Theorem 6: If f is a one-to-one continuous function defined on an interval, then its inverse function f^{-1} is also continuous.

Theorem 7: [Numerical answer] If f is a one-to-one differentiable function with inverse function f^{-1} and $f'(f^{-1}(a)) \neq 0$, then the inverse function is differentiable at *a* and

$$(f^{-1})'(a) = \frac{1}{f'(f^{-1}(a))}$$

Additional examples:

4) The function is one-to-one because for all values of *x* shown, there is only one value of $f(x)$.

6) The function is one-to-one because no horizontal line intersects more than once.

8) The function is not one-to-one because there are locations that horizontal line intersects more than once.

10) The function is on-to-one because the function is linear and for all values of *x* there is only one value of $f(x)$.

12) The function $g(x) = \cos x$ is a periodic function (wave shape). Therefore, it fails the horizontal line test.

14) The function is not one-to-one because eventually we stop growing at certain age.

18) a) It passes the horizontal line test. b) domain of f^{-1} $[-1,3]$; range of f^{-1} $[-3,3]$
c) $f^{-1}(2) = 0$ d) $f^{-1}(0) \approx -1.75$

20) $m = f(v) = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow$$

$$\frac{m_0}{m} = \sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{m_0^{\,2}}{m^2} = 1-\frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1-\frac{m_0^{\,2}}{m^2}$$

$$v^2 = c^2\left(1-\frac{m_0^{\,2}}{m^2}\right) \Rightarrow$$

$$v = \sqrt{c^2\left(1-\frac{m_0^{\,2}}{m^2}\right)}$$

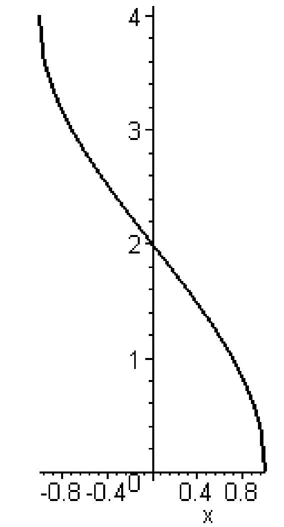
$$v = c\sqrt{1-\frac{m_0^{\,2}}{m^2}}$$

$$v = f^{-1}(m) = c\sqrt{1-\frac{m_0^{\,2}}{m^2}}$$

f^{-1} gives the velocity *v* of the particle in terms of its mass *m*.

24)
$$y = f(x) = 2x^3 + 3$$
$$y = 2x^3 + 3$$
$$y - 3 = 2x^3$$
$$\frac{y-3}{2} = x^3$$
$$\sqrt[3]{\frac{y-3}{2}} = x$$
$$f^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

30)



34) $f(x) = \sqrt{x-2} \quad a = 2$

a) The function f is same as $\sqrt{x}$ with horizontal translation of 2 units to the right. Since $\sqrt{x}$ is one-to-one, so f is one-to-one.

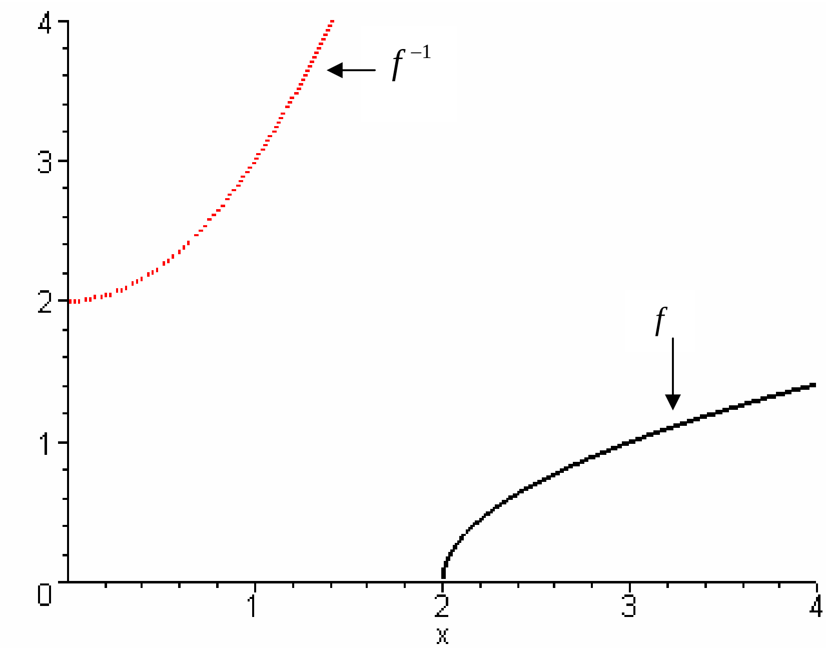
$$f'(x) = \frac{df}{dx} = \frac{1}{2\sqrt{x-2}}$$

b)
$$f^{-1}(2) = b \quad \Rightarrow \quad 2 = f(b) = \sqrt{b-2} \quad \Rightarrow \quad b = 6$$
$$f(6) = 2 \quad \Rightarrow \quad f^{-1}(2) = 6$$
$$(f^{-1})(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{\frac{1}{2\sqrt{(6)-2}}} = \frac{1}{\frac{1}{2\sqrt{4}}} = 2\sqrt{4} = 2(2) = 4$$

c)
$$y = \sqrt{x-2} \quad \Rightarrow \quad y^2 = x - 2 \quad \Rightarrow \quad y^2 + 2 = x$$
$$f^{-1}(x) = x^2 + 2, \quad x \geq 0 \quad \text{domain:} [0, \infty) \quad \text{range:} [2, \infty)$$

d) $\dfrac{df^{-1}}{dx} = 2x \quad \left.\dfrac{df^{-1}}{dx}\right|_{x=2} = 2(2) = 4$

e)



36) $f(x)=\dfrac{1}{x-1}\quad x>1\quad a=2$

a) The function *f* is same as $\dfrac{1}{x}$ with horizontal translation of 1 units to the right. Since $\dfrac{1}{x}$ is one-to-one, so *f* is one-to-one.

$$f'(x)=\frac{df}{dx}=\frac{-1}{(x-1)^2}$$

$$f^{-1}(2)=b\quad\Rightarrow\quad 2=f(b)=\frac{1}{b-1}\quad\Rightarrow\quad b=\frac{3}{2}$$

b)

$$f(\tfrac{3}{2})=2\quad\Rightarrow\quad f^{-1}(2)=\frac{3}{2}$$

$$(f^{-1})(2)=\frac{1}{f'(f^{-1}(2))}=\frac{1}{f'(\frac{3}{2})}=\frac{1}{\dfrac{-1}{((\frac{3}{2})-1)^2}}=-\left(\left(\frac{3}{2}\right)-1\right)^2=-\left(\frac{1}{2}\right)^2=\frac{-1}{4}$$

c)

$$y=\frac{1}{x-1}\quad\Rightarrow\quad x-1=\frac{1}{y}\quad\Rightarrow\quad x=\frac{1}{y}+1$$

$$f^{-1}(x)=\frac{1}{x}+1,\quad x>0\quad \text{domain: }(0,\infty)\quad \text{range: }(1,\infty)$$

d) $\dfrac{df^{-1}}{dx}=\dfrac{-1}{x^2}\quad \left.\dfrac{df^{-1}}{dx}\right|_{x=2}=\dfrac{-1}{(2)^2}=\dfrac{-1}{4}$

e)