

Getting ready for college math

In all the following $f = F'$ and $g = G'$.

1. **Product Rule** . Given F, f, G, g find $Fg + fG$

Example: If $F = (x^2 + 1)^3, G = (2x + 7)^3, f = 3(x^2 + 1)^2, g = 6(2x + 7)^2$

Find $Fg + fG$ and factor your answer completely.

2. **Quotient Rule**

Given F, f, G, g find $(F/G)' = \frac{fG - Fg}{G^2}$

Example: $F = (x^2 + 1)^m \quad G = (x^2 + x)^n$
 $f = 2mx(x^2 + 1)^{m-1} \quad g = 2n(x + 1)(x^2 + x)^{n-1}$.

3. Given $F(x)$ and $f(x) = F'(x)$. Find

$F(a), F(b), f(a), f(b)$, and $\frac{F(b) - F(a)}{b - a}$.

4. Rewrite $\frac{x^{n+1} + x^m \sqrt{x}}{x^k}$ without radical signs.

Rewrite $\frac{x^{n+1} + x^m x^{\frac{1}{2}}}{x^k}$ without negative exponents .

5. **Implicit differentiation** :

Solve $3y^2 D + y^3 x = (x^2 - 1)D + x$ for D .

6. Given $F(x)$ and $f(x)$,

- Find equation of line through $(a, F(a))$ with slope $f(a)$.

- Solve $y - F(a) = f(a)(x - a)$ for y

- Solve $\frac{y - F(a)}{x - a} = f(a)$ for y .

7. a) Let $x = a$ and $x = b$ (with $a < b$) be the solutions of $u(x) = v(x)$. Find $F(b) - F(a)$.

b) Let $x = a$ and $x = b$ and $x = c$ (with $a < b < c$) be the solutions of $u(x) = v(x)$. Find $F(b) - F(a) + F(b) - F(c)$.

8. Given F ,

simplify the difference quotient $\frac{F(a+h) - F(a)}{h}$

9. Given f (a first or second derivative)

a) Solve $f(x) = 0$.

b) In what intervals is $f(x)$ positive? negative?

10. Given F , find and simplify $\frac{F(b) - F(a)}{b - a}$

11. Suppose $F(x) = x^3 + bx^2 + cx + d$ and $f(x) = 3x^2 + 2bx + c$.

If $F(1) = 0$ and $F(2) = 4$ and $f(2) = 3$, find $F(x), f(x), f(a)$, etc.

12. **Order of operations** problems such as

- $3x(x + 1)^2 - x^2(3 - 2x)$.

- Given $f(x) = x + 1$ and $g(x) = 2 - x$ find $f(x) - 2g(x) - f(x)g(x)$.

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1. **Warmup for Product rule** Let $F = A^3$; $G = B^3$; $f = 3A^2$; and $g = 6B^2$.
Find $Fg + fG$ and factor your answer completely.

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Solution: Scroll slowly. Try each step before you look at the answer.

Substitute the given polynomials into $Fg + fG$

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Substitute the given polynomials into $Fg + fG$ $A^3(6B^2) + (3A^2)(B^3)$

Recognize as sum of two terms

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Solution: Scroll slowly. Try each step before you look at the answer.

Substitute the given polynomials into $Fg + fG$ $A^3(6B^2) + (3A^2)(B^3)$

Recognize as sum of two terms $= A^3(6B^2) + (3A^2)(B^3)$

Rewrite each term with constant at left

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Recognize as sum of two terms $= A^3(6B^2) + (3A^2)(B^3)$

Rewrite each term with constant at left $= 6A^3B^2 + 3A^2B^3$

Pull out common factor 3

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Rewrite each term with constant at left $= 6A^3B^2 + 3A^2B^3$

Pull out common factor 3 $= 3(2A^3B^2 + A^2B^3)$

Pull out the lowest common power A^2 of A

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$$\text{Rewrite each term with constant at left} \quad = 6A^3B^2 + 3A^2B^3$$

$$\text{Pull out common factor 3} \quad = 3(2A^3B^2 + A^2B^3)$$

$$\text{Pull out the lowest common power } A^2 \text{ of } A \quad = 3A^2(2A^{3-2}B^2 + B^3)$$

$$\text{Pull out lowest common power } B^2 \text{ of } B$$

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Subtract exponents	

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Pull out common factor 3	$= 3(2A^3B^2 + A^2B^3)$
Pull out the lowest common power A^2 of A	$= 3A^2(2A^{3-2}B^2 + B^3)$
Pull out lowest common power B^2 of B	$= 3A^2B^2(2A^{3-2} + B^{3-2})$
Subtract exponents	$= 3A^2B^2(2A^1 + B^1)$
This is the final answer	$= 3A^2B^2(2A + B)$

Preparation for Product rule Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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Solution: This problem was obtained by substituting $x^2 + 1$ for A and $2x + 7$ for B in the warmup problem. This problem seems harder, simply because there are more symbols.

