MATH 203 Final Exam
May 18, 2009

Instructions: Complete every question in Part I (Questions 1-7.) Complete three of the five questions in Part II (Questions 8-12). Each question is worth 10 points.

No calculators or other electronic devices may be used during the examination. Answers may be left in terms of $\sqrt{7}$, $\sin(3)$, π etc. Proctors will confiscate any devices observed in the examination room. Give reasons for all answers. This exam will last 2 hours and 15 minutes. Good luck!

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

1. A line L is given parametrically by the equations $x = 4t; y = 4; z = 2 + t$.

   (a) Compute all unit vectors parallel to the line.

   (b) Compute the coordinates of the point where the line intersects the plane P with equation $x + 2y + 3z = 0$.

   (c) Is the line L perpendicular to the plane P? Justify your answer.

2. Let $f(x, y, z) = x^2 - \cos(xy) + e^{yz}$.

   (a) At the point $(3, 0, 1)$ compute the unit vector in the direction of maximum increase of the function f and compute the rate of increase in that direction.

   (b) Compute the directional derivative of the function f at the point $(3, 0, 1)$ in the direction of the vector $\langle -5, 12, 0\rangle$.

   (c) Compute an equation for the plane tangent to the surface given by the equation $9 = f(x, y, z)$ at the point $(3, 0, 1)$.

3. A laminar region R in the first quadrant is bounded by $y = 8x, y = 8$ and $y = x^3$. It has density given by $\rho(x, y) = 3x^2$. Sketch the region and compute its mass.

4. Find and classify the critical points of $f(x, y) = 16y^2 + x^4y + 4x^2 + 4$.

5. Find the surface area of the portion of the surface $z = y^2 - x^2$ which is contained in the cylinder $x^2 + y^2 = 4$.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent and show why your answer is correct. (No credit for any part unless your reasons are given.)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n + 2}{3n^2 + 2n + 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2 + \cos(n)}{n}$

(c) $\sum_{n=1}^{\infty} \cos(\frac{5^n}{n!})$

7. Write out the first four terms <u>and</u> compute the interval of convergence (including possible endpoints) for the power series

$$\sum_{n=1}^{\infty} (-2)^n (x-1)^n.$$

PART II: Answer all parts of 3 of the following 5 questions. Each question is worth 10 points

8. Suppose the temperature T is given by $T = x^3 + y^3 - 3xy$.

   (a) Find and classify all critical points of T.

   (b) Where are the hottest points on the square plate bounded by $x = 0, x = 2$ and $y = 0, y = 3$?

9. (a) Find an equation of the plane which contains the three points $(1, 0, 1), (1, 0, 2)$ and $(2, 2, 1)$.

   (b) Show that the following limit does not exist:

$$\lim_{(x,y) \to (0,0)} \frac{xy\cos(y)}{x^2 + y^2}$$

10. (a) Let R be the solid in the first octant which is bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$. Sketch a diagram of intersection of the solid with the xz plane (that is, the plane $y = 0$).

    (b) Set up three triple integrals for the volume of the solid in part (a): one each using rectangular, cylindrical and spherical coordinates.

    (c) Use one of the three integrals of part (b) to compute the volume.

11. (a) Use known series to obtain the Maclaurin series (that is, the Taylor series centered at 0) for $f(x) = \frac{1}{1+x^2}$. Express your answer using summation notation.

    (b) Use your answer to part (a) to obtain the Maclaurin series for $g(x) = \arctan(x)$.

12. A surface is given parametrically by the equations $x = u^2 - v^2$, $y = 4uv, z = (3u - 2v)^2$. Find an equation of the plane tangent to the surface at the point where $u = v = 1$