

Decoding the rules: $\Delta x = \frac{b-a}{n}$ $x_i = a + i\Delta x$ $\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$

Midpoint Rule: $\int_a^b f(x) dx \approx M_n = \frac{\Delta x}{1} [f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + \dots + f(\bar{x}_{n-1}) + f(\bar{x}_n)]$

Trapezoidal Rule: $\int_a^b f(x) dx \approx T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \dots + 2f(x_{n-1}) + f(x_n)]$

Simpson's Rule: $\int_a^b f(x) dx \approx S_n = \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$

The value of n must be even for Simpson's rule to work.

Error Bounds: For $a \leq x \leq b$

Midpoint error: $|E_M| \leq \frac{K(b-a)^3}{24n^2}$ $|f''(x)| \leq K$

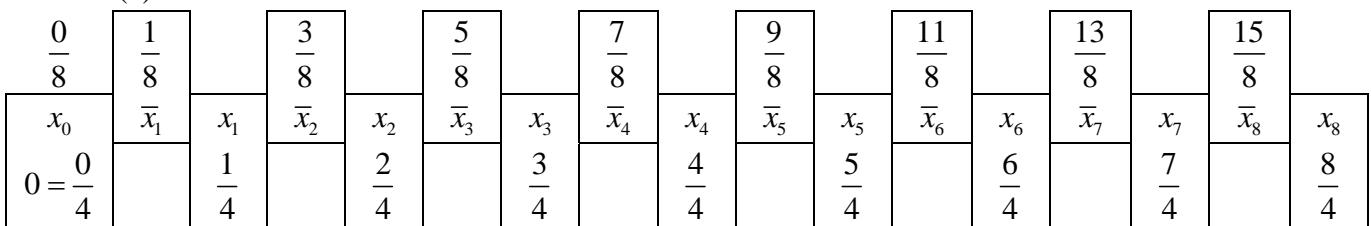
Trapezoidal error: $|E_T| \leq \frac{K(b-a)^3}{12n^2}$ $|f''(x)| \leq K$

Simpson error: $|E_S| \leq \frac{K(b-a)^5}{180n^4}$ $|f^{(4)}(x)| \leq K$

Additional examples:

8) $\int_0^2 \frac{1}{1+x^6} dx$ $n=8$

$$\Delta x = \frac{(2)-(0)}{(8)} = \frac{2}{8} = \frac{1}{4} \quad \frac{\Delta x}{2} = \frac{1}{8}$$



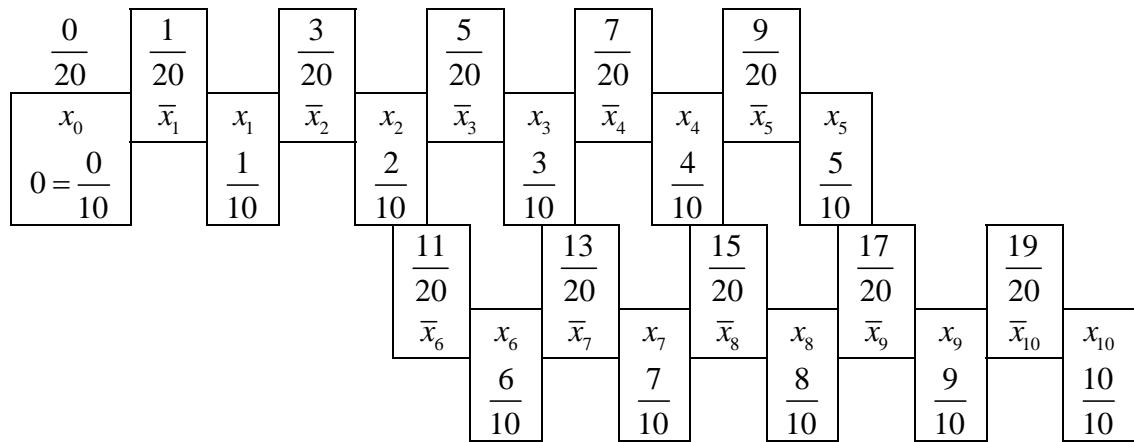
$$M_8 = \frac{\left(\frac{1}{4}\right)}{1} \left[\frac{1}{1+\left(\frac{1}{8}\right)^6} + \frac{1}{1+\left(\frac{3}{8}\right)^6} + \frac{1}{1+\left(\frac{5}{8}\right)^6} + \frac{1}{1+\left(\frac{7}{8}\right)^6} + \frac{1}{1+\left(\frac{9}{8}\right)^6} + \frac{1}{1+\left(\frac{11}{8}\right)^6} + \frac{1}{1+\left(\frac{13}{8}\right)^6} + \frac{1}{1+\left(\frac{15}{8}\right)^6} \right]$$