Substitute the given polynomials into $Fg + fG$

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$$\text{Substitute the given polynomials into } Fg + fG \quad (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3)$$

$$\text{Recognize as sum of two terms} \quad =$$

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$$\begin{aligned} \text{Substitute the given polynomials into } Fg + fG &= (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3) \\ \text{Recognize as sum of two terms} &= (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3) \\ \text{Rewrite each term with constant at left} &= \end{aligned}$$

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 \text{Recognize as sum of two terms} &= (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3) \\
 \text{Rewrite each term with constant at left} &= 6(x^2 + 1)^3(2x + 7)^2 + 3(x^2 + 1)^2(2x + 7)^3 \\
 \text{Factor out common factor 3} &=
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Factor $(x^2 + 1)^2$ from both terms.	$=$

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Subtract exponents	$=$

Preparation for Product rule Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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Subtract exponents	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1)^1 + (2x + 7)^1)$
Expand the remaining factor	$=$

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Expand the remaining factor	$= 3(x^2 + 1)^2(2x + 7)^2((2x^2 + 2) + (2x + 7))$

Preparation for Product rule Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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Subtract exponents	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1)^1 + (2x + 7)^1)$
Expand the remaining factor	$= 3(x^2 + 1)^2(2x + 7)^2((2x^2 + 2) + (2x + 7))$
Collect the remaining factor	$=$

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Rewrite each term with constant at left	$= 6(x^2 + 1)^3(2x + 7)^2 + 3(x^2 + 1)^2(2x + 7)^3$
Factor out common factor 3	$= 3(2(x^2 + 1)^3(2x + 7)^2 + (x^2 + 1)^2(2x + 7)^3)$
Factor $(x^2 + 1)^2$ from both terms.	$= 3(x^2 + 1)^2(2(x^2 + 1)^{3-2}(2x + 7)^2 + (2x + 7)^3)$
Factor $(2x + 7)^2$ from both terms.	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1)^{3-2} + (2x + 7)^{3-2})$
Subtract exponents	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1)^1 + (2x + 7)^1)$
Expand the remaining factor	$= 3(x^2 + 1)^2(2x + 7)^2((2x^2 + 2) + (2x + 7))$
Collect the remaining factor	$= \boxed{3(x^2 + 1)^2(2x + 7)^2(2x^2 + 2x + 9)}$

Preparation for Product rule Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

Solution: This problem was obtained by substituting $x^2 + 1$ for A and $2x + 7$ for B in the warmup problem. This problem seems harder, simply because there are more symbols.

$$\begin{aligned}
 \text{Substitute the given polynomials into } Fg + fG &= (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3) \\
 \text{Recognize as sum of two terms} &= (x^2 + 1)^3(6(2x + 7)^2) + (3(x^2 + 1)^2)((2x + 7)^3) \\
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 \end{aligned}$$

To be sure the boxed answer is completely factored, use the Quadratic polynomial factoring criterion to see if $2x^2 + 2x + 9$ factors.

Preparation for Product rule Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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To be sure the boxed answer is completely factored, use the Quadratic polynomial factoring criterion to see if $2x^2 + 2x + 9$ factors. It does not because $b^2 - 4ac = 2^2 - 4 \cdot 2 \cdot 9 = 4 - 72 = -68$ is not a perfect square.

This problem was a bit much. On the next slide, we show how to use abbreviations to make it easier to handle.

Using abbreviations Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

Using abbreviations Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

Solution: Substitute $A = x^2 + 1$ and $B = 2x + 7$ in F, g, f, G to get $F = A^3, G = B^3, f = 3A^2, g = 6B^2$.

Substitute the abbreviations into $Fg + fG$

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Substitute the abbreviations into $Fg + fG$ $A^3(6B^2) + (3A^2)(B^3)$

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$$\text{Substitute the abbreviations into } Fg + fG \quad A^3(6B^2) + (3A^2)(B^3)$$

$$\text{Recognize as sum of two terms} \quad = A^3(6B^2) + (3A^2)(B^3)$$

$$\text{Rewrite each term with constant at left} \quad =$$

Using abbreviations Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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Solution: Substitute $A = x^2 + 1$ and $B = 2x + 7$ in F, g, f, G to get $F = A^3, G = B^3, f = 3A^2, g = 6B^2$.

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Substitute $A = x^2 + 1$ and $B = 2x + 7$ in this answer	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1) + (2x + 7))$
Collect the remaining factor	$= \boxed{3(x^2 + 1)^2(2x + 7)^2(2x^2 + 2x + 9)}$

In calculus, some expression names are written with a prime. For example, f and f' could be the names of different polynomials, as in the next example.

Using abbreviations Let $F = (x^2 + 1)^3$; $G = (2x + 7)^3$; $f = 3(x^2 + 1)^2$; and $g = 6(2x + 7)^2$. Find $Fg + fG$ and factor your answer completely.

