MATH 203 Final Exam, May 24, 2010

Instructions: Complete every question in Part I (Questions 1-7.) Complete three of the five questions in Part II (Questions 8-12). Each question is worth 10 points.

No calculators or other electronic devices may be used during the examination. Answers may be left in terms of $\sqrt{7}$, $\sin(3)$, π, etc. Give reasons for all answers. This exam will last 2 hours and 15 minutes. Good luck!

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

1. Let P be the plane that contains the points $(2,-1,1)$, $(1,3,1)$ and $(2,1,2)$ and let Q be the plane $z = x + y$.

   (a) Parameterize the intersection of the two planes P and Q.

   (b) What is the smaller of the two angles of intersection of the planes?

2. (a) Parameterize the tangent line to the curve $\vec{r}(t) =< \frac{1}{2t}, e^{t^2}, t^{1/3} >$ at the point $(\frac{-1}{2}, e, -1)$.

   (b) Find an equation of the tangent plane to the surface $z = x^2 - 4y^2 + 3$ at the point $(2,-1,3)$.

3. Find all local maxima and minima and all saddle points of the function $f(x,y) = x^2 - 4xy + 6x - 8y + 2y^2 + 10$.

4. A laminar region R lies the first quadrant, includes the origin, and is bounded by $y = 2x, y = 1$ and $y = x^3$. It has density given by $\rho(x,y) = 24x^2$. Sketch the region and compute its mass.

5. Find $\int\int\int_R x\,dV$, where R the solid bounded on top by $z = x$, on the bottom by the triangle with vertices $(0,0,0)$, $(2,2,0)$ and $(2,0,0)$ and on the sides by vertical planes through the sides of the triangle.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent and show why your answer is correct. (No credit for any part unless your reasons are given.)

   (a) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{4}{\sqrt{k+5}}$

   (b) $\displaystyle\sum_{k=0}^{\infty} \frac{e^{3k}}{k^{3k}}$

   (c) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k^3\sqrt{k+4}}{k^3+1}$

7. Find the interval of convergence including possible endpoints for the power series

$$\sum_{k=1}^{\infty} \frac{k^2(x+3)^k}{2^k}.$$

Part II: (30 points) Solve 3 complete problems out of 5

8. (a) Find the Maclaurin series for $\frac{2t}{1+t^2}$. Express your answer in summation notation.

   (b) Use your answer for part (a) to find the Maclaurin series for $\ln(x^2+1)$.

9. Sketch the region of integration and change the method of integration to evaluate:

$$\int_{-2}^{0} \int_{0}^{\sqrt{4-y^2}} e^{(x^2+y^2)}\, dx\, dy$$

10. Find the surface area of that part of the surface $z = 4 - x^2 - y^2$ that lies above the plane $z = 1$.

11. Find the mass of the sphere $\rho = 2\cos(\phi)$ of radius 1 centered at $(0, 0, 1)$, given that the density at each point is given by $\delta(x, y, z) = x^2 + y^2 + z^2$.

12. For $P = T\sqrt{V}$, use differentials (that is, the best linear approximation) to estimate the value of P when $V = 99.8$ and $T = 10.3$.

13. (a) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + xy^2}{x^2 + y^2}$$

   (b) Classify and sketch the quadric surface $x^2+2x+y^2-z^2+2z+1 = 0$, labeling at least 3 points on the surface. Show the trace of the graph in 3 planes.