Introduction

It is impossible to study the biological sciences today without a grounding in a substantial body of mathematics. Much active biological research relies heavily on mathematical and computational techniques. Consider for instance that:

- Differential equation models for the replication of HIV were used to develop current drug therapy protocols. These models are similar to, though more elaborate, than the examples we will study in Chapters 5 and 6 of the text.

- Clinical trials and epidemiology studies use statistical methods, from the planning stage to final data analysis. Part II of the text contains an introduction to these matters.

- Gene sequencing uses sophisticated mathematical algorithms to reconstruct the nucleotide ordering on the genome from the fragments analyzed by sequencing machines.

The purpose of this text is to begin the process of developing a working familiarity with some of these key mathematical concepts. As your studies in the life sciences become more specialized you will naturally find that certain mathematical fields receive more emphasis than do others. We hope that we will have laid a foundation that will allow you to deepen your understanding as the need to do so arises.

The course focuses on two principal mathematical themes: modeling using differential and difference equations to create mathematical mini-worlds that describe biological processes, and statistics, with its attendant basis in probability theory. The reader who perseveres to the end will see how the latter theory also infuses the earlier models, allowing us to incorporate the effects of chance and uncertainty, albeit having to give up a degree of predictability.

Throughout, we make constant use of Microsoft *Excel*® to illustrate many complicated mathematical ideas and to perform computations that are not feasible to do otherwise. Many other software platforms could have been chosen for this purpose. Indeed, many of the computations could be done on graphing calculators. *Excel* was chosen because it (or similar spreadsheet based programs) is the computational package most commonly used by professionals working in the life sciences.

The reader is not assumed to know how to use *Excel*, though learning the basics is certainly implicit in the course design and the exercises. Particularly at the beginning of the course when the mathematics sometimes required a somewhat more sophisticated computational treatment, we have developed *Visual Basic* packages that allow the user to

concentrate principally on the mathematics and not be distracted by excessive interaction with *Excel*. Similarly, in writing the text, except for specific sections marked "Tech Notes", *Excel* is used simply as a tool to illustrate the mathematical concepts.

We assume the reader has had at least one semester of calculus, covering the basics of differentiation and integration. The first chapter is a quick review of this material, heavily slanted to the algebraic aspects. We soon leave this familiar territory, to which the reader has probably become acclimated over many years of instruction, to a different landscape, where the algebra blends with geometry, combinatorics and numerical computation. This is a richer but more ambiguous environment. The tasks are not so easily discerned and the meaning is often the answer.

Although reading a mathematics text is not easy, you will gain immeasurably from your efforts to do so. It is a good idea to read the relevant sections before a class and then again, in conjunction with your notes, when you attempt the homework. A steady diet of the latter is the best way to build mathematical muscle, which develops slowly over time. Mathematics texts often seem impenetrable to students. We have tried our best to minimize heavy-handed notation and dense algebraic computations (except in some occasional asides). We apologize for any obscurities (as well as errors) that remain, and would welcome feedback from any reader who would be gracious enough to provide us with his or her comments.

There are a few stylistic conventions that we have adopted with which the reader ought to be familiar. The statements of theorems, definitions, examples and so forth are boxed. We use the symbol ∎ to signify the end of the related discussion. Occasionally we wish to emphasize a point more emphatically and we do so by using an enlarged version of the symbol ⚡ placed in the margin. Exercises that require the use of a computer are designated with the symbol 🖥. Finally, although mathematics and science are serious and in their own way enjoyable undertakings, we can certainly benefit from seeing our endeavors through the eyes of popular culture. For some irreverant and perhaps irrelevant snapshots we offer a choice quote from *The Simpsons* at the head of each chapter … enjoy.

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