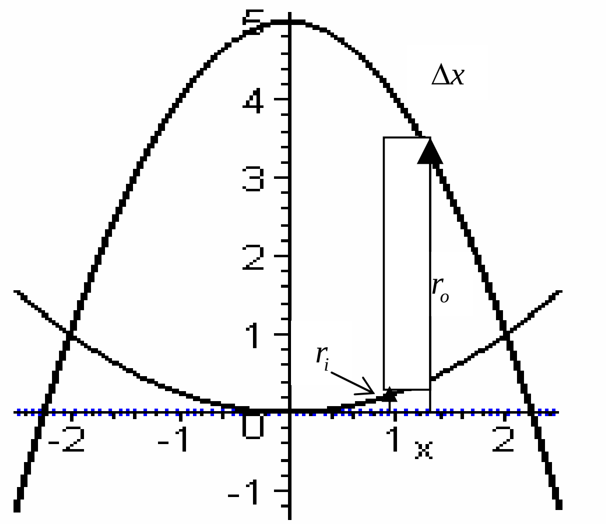
6) $y = \frac{1}{4}x^2 \quad y = 5 - x^2$; about x-axis



$$\frac{1}{4}x^2 = 5 - x^2$$
$$\frac{5}{4}x^2 - 5 = 0$$
$$\frac{5}{4}(x^2 - 4) = 0$$
$$\frac{5}{4}(x+2)(x-2) = 0$$
$$x + 2 = 0 \quad x - 2 = 0$$
$$x = -2 \quad x = 2$$

interval: $-2 \le x \le 2$

$$r_o = (5 - x^2) + 0 = (5 - x^2) \quad r_i = \left(\frac{1}{4}x^2\right) + 0 = \left(\frac{1}{4}x^2\right)$$

$$A_o = \pi(5 - x^2)^2 = \pi(25 - 10x^2 + x^4) \quad A_i = \pi\left(\frac{1}{4}x^2\right)^2 = \pi\left(\frac{1}{16}x^4\right)$$

$$A = A_o - A_i = \pi(25 - 10x^2 + x^4) - \pi\left(\frac{1}{16}x^4\right) = \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right)$$

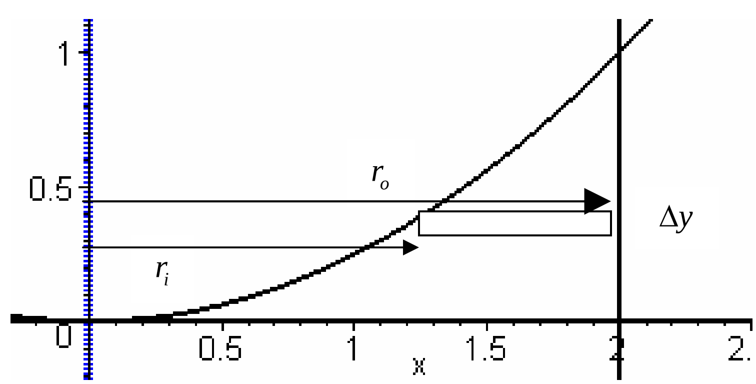
$$\Delta V = A\Delta x = \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right)\Delta x$$

$$V = \int_{-2}^{2} \pi\left(25 - 10x^2 + \frac{15}{16}x^4\right) dx = \pi\left[25x - \frac{10}{3}x^3 + \frac{3}{16}x^5 + C\right]_{-2}^{2}$$

$$= \pi\left\{\left[25(2) - \frac{10}{3}(2)^3 + \frac{3}{16}(2)^5 + C\right] - \left[25(-2) - \frac{10}{3}(-2)^3 + \frac{3}{16}(-2)^5 + C\right]\right\}$$

$$= \pi\left\{\left[50 - \frac{80}{3} + 6\right] - \left[-50 + \frac{80}{3} - 6\right]\right\} = \pi\left\{100 - \frac{160}{3} + 12\right\} = \pi\left\{\frac{300}{3} - \frac{160}{3} + \frac{36}{3}\right\} = \frac{176\pi}{3}\ units^3$$

8) $y = \frac{1}{4}x^2 \quad x = 2 \quad y = 0$; about y-axis



$$y = \frac{1}{4}(2)^2 = 1$$

interval: $0 \le y \le 1$

$$y = \frac{1}{4}x^2 \quad \Rightarrow \quad \begin{matrix} 4y = x^2 \\ x = \sqrt{4y} \end{matrix} \quad x = 2$$

$$r_o = (2) + 0 = (2) \quad r_i = (\sqrt{4y}) + 0 = (\sqrt{4y})$$

$$A_o = \pi(2)^2 = \pi(4) \quad A_i = \pi(\sqrt{4y})^2 = \pi(4y)$$

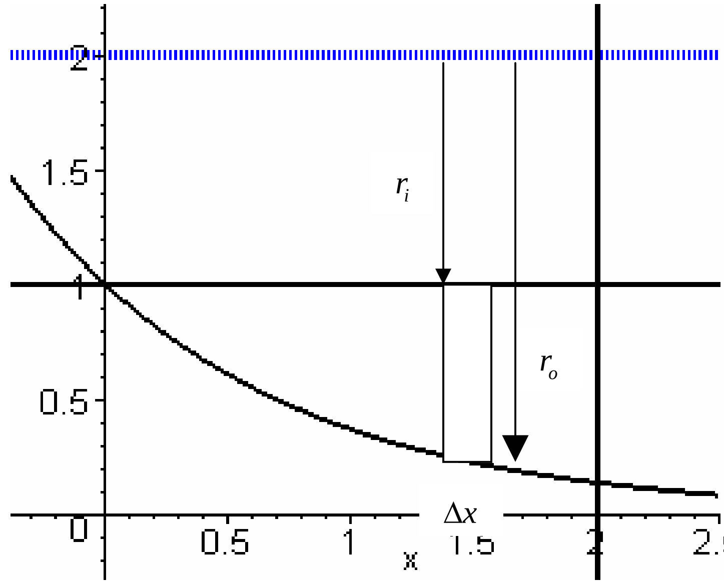
$$A = A_o - A_i = \pi(4) - \pi(4y) = \pi(4 - 4y)$$

