City College of New York   MATH 20900   Spring 2014
Department of Mathematics   Final Examination

1) Turn-off cell phones and put them and all notes out of sight.

2) CALCULATORS are allowed, NO scrap paper (use sheets provided)

3) Points will be deducted if a solution is given without written proof of your work

4) If you need additional space to answer a question, please use the facing side of each sheet.

5) Note: See page 2 for Normal Distribution tables.

<u>SHOW ALL WORK</u>

Completely answer 10 out of 12 questions (1 question worth 10 points in each page). If you answer more than 10, cross-out work not to be graded.

1)   [10 points] Solve the differential equation $\dfrac{dy}{dx} = \dfrac{e^y}{x^2}$ subject to the condition $y(1) = 0$. Solve the equation explicitly and compute the exact value of $y(e^2)$. (Compute your answer to 3 decimal places.)

2)   [10 points] Consider the initial value problem $\dfrac{dy}{dx} = \dfrac{2x}{y^2}$, where $y(0) = 1$. Use the Modified Euler's method with 2 subintervals to find the approximate value of $y(1)$. (Compute your answer to 3 decimal places.)

3)   [10 points] Use geometric analysis to analyze the differential equation $\dfrac{dy}{dt} = y(6 - y) - 5$. Your answer must show:
   a) your solution for the steady states and inflection points of a solution curve $y(t)$.
   b) a graph of $g(y)$ vs. y, where $g(y)$ is the derivative of y as a function of y. Your steady states and stability arrows must be indicated on the graph.
   c) sketches of the solution curve $y(t)$ for the initial value problem $y(0) = 2$. The graph must show the proper concavity and the steady states and inflection point must be indicated.
   d) a description of the long term behavior of the solution curve from part c).

4)   [10 points] The height y in inches, and weight x in pounds of four people are recorded in a table.

Weight x	150	180	200	220
Height y	69	64	70	68

   a) Compute the quantities $\bar{x}$, $\bar{y}$, s_x, s_y, c_{xy}, and r and use these to find the regression line for the bivariate data. (Compute your answer to 3 decimal places.)
   b) Suppose a person weighs 195 pounds, what estimate would you give for the person's height? (Compute your answer to 3 decimal places.)

5)   [10 points] The average time students take to complete a math 209 final exam is 105 minutes with a standard deviation of 16 minutes. Assume that the time students take to complete a math 209 final is normally distributed.
   a) What percentage of students take more than 120 minutes to complete a math 209 final? (Compute your answer to 4 decimal places or percentage.)
   b) The total time given for a math 209 final is 2 hours and 15 minutes. What is the probability that a student will run out of time and not complete the final? (Compute your answer to 4 decimal places or percentage.)

6)   [10 points] Pirate Captain Jack Sparrow swore to his crew that he will be merciful to a disobedient deckhand. He puts three different types of coins (5 brass coins, 3 silver coins, and 1 gold coin) into his hat and asks the deckhand to randomly select 2 coins **without replacement** (he does not tell his crew how many of each type of coin he placed in his hat). He then passes the judgment that if the deck hand picks two gold coins, he shall be completely forgiven. Otherwise, if he picks two coins of the same type, then he will be left on a deserted island. If the deckhand picks two different types of coins, he must walk the plank. Find the following probabilities and write them as **simplified fractions**.
   a) What is the probability that the deckhand will be forgiven?
   b) What is the probability that the deckhand will select two silver coins?
   c) What is the probability that the deckhand will **not** have to walk the plank?
   d) What is the probability that the deckhand will **not** be left on a deserted island?

7)   [10 points] An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:
$$\frac{dN_1}{dt} = 0.20N_1(1 - \frac{N_1}{60} - \frac{N_2}{60})$$
$$\frac{dN_2}{dt} = 0.10N_2(1 - \frac{N_2}{90} - \frac{N_1}{30})$$
   a) Find all steady-state solutions of this system.
   b) In the long-term to which of the possible steady state solutions will the populations tend? Explain

8)   [10 points] A lake holds a population of 600,000 trout. The natural growth rate of the trout population is 5%. Fishing removes 35,000 trout each year from the lake. Set up a differential equation modeling the fish population as a function of time. Solve it to find out <u>if</u> and <u>when</u> the fish population will be completely depleted. (Compute your answer to 3 decimal places.)

9)   Final grades of 150 students from six math courses were recorded. The table below
      summarizes the distribution of their final grade.

Grade range	(50, 60]	(60, 70]	(70, 80]	(80, 90]	(90, 100]
# of grades in range	23	22	37	45	23

      a) [2 points]Based on the data, prepare a <u>relative</u> frequency histogram for the grades.
      b) [4 points]Based on data, estimate as accurately as you can the <u>median</u> number of grades.
          (Compute your answer to 3 decimal places, if necessary.)
      c) [4 points]Based on the data estimate as accurately as you can the <u>average</u> number of grades.
          (Compute your answer to 3 decimal places, if necessary.)

10)   New York Yankees short stop, Derek Jeter #2, has a career batting average of 0.312 [as of first
      week of May 2014] (batting average is the ratio of number of hits over the total number of at
      bats appearance).
      a) [5 points] Assume that in the current season, Jeter will have 650 at bats. Using normal
          distribution, estimate the probability that Jeter will have between 180 to 210 hits? (Compute
          your answer to 4 decimal places.)
      b) [5 points] Assume that on this years Subway Series games (Yankees vs. Mets), Jeter will
          have 20 at bats. What is the actual probability that Jeter will have between 7 and 10 hits?
          (Compute your answer to 4 decimal places.)

11)   Answer parts a) and b) below based on the following information: Blood types O, A, B and AB
      occur in the population with frequencies 0.43, 0.41, 0.11, and 0.05 respectively. A person may
      receive blood from another person with the same Rh factor and with the same type or type O.
      Within all the blood types, the Rh factors, (plus and minus), occur with frequencies 0.86 and
      0.14 respectively.
      a) [2 points] What is the probability that a randomly selected individual can be a donor for a
          person of type B, Rh+?(Compute your answer to 4 decimal places.)
      b) [8 points] A hospital needs eight units of blood to give to a type A, Rh+ recipient. An
          individual can donate only one unit. If 10 people arrive at the blood donation center, what is
          the probability that the hospital will be able to obtain the blood that it needs? Be careful to
          consider all possibilities that will satisfy the hospital's needs. Show details of the
          calculation using appropriate probability formulas. (Compute your answer to 4 decimal
          places.)

12)   a) [5 points] A college requires all freshmen to take Math and English courses. Records show
          that 24% receive an A in English course, while only 18% receive an A in Math course.
          Altogether, 35.7% of the students get an A in Math course or English course. What is the
          probability that a student who receives an A in Math course will also receive an A in
          English course? (Write your answers to 3 decimal places)
      b) [5 points] A study of weight (y) vs. height (x) of adult males gave a regression line of
          $y = -180 + 5.2x$, where x is measured in inches and y in pounds. The standard deviations
          for x and y were found to be $s_x = 2.5$ and $s_y = 16$. Find the correlation coefficient and what
          would be the weight for a male whose height is 67 inches?