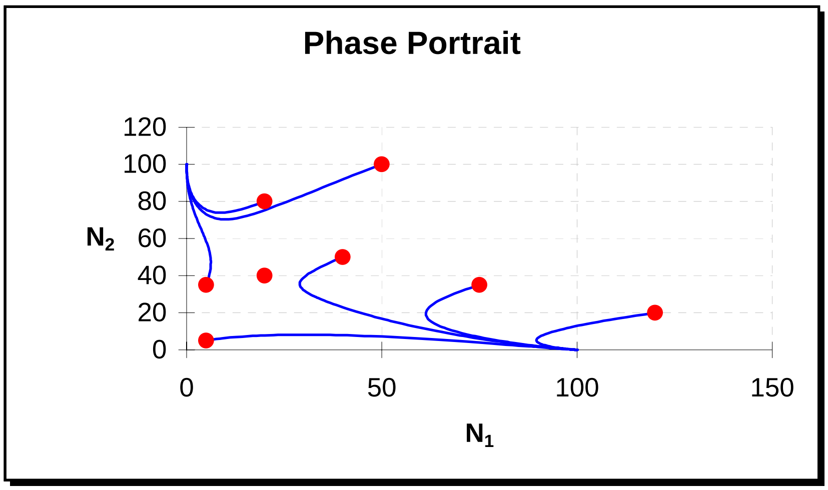
(i) $N_0 = 100$ (ii) $N_0 = 300$ (iii) $N_0 = 400$

10. a) Consider the competition model described by the system of differential equations:

$$\frac{dN_1}{dt} = .25N_1\left(1 - \frac{N_1}{100} - \frac{N_2}{50}\right)$$
$$\frac{dN_2}{dt} = .10N_2\left(1 - \frac{N_2}{100} - \frac{3N_1}{100}\right).$$

Find the steady-state solutions of the system.

b) A collection of phase curves for this system is shown below. Based on these graphs how would you characterize the stability of each of the steady-state solutions you found in a)? Briefly explain your response.



c) Would you characterize the species modeled by this system as competitive or not? Justify your answer.