$$\Delta V = A\Delta y = \pi(4 - 4y)\Delta y$$

$$V = \int_0^1 \pi(4 - 4y)\, dy = \pi\left[4y - 2y^2 + C\right]_0^1 = \pi\left\{\left[4(1) - 2(1)^2 + C\right] - \left[4(0) - 2(0)^2 + C\right]\right\}$$

$$= \pi\left\{[4 - 2] - [0]\right\} = 2\pi\ units^3$$

10) $y = e^{-x} \quad y = 1 \quad x = 2$; about $y = 2$



interval: $0 \le x \le 2$

$r_o = 2 - (e^{-x}) = (2 - e^{-x})$ $\quad r_i = 2 - (1) = (1)$

$A_o = \pi(2 - e^{-x})^2 = \pi(4 - 4e^{-x} + e^{-2x})$ $\quad A_i = \pi(1)^2 = \pi(1)$

$A = A_o - A_i = \pi(4 - 4e^{-x} + e^{-2x}) - \pi(1) = \pi(3 - 4e^{-x} + e^{-2x})$

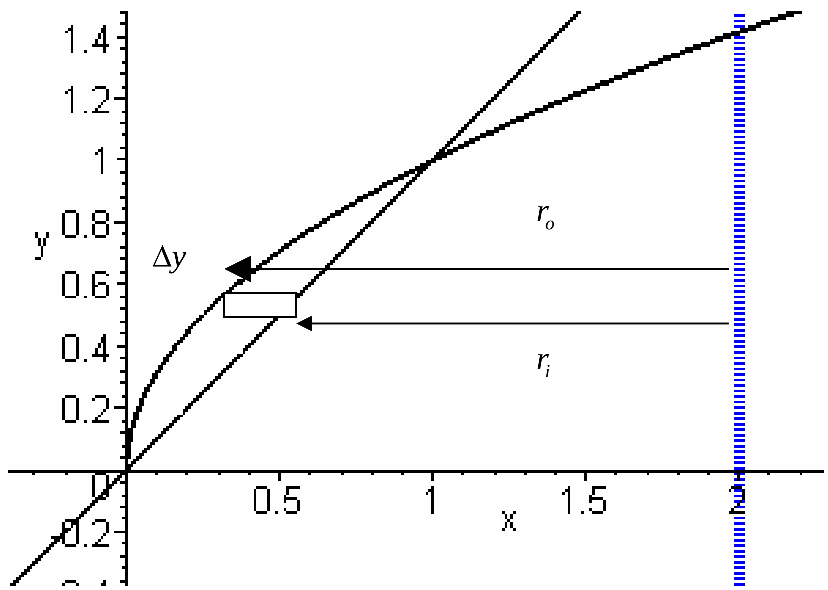
$\Delta V = A\Delta x = \pi(3 - 4e^{-x} + e^{-2x})\Delta x$

$$V = \int_0^2 \pi(3 - 4e^{-x} + e^{-2x})\, dx = \pi\left[3x + 4e^{-x} - \frac{1}{2}e^{-2x} + C\right]_0^2 = \pi\left[3x + \frac{4}{e^x} - \frac{1}{2e^{2x}} + C\right]_0^2$$

$$= \pi\left\{\left[3(2) + \frac{4}{e^{(2)}} - \frac{1}{2e^{2(2)}} + C\right] - \left[3(0) + \frac{4}{e^{(0)}} - \frac{1}{2e^{2(0)}} + C\right]\right\}$$

$$= \pi\left\{\left[6 + \frac{4}{e^2} - \frac{1}{2e^4}\right] - \left[0 + \frac{4}{1} - \frac{1}{2(1)}\right]\right\} = \pi\left\{6 + \frac{4}{e^2} - \frac{1}{2e^4} - \frac{7}{2}\right\} = \pi\left(\frac{5}{2} + \frac{4}{e^2} - \frac{1}{2e^4}\right) units^3$$

12) $y = x \quad y = \sqrt{x}$; about $x = 2$



$y = x \quad y = \sqrt{x}$

$x = y \quad x = y^2$

$y = y^2$

$0 = y^2 - y \quad \Rightarrow \quad y = 0 \quad y - 1 = 0$

$0 = y(y-1) \qquad y = 1$

interval: $0 \le y \le 1$

$r_o = 2 - (y^2) = (2 - y^2)$ $\quad r_i = 2 - (y) = (2 - y)$

$A_o = \pi(2 - y^2)^2 = \pi(4 - 4y^2 + y^4)$ $\quad A_i = \pi(2 - y)^2 = \pi(4 - 4y + y^2)$

