The City College Department of Mathematics
Spring 2008

MATH 20900 Final Exam

1) Turn-off cell phones and put them and all notes out of sight.

2) CALCULATORS are allowed, NO scrap paper (use sheets provided)

3) Points will be deducted if a solution is given without written proof of your work

4) If you need additional space to answer a question, please use the facing side of each sheet.

5) Note: See behind page 1 and page 2 for Normal Distribution tables.

<u>SHOW ALL WORK</u>

PART 1: [pages 3 to 8] Answer ALL questions in this part. (60 points) If you need additional space to answer a question, please use the facing side of each sheet.

1)   [10 points] Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1+2y}{1+x}$ subject to the initial condition $y(0) = 3$.

2)   [10 points] Consider the initial value problem $\dfrac{dy}{dx} = x^2 - y$, where $y(0) = 1$. Use the Euler's method with step size 0.25 to find the approximate value of $y(1)$. (Compute your answer to 3 decimal places.)

3)   [10 points] Use geometric analysis to analyze the differential equation $\dfrac{dy}{dt} = -y^2 + 9y - 14$.

Your answer should include:
a) a graph of $g(y)$ vs. y, where $g(y)$ is the derivative of y as a function of y.
b) sketches of the solution curves $y(t)$ for the initial value problem $y(0) = 3$ and all steady state solutions.
c) show concavity and show the y of all inflection points for solution curves.
d) stability analysis for all steady state solutions.

4)   [4 points] a) Put the three graphs below in order of increasing correlation coefficient r.

(smallest r) _____   _____   _____ (largest r)

Data set A	Data set B	Data set C
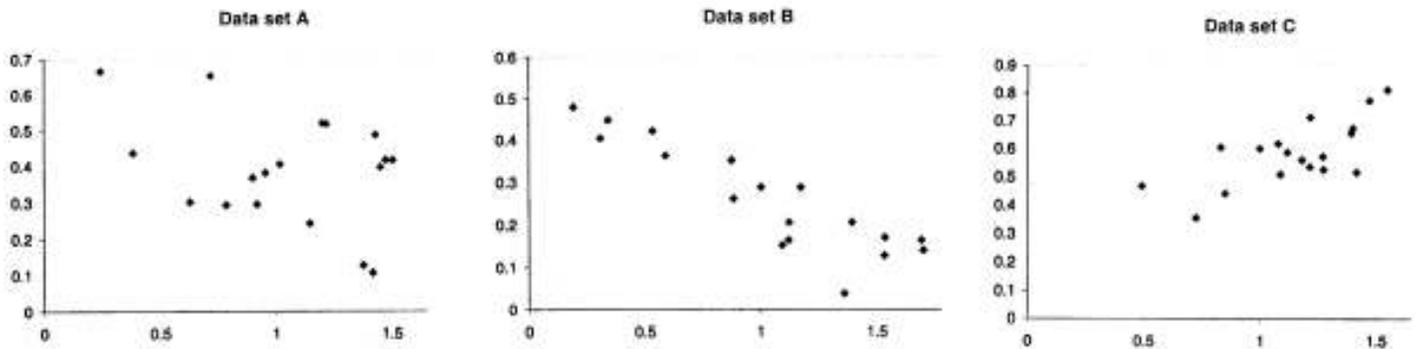
[6 points] b) Sketch your best guess as to the regression line of the data set B, and write down the equation of that line.

5)   [10 points] A milk company unconditionally guarantees that an unopened, properly refrigerated container of its milk will be fresh for 4 days after the date stamped on the carton. The company's data shows that unopened containers remain fresh on average for 6 days, with a standard deviation of 0.43 days. Assume that the length of time milk stays fresh follows a normal distribution. What percent of the milk containers will spoil in fewer than 5 days after the date stamped?

6)   A bowl contains 5 red balls, 6 blue balls and 4 white balls. You pick two balls without replacement.
        a) [3 points] What is the probability that both balls are blue? (Answer may be left as a fraction or as a three-place decimal.)

        b) [4 points] What is the probability that both balls are the same color? (Answer may be left as a fraction or as a three-place decimal.)

        c) [3 points] What is the probability that both balls have different colors? (Answer may be left as a fraction or as a three-place decimal.)

PART 2: [pages 9 to 14] Answer any 4 complete questions. Each question is worth 10 points.
If you answer more than 4 questions, cross out work you do not want graded or only the first 4 questions will be graded.
If you need additional space to answer a question, please use the facing side of each sheet.

7)   [10 points] An ecosystem containing two species is modeled by the system of differential equations given below, where N_1 and N_2 denote the number of members of each species and the rates are annual rates of change of the species populations:

$$\frac{dN_1}{dt} = 0.03N_1(1 - \frac{N_1}{100} - \frac{3N_2}{100})$$

$$\frac{dN_2}{dt} = 0.05N_2(1 - \frac{N_2}{50} - \frac{N_1}{50})$$

a) Find all steady-state solutions of this system.

b) Based on the above model, would you characterize the species as competitive? In the long-term to which of the possible steady state solutions will the populations tend? Explain

8)   [10 points] A shrimp farm in Thailand starts with 3,000,000 shrimp. The natural growth rate of shrimp is 12% per year. The owner (perhaps unwisely) harvests 420,000 shrimp each year. Set up a differential equation modeling the fish population as a function of time. Solve it to find out if and when the fish population will be completely depleted. (Compute your answer to 3 decimal places.)

9)   [10 points] 135 light bulbs were randomly selected from a large batch and placed through a simulation of everyday use until they burned out. The table below summarizes the distribution of their lifetime (=number of hours until burn out). For example, 25 bulbs lasted between a little more than 900 hours and up to 1000 hours.

Hours until burnout	(800, 900]	(900, 1000]	(1000, 1100]	(1100, 1200]	(1200, 1300]
# of bulbs	10	25	35	40	25

a) Based on the data, prepare a <u>relative</u> frequency histogram for the time until burnout for bulbs from the sample.

b) Based on data, estimate as accurately as you can the <u>median</u> number of hours that a sample bulb lasted until burnout. (Compute your answer to 3 decimal places.)

c) Based on the data, estimate as accurately as you can the <u>average</u> number of hours that a sample bulb lasted until burnout. (Compute your answer to 3 decimal places.)

10)   [10 points] Assume that 7 % of cars pulled over at a police check point are found to have a vehicular infraction.

   a) If 25 cars are stopped at a check point what is the probability that 2 or more cars will be found to have some infraction? (Compute your answer to 3 decimal places.)

   b) The local precinct is aiming to give out at least 20 tickets for vehicular infractions to cars stopped at the check point. If 200 cars pass through the check point, use a normal distribution to estimate the probability that the police will reach their target. (Compute your answer to 3 decimal places.)

11)   [10 points] A basketball player, LeBron James, has a career free throw percentage of 0.73 [as of May 2008] (this number is the ratio of free throws made over the number of free throws attempted). At a random game he will have 21 free throw attempts; Let X be the random variable which is the number of free throws made during the game. Find the probability distribution that he makes 10 to 14 free throws in a game, the expected value and the standard deviation for this game. (Compute your answer to 3 decimal places.)

12)   [10 points] To boost sinking sales, a cola company has decided to market a new product: invisible cola. To test the popularity of the new product, it has offered invisible cola to 40 consumers and found that 2 people would buy it. a) If invisible cola is offered for sale, what it the probability that more than 10% of the consumers will buy it? b) Find a 90% confidence interval (confidence coefficient = 1.65) for the expected percentage of consumers who will buy invisible cola.