$$T_8 = \frac{\left(\frac{1}{4}\right)}{2} \left[\frac{1}{1+\left(\frac{1}{4}\right)^6} + 2 \left(\frac{1}{1+\left(\frac{2}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{3}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{4}{4}\right)^6} \right) \right. \\ \left. + 2 \left(\frac{1}{1+\left(\frac{5}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{6}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{7}{4}\right)^6} \right) + \frac{1}{1+\left(\frac{8}{4}\right)^6} \right]$$

$$S_8 = \frac{\left(\frac{1}{4}\right)}{3} \left[\frac{1}{1+\left(\frac{1}{4}\right)^6} + 4 \left(\frac{1}{1+\left(\frac{2}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{3}{4}\right)^6} \right) + 4 \left(\frac{1}{1+\left(\frac{4}{4}\right)^6} \right) \right. \\ \left. + 2 \left(\frac{1}{1+\left(\frac{5}{4}\right)^6} \right) + 4 \left(\frac{1}{1+\left(\frac{6}{4}\right)^6} \right) + 2 \left(\frac{1}{1+\left(\frac{7}{4}\right)^6} \right) + \frac{1}{1+\left(\frac{8}{4}\right)^6} \right]$$

12) $\int_0^1 \sin(x^3) dx \quad n=10$

$$\Delta x = \frac{(1)-(0)}{(10)} = \frac{1}{10} = \frac{2}{20} \quad \frac{\Delta x}{2} = \frac{1}{20}$$



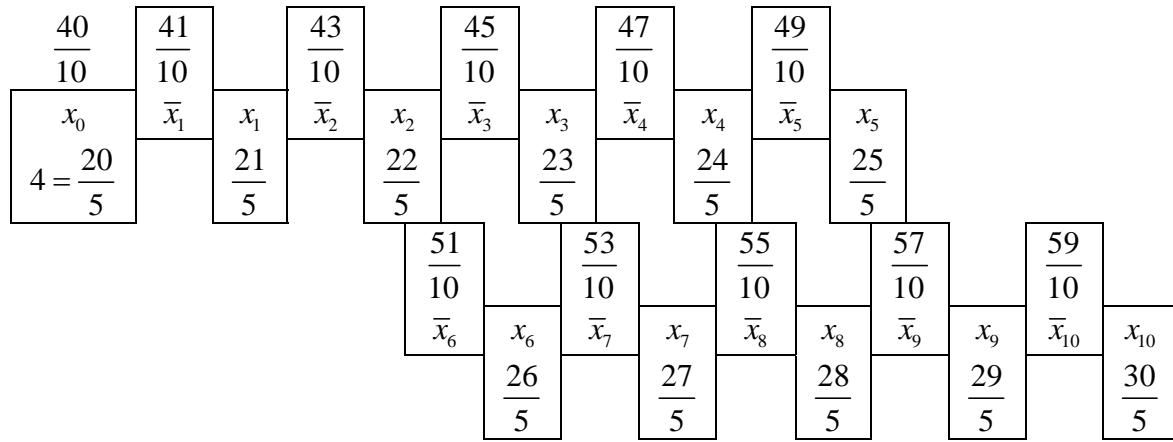
$$M_{10} = \frac{\left(\frac{1}{10}\right)}{1} \left[\sin\left(\frac{1}{20}\right) + \sin\left(\frac{3}{20}\right) + \sin\left(\frac{5}{20}\right) + \sin\left(\frac{7}{20}\right) + \sin\left(\frac{9}{20}\right) \right. \\ \left. + \sin\left(\frac{11}{20}\right) + \sin\left(\frac{13}{20}\right) + \sin\left(\frac{15}{20}\right) + \sin\left(\frac{17}{20}\right) + \sin\left(\frac{19}{20}\right) \right]$$