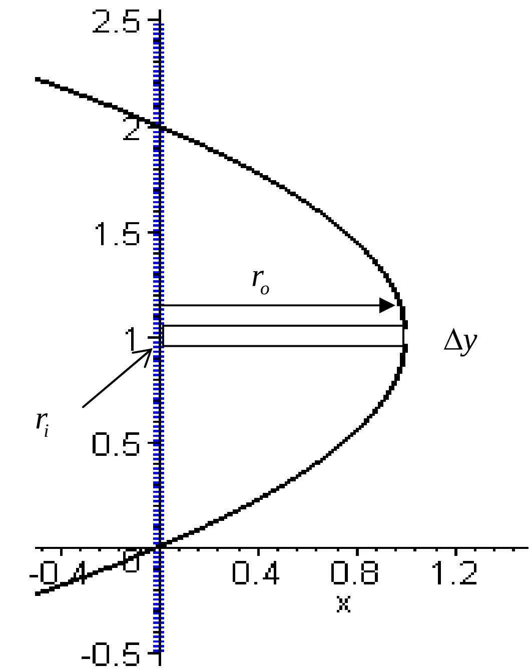
$A = A_o - A_i = \pi(4 - 4y^2 + y^4) - \pi(4 - 4y + y^2) = \pi(y^4 - 5y^2 + 4y)$

$\Delta V = A\Delta y = \pi(y^4 - 5y^2 + 4y)\Delta y$

$$V = \int_0^1 \pi(y^4 - 5y^2 + 4y)\, dy = \pi\left[\frac{1}{5}y^5 - \frac{5}{3}y^3 + 2y^2 + C\right]_0^1$$

$$= \pi\left\{\left[\frac{1}{5}(1)^5 - \frac{5}{3}(1)^3 + 2(1)^2 + C\right] - \left[\frac{1}{5}(0)^5 - \frac{5}{3}(0)^3 + 2(0)^2 + C\right]\right\} = \pi\left\{\left[\frac{1}{5} - \frac{5}{3} + 2\right] - [0]\right\} = \frac{8\pi}{15}\, units^3$$

14) $x = 2y - y^2 \quad x = 0$; about y-axis



$$0 = 2y - y^2 \qquad \Rightarrow \quad y = 0 \qquad 2 - y = 0$$
$$0 = y(2-y) \qquad\qquad\qquad y = 2$$

interval: $0 \le y \le 2$

$r_o = (2y - y^2) + 0 \quad r_i = (0) + 0$
$\quad = (2y - y^2) \qquad\quad = (0)$

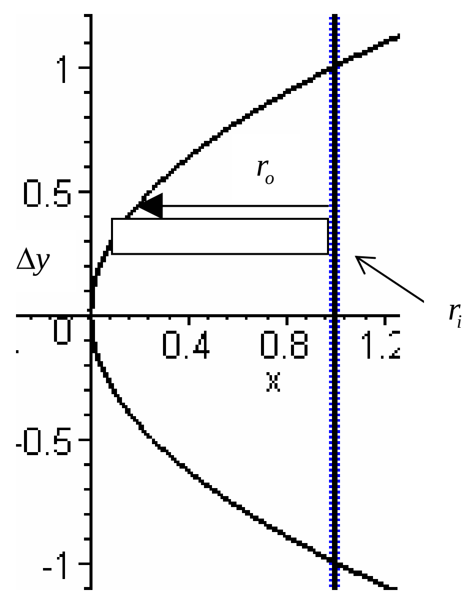
$A_o = \pi(2y - y^2)^2 \qquad A_i = \pi(0)^2$
$\quad = \pi(4y^2 - 4y^3 + y^4) \qquad = \pi(0)$

$A = A_o - A_i = \pi(4y^2 - 4y^3 + y^4) - \pi(0)$
$\quad = \pi(4y^2 - 4y^3 + y^4)$

$\Delta V = A\Delta y = \pi(4y^2 - 4y^3 + y^4)\Delta y$

$$V = \int_0^2 \pi(4y^2 - 4y^3 + y^4)\,dy = \pi\left[\frac{4}{3}y^3 - y^4 + \frac{1}{5}y^5 + C\right]_0^2$$
$$= \pi\left\{\left[\frac{4}{3}(2)^3 - (2)^4 + \frac{1}{5}(2)^5 + C\right] - \left[\frac{4}{3}(0)^3 - (0)^4 + \frac{1}{5}(0)^5 + C\right]\right\}$$
$$= \pi\left\{\left[\frac{2}{3}(2)^4 - (2)^4 + \frac{2}{5}(2)^4\right] - [0]\right\} = \pi(2)^4\left\{\frac{2}{3} - 1 + \frac{2}{5}\right\} = \pi(2)^4\left\{\frac{10}{15} - \frac{15}{15} + \frac{6}{15}\right\} = \pi(2)^4\left\{\frac{1}{15}\right\} = \frac{16\pi}{15}\ units^3$$

