Derivatives of General Exponential Functions: The best technique for finding derivatives of general exponential functions without memorizing multiple rules is the following:
Use the method of Logarithmic Differentiation (see end of Chapter 5 Section 2) and then appply implicit differentiation to solve for the derivative.

Type 1: (where *a* is a constant) $y = a^x$

$$\begin{aligned} y &= a^x \\ \ln y &= \ln a^x \\ \ln y &= x \ln a \\ \frac{1}{y}\frac{dy}{dx} &= (\ln a)[1] \\ \frac{dy}{dx} &= (\ln a) y = (\ln a) a^x \end{aligned}$$

Type 2: (where both base and exponent contains variable) $y = x^x$

$$\begin{aligned} y &= x^x \\ \ln y &= \ln x^x \\ \ln y &= x \ln x \\ \frac{1}{y}\frac{dy}{dx} &= [1](\ln x) + (x)\left[\frac{1}{x}(1)\right] \\ \frac{dy}{dx} &= \{\ln x + 1\} y = \{\ln x + 1\} x^x \end{aligned}$$

Derivatives of General Logarithmic Functions: Use the change of base formula to transform general base logarithm into base *e* form. $\log_b x = \dfrac{\ln x}{\ln b}$

$$\begin{aligned} y &= \log_b x = \frac{\ln x}{\ln b} = \frac{1}{\ln b} \ln x \\ \frac{dy}{dx} &= \frac{1}{\ln b}\left[\frac{1}{x}(1)\right] = \frac{1}{x \ln b} \end{aligned}$$

The only integral covered in this section is Integral of Exponential function where *a* is a constant.

$$\int a^x \, dx = \frac{a^x}{\ln a} + C \quad a \neq 1$$

Additional examples:

4) $x^{\sqrt{5}} = e^{\ln(x^{\sqrt{5}})} = e^{\sqrt{5}\ln x}$

6) $(\tan x)^{\sec x} = e^{\ln\left((\tan x)^{\sec x}\right)} = e^{(\sec x)\ln(\tan x)}$

8) a) $\log_{10}\sqrt{10} = \log_{10}\left(10^{1/2}\right) = \frac{1}{2}\log_{10} 10 = \frac{1}{2}(1) = \frac{1}{2}$

b) $\log_8 320 - \log_8 5 = \log_8\left(\frac{320}{5}\right) = \log_8(64) = \log_8(8^2) = 2\log_8(8) = 2(1) = 2$

10) a) $\log_a \frac{1}{a} = \log_a(a^{-1}) = -1\log_a(a) = -1(1) = -1$

b) $10^{(\log_{10} 4 + \log_{10} 7)} = 10^{\log_{10}((4)(7))} = 10^{\log_{10}(28)} = 28$

26) $F(t) = 3^{\cos 2t}$

$$y = F(t) = 3^{\cos 2t}$$
$$\ln y = \ln(3^{\cos 2t})$$
$$\ln y = (\cos 2t)(\ln 3)$$
$$\frac{1}{y}\frac{dy}{dt} = [-\sin 2t(2)](\ln 3)$$
$$\frac{dy}{dt} = \{-2(\ln 3)\sin 2t\}y = \{-2(\ln 3)\sin 2t\}3^{\cos 2t}$$

28) $G(u) = (1+10^{\ln u})^6$

$$p = 10^{\ln u}$$
$$\ln p = \ln(10^{\ln u})$$
$$\ln p = (\ln u)(\ln 10)$$
$$\frac{1}{p}\frac{dp}{du} = \left[\frac{1}{u}(1)\right](\ln 10)$$
$$\frac{dp}{du} = \left\{\frac{\ln 10}{u}\right\}p = \left\{\frac{\ln 10}{u}\right\}10^{\ln u}$$

$$\frac{dG}{du} = 6(1+10^{\ln u})^5\left[\left\{\frac{\ln 10}{u}\right\}10^{\ln u}\right]$$
$$= \frac{6(\ln 10)10^{\ln u}(1+10^{\ln u})^5}{u}$$

34) $y = \sqrt{x}^{\,x}$

$$\ln y = \ln\left(\sqrt{x}\right)^x$$
$$\ln y = x\ln(x^{1/2})$$
$$\frac{1}{y}\frac{dy}{dx} = [1](\ln(x^{1/2})) + (x)\left[\frac{1}{x^{1/2}}\left(\frac{1}{2}x^{-1/2}\right)\right]$$
$$\frac{dy}{dx} = \left\{\ln(\sqrt{x}) + x\left[\frac{1}{2x}\right]\right\}y = \left\{\ln(\sqrt{x}) + \frac{1}{2}\right\}\left(\sqrt{x}\right)^x$$

38) $y = (\ln x)^{\cos x}$

$$\ln y = \ln((\ln x)^{\cos x})$$

$$\ln y = (\cos x)\ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = [-\sin x](\ln(\ln x)) + (\cos x)\left[\frac{1}{\ln x}\left(\frac{1}{x}(1)\right)\right]$$

$$\frac{dy}{dx} = \left\{\frac{\cos x}{x\ln x} - \sin x\ln(\ln x)\right\}y = \left\{\frac{\cos x}{x\ln x} - \sin x\ln(\ln x)\right\}(\ln x)^{\cos x}$$

42) $\int (x^5 + 5^x)\, dx = \frac{1}{6}x^6 + \frac{1}{\ln 5}5^x + C$

46) $\int \frac{2^x}{2^x+1}\, dx$

$$u = 2^x + 1 \qquad du = (\ln 2)2^x\, dx \qquad \frac{1}{\ln 2}\, du = 2^x\, dx$$

$$\int \frac{2^x}{2^x+1}\, dx = \int \frac{1}{u}\left(\frac{1}{\ln 2}\, du\right) = \frac{1}{\ln 2}\ln|u| + C = \frac{1}{\ln 2}\ln\left|2^x+1\right| + C = \frac{1}{\ln 2}\ln(2^x+1) + C$$