$$T_{10} = \frac{\left(\frac{1}{10}\right)}{2} \left[\sin\left(\frac{0}{10}\right) + 2\left(\sin\left(\frac{1}{10}\right)\right) + 2\left(\sin\left(\frac{2}{10}\right)\right) + 2\left(\sin\left(\frac{3}{10}\right)\right) + 2\left(\sin\left(\frac{4}{10}\right)\right) + 2\left(\sin\left(\frac{5}{10}\right)\right) + 2\left(\sin\left(\frac{6}{10}\right)\right) + 2\left(\sin\left(\frac{7}{10}\right)\right) + 2\left(\sin\left(\frac{8}{10}\right)\right) + 2\left(\sin\left(\frac{9}{10}\right)\right) + \sin\left(\frac{10}{10}\right) \right]$$

$$S_{10} = \frac{\left(\frac{1}{10}\right)}{2} \left[\sin\left(\frac{0}{10}\right) + 4\left(\sin\left(\frac{1}{10}\right)\right) + 2\left(\sin\left(\frac{2}{10}\right)\right) + 4\left(\sin\left(\frac{3}{10}\right)\right) + 2\left(\sin\left(\frac{4}{10}\right)\right) + 4\left(\sin\left(\frac{5}{10}\right)\right) + 2\left(\sin\left(\frac{6}{10}\right)\right) + 4\left(\sin\left(\frac{7}{10}\right)\right) + 2\left(\sin\left(\frac{8}{10}\right)\right) + 4\left(\sin\left(\frac{9}{10}\right)\right) + \sin\left(\frac{10}{10}\right) \right]$$

14) $\int_4^6 \ln(x^3 + 2) dx \quad n = 10$

$$\Delta x = \frac{(6)-(4)}{(10)} = \frac{2}{10} = \frac{1}{5} \quad \frac{\Delta x}{2} = \frac{1}{10}$$



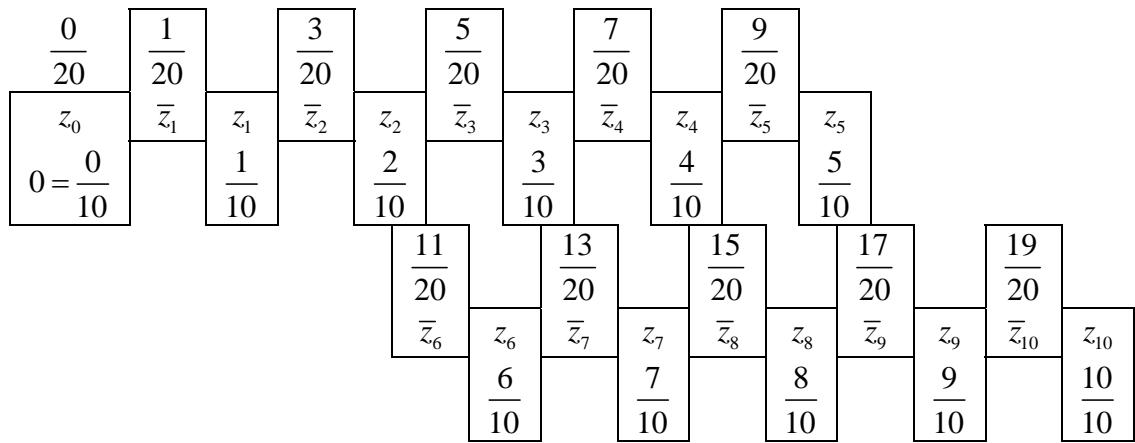
$$M_{10} = \frac{\left(\frac{1}{5}\right)}{1} \left[\ln\left(\left(\frac{41}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{43}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{45}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{47}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{49}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{51}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{53}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{55}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{57}{10}\right)^3 + 2\right) + \ln\left(\left(\frac{59}{10}\right)^3 + 2\right) \right]$$