16) $x = y^2 \quad x = 1$; about $x = 1$



$$1 = y^2$$
$$0 = y^2 - 1 \qquad \Rightarrow \quad y + 1 = 0 \quad y - 1 = 0$$
$$0 = (y+1)(y-1) \qquad y = -1 \qquad y = 1$$

interval: $-1 \le y \le 1$

$r_o = 1 - (y^2) = (1 - y^2) \quad r_i = 1 - (1) = (0)$

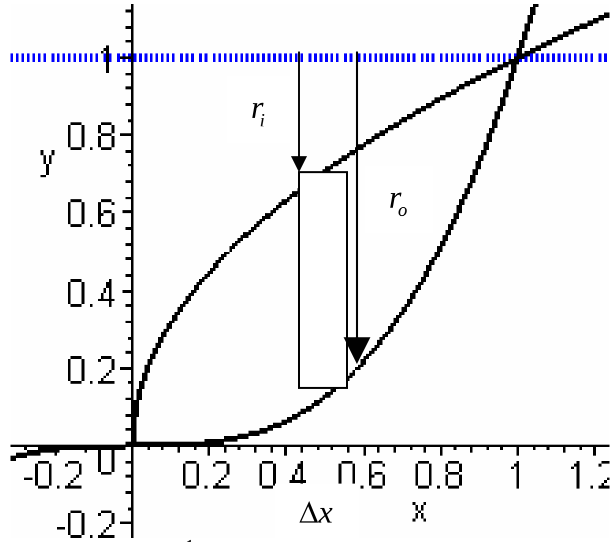
$A_o = \pi(1 - y^2)^2 = \pi(1 - 2y^2 + y^4) \quad A_i = \pi(0)^2 = \pi(0)$

$A = A_o - A_i = \pi(1 - 2y^2 + y^4) - \pi(0) = \pi(1 - 2y^2 + y^4)$

$\Delta V = A\Delta y = \pi(1 - 2y^2 + y^4)\Delta y$

$$V = \int_{-1}^1 \pi(1 - 2y^2 + y^4)\,dy = \pi\left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5 + C\right]_{-1}^1$$
$$= \pi\left\{\left[(1) - \frac{2}{3}(1)^3 + \frac{1}{5}(1)^5 + C\right] - \left[(-1) - \frac{2}{3}(-1)^3 + \frac{1}{5}(-1)^5 + C\right]\right\} = \pi\left\{\left[1 - \frac{2}{3} + \frac{1}{5}\right] - \left[-1 + \frac{2}{3} - \frac{1}{5}\right]\right\}$$
$$= \pi\left\{2 - \frac{4}{3} + \frac{2}{5}\right\} = \pi\left\{\frac{30}{15} - \frac{20}{15} + \frac{6}{15}\right\} = \frac{16\pi}{15}\ units^3$$

18) $y = x^3 \quad y = \sqrt{x}$; about $y = 1$



interval: $0 \le x \le 1$

$r_o = 1 - (x^3) = (1 - x^3) \quad r_i = 1 - (\sqrt{x}) = (1 - \sqrt{x})$

$A_o = \pi(1 - x^3)^2 \qquad A_i = \pi(1 - \sqrt{x})^2$

$= \pi(1 - 2x^3 + x^6) \qquad = \pi(1 - 2\sqrt{x} + x)$

$A = A_o - A_i = \pi(1 - 2x^3 + x^6) - \pi(1 - 2\sqrt{x} + x)$

$= \pi(x^6 - 2x^3 - x + 2\sqrt{x})$

$\Delta V = A\Delta x$

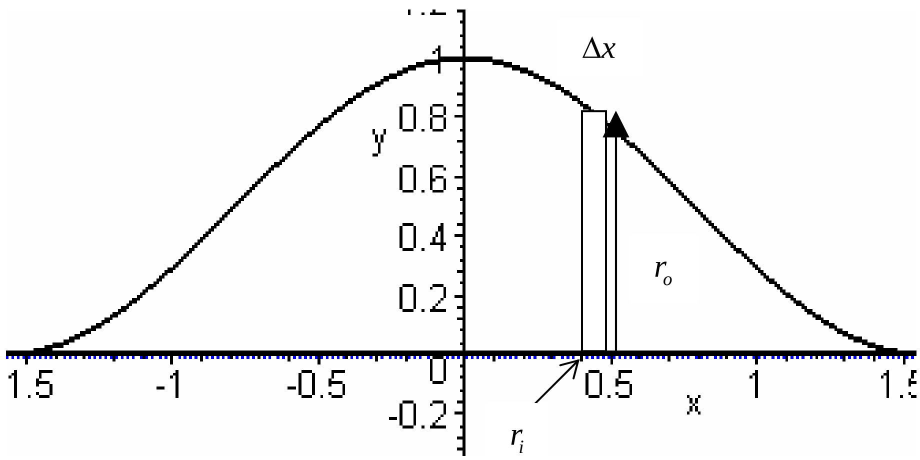
$= \pi(x^6 - 2x^3 - x + 2\sqrt{x})\Delta x$

$$V = \int_0^1 \pi(x^6 - 2x^3 - x + 2\sqrt{x})\, dx = \pi\left[\frac{1}{7}x^7 - \frac{1}{2}x^4 - \frac{1}{2}x^2 + \frac{4}{3}(\sqrt{x})^3 + C\right]_0^1$$

$$= \pi\left\{\left[\frac{1}{7}(1)^7 - \frac{1}{2}(1)^4 - \frac{1}{2}(1)^2 + \frac{4}{3}(\sqrt{(1)})^3 + C\right] - \left[\frac{1}{7}(0)^7 - \frac{1}{2}(0)^4 - \frac{1}{2}(0)^2 + \frac{4}{3}(\sqrt{(0)})^3 + C\right]\right\}$$

$$= \pi\left\{\left[\frac{1}{7} - \frac{1}{2} - \frac{1}{2} + \frac{4}{3}\right] - [0]\right\} = \pi\left\{\frac{1}{7} - 1 + \frac{4}{3}\right\} = \pi\left\{\frac{1}{7} + \frac{1}{3}\right\} = \pi\left\{\frac{3}{21} + \frac{7}{21}\right\} = \frac{10\pi}{21}\ units^3$$

20) $y = 0 \quad y = \cos^2 x \quad \frac{-\pi}{2} \le x \le \frac{\pi}{2}$



a) about x-axis

interval: $\frac{-\pi}{2} \le x \le \frac{\pi}{2}$

$r_o = (\cos^2 x) + 0 \quad r_i = (0) + 0$

$= (\cos^2 x) \qquad = (0)$

$A_o = \pi(\cos^2 x)^2 \quad A_i = \pi(0)^2$

$= \pi(\cos^4 x) \qquad = \pi(0)$

$A = A_o - A_i = \pi(\cos^4 x) - \pi(0) = \pi(\cos^4 x)$

$\Delta V = A\Delta x = \pi(\cos^4 x)\Delta x = \pi(\cos^2 x)^2 \Delta x$

$$= \pi\left(\frac{1}{2}(1 + \cos(2x))\right)^2 \Delta x$$

$$= \frac{\pi}{4}(1 + 2\cos(2x) + \cos^2(2x))\Delta x = \frac{\pi}{4}\left(1 + 2\cos(2x) + \left(\frac{1}{2}(1 + \cos(4x))\right)\right)\Delta x$$

$$= \frac{\pi}{4}\left(\frac{3}{2} + 2\cos(2x) + \frac{1}{2}\cos(4x)\right)\Delta x$$

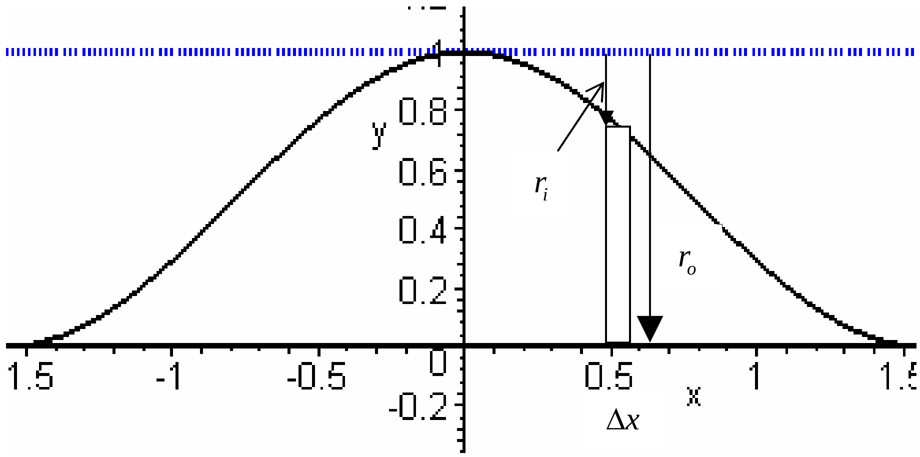
$$V=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\pi}{4}\left(\frac{3}{2}+2\cos(2x)+\frac{1}{2}\cos(4x)\right)dx=\frac{\pi}{4}\left[\frac{3}{2}x+\sin(2x)+\frac{1}{8}\sin(4x)+C\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{\pi}{4}\left\{\left[\frac{3}{2}\left(\frac{\pi}{2}\right)+\sin\left(2\left(\frac{\pi}{2}\right)\right)+\frac{1}{8}\sin\left(4\left(\frac{\pi}{2}\right)\right)+C\right]-\left[\frac{3}{2}\left(\frac{-\pi}{2}\right)+\sin\left(2\left(\frac{-\pi}{2}\right)\right)+\frac{1}{8}\sin\left(4\left(\frac{-\pi}{2}\right)\right)+C\right]\right\}$$

$$=\frac{\pi}{4}\left\{\left[\frac{3\pi}{4}\right]-\left[\frac{-3\pi}{4}\right]\right\}=\frac{\pi}{4}\left\{\frac{6\pi}{4}\right\}=\frac{\pi}{4}\left\{\frac{3\pi}{2}\right\}=\frac{3\pi^2}{8}\,units^3$$

b) about $y=1$

interval: $\frac{-\pi}{2}\le x\le\frac{\pi}{2}$



$r_o=1-(0)$
$=(1)$

$r_i=1-(\cos^2 x)$
$=(1-\cos^2 x)$
$=\sin^2 x$

$A_o=\pi(1)^2$
$=\pi(1)$

$A_i=\pi(\sin^2 x)^2$
$=\pi(\sin^4 x)$

$A=A_o-A_i=\pi(1)-\pi(\sin^4 x)$
$=\pi(1-\sin^4 x)$

$$\Delta V=A\Delta x=\pi(1-\sin^4 x)\Delta x=\pi(1-(\sin^2 x)^2)\Delta x=\pi\left(1-\left(\frac{1}{2}(1-\cos(2x))\right)^2\right)\Delta x$$

$$=\pi\left(1-\frac{1}{4}(1-2\cos(2x)+\cos^2(2x))\right)\Delta x=\pi\left(\frac{3}{4}+\frac{1}{2}\cos(2x)-\frac{1}{4}\cos^2(2x)\right)\Delta x$$

$$=\pi\left(\frac{3}{4}+\frac{1}{2}\cos(2x)-\frac{1}{4}\left(\frac{1}{2}(1+\cos(4x))\right)\right)\Delta x=\pi\left(\frac{5}{8}+\frac{1}{2}\cos(2x)-\frac{1}{8}\cos(4x)\right)\Delta x$$