$$T_{10} = \frac{\left(\frac{1}{5}\right)}{2} \left[\ln\left(\left(\frac{20}{5}\right)^3 + 2\right) + 2\left(\ln\left(\left(\frac{21}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{22}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{23}{5}\right)^3 + 2\right)\right) \right. \\ \left. + 2\left(\ln\left(\left(\frac{24}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{25}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{26}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{27}{5}\right)^3 + 2\right)\right) \right. \\ \left. + 2\left(\ln\left(\left(\frac{28}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{29}{5}\right)^3 + 2\right)\right) + \ln\left(\left(\frac{30}{5}\right)^3 + 2\right) \right]$$

$$S_{10} = \frac{\left(\frac{1}{5}\right)}{2} \left[\ln\left(\left(\frac{20}{5}\right)^3 + 2\right) + 4\left(\ln\left(\left(\frac{21}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{22}{5}\right)^3 + 2\right)\right) + 4\left(\ln\left(\left(\frac{23}{5}\right)^3 + 2\right)\right) \right. \\ \left. + 2\left(\ln\left(\left(\frac{24}{5}\right)^3 + 2\right)\right) + 4\left(\ln\left(\left(\frac{25}{5}\right)^3 + 2\right)\right) + 2\left(\ln\left(\left(\frac{26}{5}\right)^3 + 2\right)\right) + 4\left(\ln\left(\left(\frac{27}{5}\right)^3 + 2\right)\right) \right. \\ \left. + 2\left(\ln\left(\left(\frac{28}{5}\right)^3 + 2\right)\right) + 4\left(\ln\left(\left(\frac{29}{5}\right)^3 + 2\right)\right) + \ln\left(\left(\frac{30}{5}\right)^3 + 2\right) \right]$$

16) $\int_0^1 \sqrt{z} e^{-z} dx = \int_0^1 \frac{\sqrt{z}}{e^z} dx \quad n=10$

$$\Delta x = \frac{(1)-(0)}{(10)} = \frac{1}{10} = \frac{2}{20} \quad \frac{\Delta x}{2} = \frac{1}{20}$$



$$M_{10} = \frac{1}{10} \left[\frac{\sqrt{\left(\frac{1}{20}\right)}}{e^{\left(\frac{1}{20}\right)}} + \frac{\sqrt{\left(\frac{3}{20}\right)}}{e^{\left(\frac{3}{20}\right)}} + \frac{\sqrt{\left(\frac{5}{20}\right)}}{e^{\left(\frac{5}{20}\right)}} + \frac{\sqrt{\left(\frac{7}{20}\right)}}{e^{\left(\frac{7}{20}\right)}} + \frac{\sqrt{\left(\frac{9}{20}\right)}}{e^{\left(\frac{9}{20}\right)}} \right. \\ \left. + \frac{\sqrt{\left(\frac{11}{20}\right)}}{e^{\left(\frac{11}{20}\right)}} + \frac{\sqrt{\left(\frac{13}{20}\right)}}{e^{\left(\frac{13}{20}\right)}} + \frac{\sqrt{\left(\frac{15}{20}\right)}}{e^{\left(\frac{15}{20}\right)}} + \frac{\sqrt{\left(\frac{17}{20}\right)}}{e^{\left(\frac{17}{20}\right)}} + \frac{\sqrt{\left(\frac{19}{20}\right)}}{e^{\left(\frac{19}{20}\right)}} \right]$$