$$=\frac{\pi}{8}(5+4\cos(2x)-\cos(4x))\Delta x$$

$$V=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\frac{\pi}{8}(5+4\cos(2x)-\cos(4x))\,dx=\frac{\pi}{8}\left[5x+2\sin(2x)-\frac{1}{4}\sin(4x)+C\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{\pi}{8}\left\{\left[5\left(\frac{\pi}{2}\right)+2\sin\left(2\left(\frac{\pi}{2}\right)\right)-\frac{1}{4}\sin\left(4\left(\frac{\pi}{2}\right)\right)+C\right]-\left[5\left(\frac{-\pi}{2}\right)+2\sin\left(2\left(\frac{-\pi}{2}\right)\right)-\frac{1}{4}\sin\left(4\left(\frac{-\pi}{2}\right)\right)+C\right]\right\}$$

$$=\frac{\pi}{8}\left\{\left[\frac{5\pi}{2}\right]-\left[\frac{-5\pi}{2}\right]\right\}=\frac{\pi}{8}\left\{\frac{10\pi}{2}\right\}=\frac{\pi}{8}\{5\pi\}=\frac{5\pi}{8}\,units^3$$

22) $y = x^2 \quad x^2 + y^2 = 1 \quad y \ge 0$

intersection points:

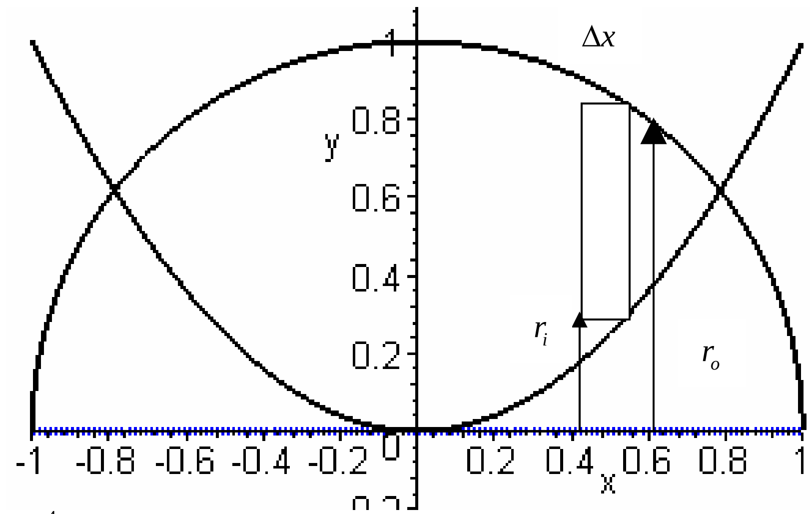
$$x^2 + (x^2)^2 = 1$$
$$x^4 + x^2 - 1 = 0$$
$$x^2 = \frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\Rightarrow \quad x^2 = \frac{-1-\sqrt{5}}{2} \qquad x^2 = \frac{-1+\sqrt{5}}{2}$$
$$x = \pm\sqrt{\frac{-1-\sqrt{5}}{2}} \qquad x = \pm\sqrt{\frac{-1+\sqrt{5}}{2}}$$

discard

$$x = -\sqrt{\frac{\sqrt{5}-1}{2}} \quad x = +\sqrt{\frac{\sqrt{5}-1}{2}}$$

a) about x-axis



interval: $-\sqrt{\frac{\sqrt{5}-1}{2}} \le x \le +\sqrt{\frac{\sqrt{5}-1}{2}}$

$$y = x^2 \qquad x^2 + y^2 = 1 \qquad y \ge 0$$
$$y = \sqrt{1-x^2}$$

$$r_o = (\sqrt{1-x^2}) + 0 \qquad r_i = (x^2) + 0$$
$$= (\sqrt{1-x^2}) \qquad = (x^2)$$
$$A_o = \pi(\sqrt{1-x^2})^2 \qquad A_i = \pi(x^2)^2$$
$$= \pi(1-x^2) \qquad = \pi(x^4)$$
$$A = A_o - A_i = \pi(1-x^2) - \pi(x^4) = \pi(1-x^2-x^4)$$
$$\Delta V = A\Delta x = \pi(1-x^2-x^4)\Delta x$$

$$V = \int_{-\sqrt{\frac{\sqrt{5}-1}{2}}}^{+\sqrt{\frac{\sqrt{5}-1}{2}}} \pi(1-x^2-x^4)\,dx = \pi\left[x - \frac{1}{3}x^3 - \frac{1}{5}x^5 + C\right]_{-\sqrt{\frac{\sqrt{5}-1}{2}}}^{+\sqrt{\frac{\sqrt{5}-1}{2}}}$$

$$= \pi\left\{\left[\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right) - \frac{1}{3}\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^3 - \frac{1}{5}\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^5 + C\right] - \left[\left(-\sqrt{\frac{\sqrt{5}-1}{2}}\right) - \frac{1}{3}\left(-\sqrt{\frac{\sqrt{5}-1}{2}}\right)^3 - \frac{1}{5}\left(-\sqrt{\frac{\sqrt{5}-1}{2}}\right)^5 + C\right]\right\}$$

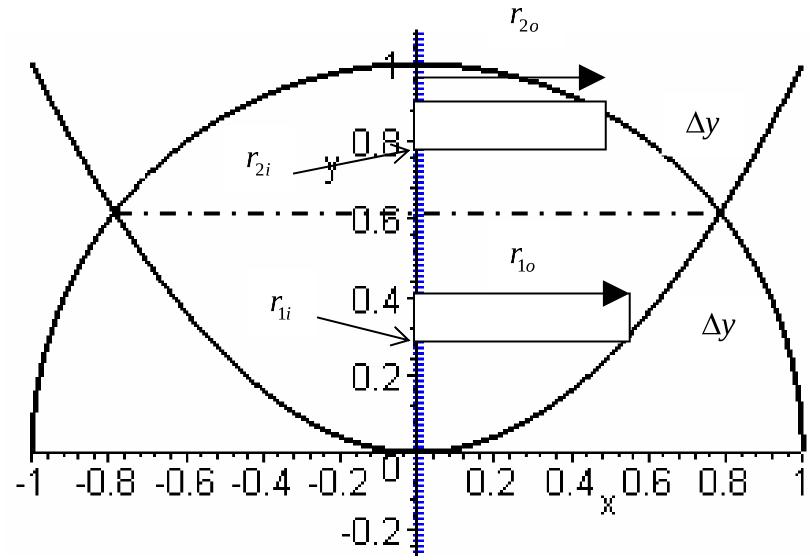
$$= \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{\left[1 - \frac{1}{3}\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^2 - \frac{1}{5}\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^4\right] - \left[-1 + \frac{1}{3}\left(-\sqrt{\frac{\sqrt{5}-1}{2}}\right)^2 + \frac{1}{5}\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)^4\right]\right\}$$

$$= \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{2 - \frac{2}{3}\left(\frac{\sqrt{5}-1}{2}\right) - \frac{2}{5}\left(\frac{\sqrt{5}-1}{2}\right)^2\right\} = \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{2 - \frac{\sqrt{5}-1}{3} - \frac{(5-2\sqrt{5}+1)}{10}\right\}$$

$$= \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{2 - \frac{\sqrt{5}-1}{3} - \frac{3-\sqrt{5}}{5}\right\} = \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{\frac{30}{15} - \frac{5\sqrt{5}-5}{15} - \frac{9-3\sqrt{5}}{15}\right\} = \pi\left(\sqrt{\frac{\sqrt{5}-1}{2}}\right)\left\{\frac{26-2\sqrt{5}}{15}\right\} \text{units}^3$$

b) about y-axis

This volume must be computed in 2 pieces.



$$y = \left(+\sqrt{\frac{\sqrt{5}-1}{2}}\right)^2 = \frac{\sqrt{5}-1}{2}$$

interval 1: $0 \le y \le \frac{\sqrt{5}-1}{2}$

$y = x^2 \quad x^2 + y^2 = 1 \quad y \ge 0$

$x = \sqrt{y} \quad x = \sqrt{1-y^2}$

$r_{1o} = (\sqrt{y}) + 0 = (\sqrt{y}) \quad r_{1i} = (0) + 0 = (0)$

$A_{1o} = \pi(\sqrt{y})^2 = \pi(y) \quad A_{1i} = \pi(0)^2 = \pi(0)$

$A_1 = A_{1o} - A_{1i} = \pi(y) - \pi(0) = \pi(y)$

$\Delta V_1 = A_1 \Delta y = \pi(y)\Delta y$

$$V_1 = \int_0^{\frac{\sqrt{5}-1}{2}} \pi(y)\,dy = \pi\left[\frac{1}{2}y^2 + C\right]_0^{\frac{\sqrt{5}-1}{2}} = \pi\left\{\left[\frac{1}{2}\left(\frac{\sqrt{5}-1}{2}\right)^2 + C\right] - \left[\frac{1}{2}(0)^2 + C\right]\right\} = \frac{\pi}{2}\left\{\frac{5-2\sqrt{5}+1}{4}\right\}$$

$$= \frac{\pi}{2}\left\{\frac{6-2\sqrt{5}}{4}\right\} = \pi\left(\frac{3-\sqrt{5}}{4}\right) units^3$$