$$T_{10} = \frac{1}{2} \left[\frac{\sqrt{\left(\frac{0}{10}\right)}}{e^{\left(\frac{0}{10}\right)}} + 2 \left(\frac{\sqrt{\left(\frac{1}{10}\right)}}{e^{\left(\frac{1}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{2}{10}\right)}}{e^{\left(\frac{2}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{3}{10}\right)}}{e^{\left(\frac{3}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{4}{10}\right)}}{e^{\left(\frac{4}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{5}{10}\right)}}{e^{\left(\frac{5}{10}\right)}} \right) \right. \\ \left. + 2 \left(\frac{\sqrt{\left(\frac{6}{10}\right)}}{e^{\left(\frac{6}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{7}{10}\right)}}{e^{\left(\frac{7}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{8}{10}\right)}}{e^{\left(\frac{8}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{9}{10}\right)}}{e^{\left(\frac{9}{10}\right)}} \right) + \frac{\sqrt{\left(\frac{10}{10}\right)}}{e^{\left(\frac{10}{10}\right)}} \right]$$

$$S_{10} = \frac{1}{2} \left[\frac{\sqrt{\left(\frac{0}{10}\right)}}{e^{\left(\frac{0}{10}\right)}} + 4 \left(\frac{\sqrt{\left(\frac{1}{10}\right)}}{e^{\left(\frac{1}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{2}{10}\right)}}{e^{\left(\frac{2}{10}\right)}} \right) + 4 \left(\frac{\sqrt{\left(\frac{3}{10}\right)}}{e^{\left(\frac{3}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{4}{10}\right)}}{e^{\left(\frac{4}{10}\right)}} \right) + 4 \left(\frac{\sqrt{\left(\frac{5}{10}\right)}}{e^{\left(\frac{5}{10}\right)}} \right) \right. \\ \left. + 2 \left(\frac{\sqrt{\left(\frac{6}{10}\right)}}{e^{\left(\frac{6}{10}\right)}} \right) + 4 \left(\frac{\sqrt{\left(\frac{7}{10}\right)}}{e^{\left(\frac{7}{10}\right)}} \right) + 2 \left(\frac{\sqrt{\left(\frac{8}{10}\right)}}{e^{\left(\frac{8}{10}\right)}} \right) + 4 \left(\frac{\sqrt{\left(\frac{9}{10}\right)}}{e^{\left(\frac{9}{10}\right)}} \right) + \frac{\sqrt{\left(\frac{10}{10}\right)}}{e^{\left(\frac{10}{10}\right)}} \right]$$

20) $\int_0^1 e^{x^2} dx \quad |E_s| \leq 0.00001$

$$y = e^{x^2} \quad f^{(4)}(x) = \frac{d^4 y}{dx^4} = e^{x^2} (12 + 48x^2 + 16x^4) \quad |E_s| \leq \frac{K(b-a)^5}{180n^4} \quad |f^{(4)}(x)| \leq K$$

$$|f^{(4)}(x)| \leq K = |e^{(1)^2} (12 + 48(1)^2 + 16(1)^4)| = 76e$$

$$\frac{1}{100000} = \frac{(76e)(1)^5}{180n^4} \quad n = \sqrt[4]{\frac{380000e}{9}}$$

$$0.00001 \leq \frac{(76e)((1)-(0))^5}{180n^4} \Rightarrow n^4 = \frac{(76e)(100000)}{180} \Rightarrow n = 18.40597773$$

$$n^4 = \frac{380000e}{9} \quad n \approx 20$$