interval 2: $\frac{\sqrt{5}-1}{2} \le y \le 1$

$r_{2o} = (\sqrt{1-y^2}) + 0 = (\sqrt{1-y^2}) \quad r_{2i} = (0) + 0 = (0)$

$A_{2o} = \pi(\sqrt{1-y^2})^2 = \pi(1-y^2) \quad A_{2i} = \pi(0)^2 = \pi(0)$

$A_2 = A_{2o} - A_{2i} = \pi(1-y^2) - \pi(0) = \pi(1-y^2)$

$\Delta V_2 = A_2 \Delta y = \pi(1-y^2)\Delta y$

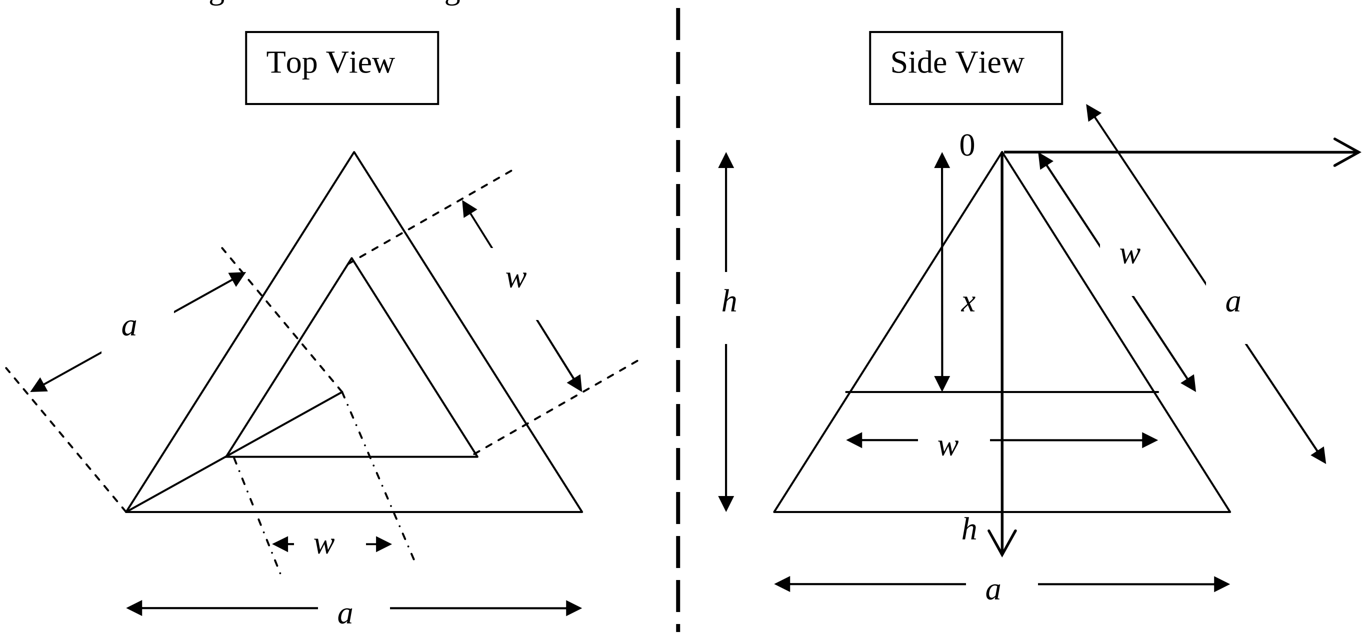
$$V_2 = \int_{\frac{\sqrt{5}-1}{2}}^{1} \pi(1-y^2)\,dy = \pi\left[y - \frac{1}{3}y^3 + C\right]_{\frac{\sqrt{5}-1}{2}}^{1} = \pi\left\{\left[(1) - \frac{1}{3}(1)^3 + C\right] - \left[\left(\frac{\sqrt{5}-1}{2}\right) - \frac{1}{3}\left(\frac{\sqrt{5}-1}{2}\right)^3 + C\right]\right\}$$

$$= \pi\left\{\left[1-\frac{1}{3}\right] - \left(\frac{\sqrt{5}-1}{2}\right)\left[1 - \frac{1}{3}\left(\frac{\sqrt{5}-1}{2}\right)^2\right]\right\} = \pi\left\{\left[\frac{2}{3}\right] - \left(\frac{\sqrt{5}-1}{2}\right)\left[1 - \frac{1}{3}\left(\frac{6-2\sqrt{5}}{4}\right)\right]\right\}$$

$$= \pi\left\{\left[\frac{2}{3}\right] - \left(\frac{\sqrt{5}-1}{2}\right)\left[\frac{6}{6} - \frac{3-\sqrt{5}}{6}\right]\right\} = \pi\left\{\left[\frac{2}{3}\right] - \left(\frac{\sqrt{5}-1}{2}\right)\left[\frac{3+\sqrt{5}}{6}\right]\right\} = \pi\left\{\left[\frac{2}{3}\right] - \left(\frac{1+\sqrt{5}}{6}\right)\right\} = \pi\left(\frac{3-\sqrt{5}}{6}\right) units^3$$

$$V = V_1 + V_2 = \pi\left(\frac{3-\sqrt{5}}{4}\right) + \pi\left(\frac{3-\sqrt{5}}{6}\right) = \pi\left\{\left(\frac{9-3\sqrt{5}}{12}\right) + \left(\frac{6-2\sqrt{5}}{12}\right)\right\} = \pi\left(\frac{15-5\sqrt{5}}{12}\right) units^3$$

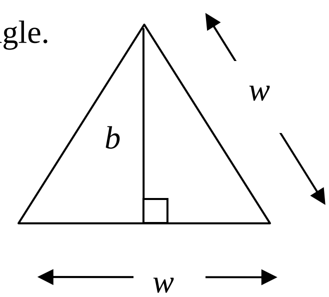
36) For the 3D representation, see the figure in the text. Also, setting the positive *x*-axis to go down we have the following cross section diagrams.



From the side view we can set up a ratio to obtain:

$$\frac{w}{a}=\frac{x}{h} \quad \Rightarrow \quad w=\frac{a}{h}x$$

From Top View, we can see that we need to set up an area of triangle.



$$\frac{b}{w}=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2} \quad \Rightarrow \quad b=\frac{\sqrt{3}}{2}w$$

$$A_{\vartriangle}=\frac{1}{2}bw=\frac{1}{2}\left(\frac{\sqrt{3}}{2}w\right)w=\frac{\sqrt{3}}{4}w^2=\frac{\sqrt{3}}{4}\left(\frac{a}{h}x\right)^2=\frac{\sqrt{3}a^2}{4h^2}x^2$$

$$\Delta V=A_{\vartriangle}\Delta x=\left(\frac{\sqrt{3}a^2}{4h^2}x^2\right)\Delta x$$

$$V=\int_0^h\left(\frac{\sqrt{3}a^2}{4h^2}x^2\right)dx=\left[\frac{\sqrt{3}a^2}{12h^2}x^3+C\right]_0^h=\left[\frac{\sqrt{3}a^2}{12h^2}(h)^3+C\right]-\left[\frac{\sqrt{3}a^2}{12h^2}(0)^3+C\right]$$

$$=\left[\frac{\sqrt{3}a^2h}{12}\right]-[0]=\frac{a^2h}{4\sqrt{3}}\,units^3$$