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Substitute $A = x^2 + 1$ and $B = 2x + 7$ in this answer	$= 3(x^2 + 1)^2(2x + 7)^2(2(x^2 + 1) + (2x + 7))$
Collect the remaining factor	$= \boxed{3(x^2 + 1)^2(2x + 7)^2(2x^2 + 2x + 9)}$

In calculus, some expression names are written with a prime. For example, f and f' could be the names of different polynomials, as in the next example.

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

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Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

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Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$ $\frac{(8x(x^2 + 1)^3)(2(2x + 7)^3) - (x^2 + 1)^4(6(2x + 7)^2)}{(2(2x + 7)^3)^2}$

Recognize as sum of two terms

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Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$ $\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$

Recognize as sum of two terms $= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$

Rewrite each term with constant at left

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$
$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms
$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

Pull out common factor 2 from numerator

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms

$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left

$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

Pull out common factor 2 from numerator

$$= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms

$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left

$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

Pull out common factor 2 from numerator

$$= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6}$$

Cancel constant factor from numerator and denominator

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms

$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left

$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

Pull out common factor 2 from numerator

$$= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6}$$

Cancel constant factor from numerator and denominator

$$= \frac{\cancel{2}(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{\cancel{4}(2x+7)^6}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$
$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms

$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left

$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

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Cancel constant factor from numerator and denominator

$$= \frac{\cancel{2}(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{\cancel{4}(2x+7)^6}$$

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

Recognize as sum of two terms

Rewrite each term with constant at left

Pull out common factor 2 from numerator

Cancel constant factor from numerator and denominator

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

$$\begin{aligned} & \frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6} \\ &= \frac{\cancel{2}(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{\cancel{4}(2x+7)^6} \\ &= \frac{(x^2+1)^3(8x(2x+7)^3 - (x^2+1)(3(2x+7)^2))}{2(2x+7)^6} \end{aligned}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

Recognize as sum of two terms

Rewrite each term with constant at left

Pull out common factor 2 from numerator

Cancel constant factor from numerator and denominator

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

Pull out least power $(2x + 7)^2$ of $2x + 7$ from both terms.

$$\begin{aligned} & \frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{4(2x+7)^6} \\ &= \frac{(x^2+1)^3(8x(2x+7)^3 - (x^2+1)(3(2x+7)^2))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(2x+7)^2(8x(2x+7) - (x^2+1)(3))}{2(2x+7)^6} \end{aligned}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$
$$\frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Recognize as sum of two terms

$$= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2}$$

Rewrite each term with constant at left

$$= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2}$$

Pull out common factor 2 from numerator

$$= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6}$$

Cancel constant factor from numerator and denominator

$$= \frac{2(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{4(2x+7)^6}$$

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

$$= \frac{(x^2+1)^3(8x(2x+7)^3 - (x^2+1)(3(2x+7)^2))}{2(2x+7)^6}$$

Pull out least power $(2x + 7)^2$ of $2x + 7$ from both terms.

$$= \frac{(x^2+1)^3(2x+7)^2(8x(2x+7) - (x^2+1)(3))}{2(2x+7)^6}$$

Cancel powers of $2x + 7$ and rewrite the remaining factor

$$= \frac{(x^2+1)^3(2x+7)^2(16x^2+56x-(3x^2+3))}{2(2x+7)^6}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

Recognize as sum of two terms

Rewrite each term with constant at left

Pull out common factor 2 from numerator

Cancel constant factor from numerator and denominator

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

Pull out least power $(2x + 7)^2$ of $2x + 7$ from both terms.

Cancel powers of $2x + 7$ and rewrite the remaining factor

Distribute the minus sign

$$\begin{aligned} & \frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{4(2x+7)^6} \\ &= \frac{(x^2+1)^3(8x(2x+7)^3 - (x^2+1)(3(2x+7)^2))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(2x+7)^2(8x(2x+7) - (x^2+1)(3))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(2x+7)^2(16x^2+56x - (3x^2+3))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(16x^2+56x-3x^2-3)}{2(2x+7)^{6-2}} \end{aligned}$$

1. **Preparation for Quotient rule:** Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Substitute the given polynomials into $\frac{f'g - fg'}{g^2}$

Recognize as sum of two terms

Rewrite each term with constant at left

Pull out common factor 2 from numerator

Cancel constant factor from numerator and denominator

Pull out least power $(x^2 + 1)^3$ of $x^2 + 1$ from both terms.

Pull out least power $(2x + 7)^2$ of $2x + 7$ from both terms.

Cancel powers of $2x + 7$ and rewrite the remaining factor

Distribute the minus sign

Collect the remaining factor

$$\begin{aligned} & \frac{(8x(x^2+1)^3)(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{8x(x^2+1)^3(2(2x+7)^3) - (x^2+1)^4(6(2x+7)^2)}{(2(2x+7)^3)^2} \\ &= \frac{16x(x^2+1)^3(2x+7)^3 - 6(x^2+1)^4(2x+7)^2}{2^2((2x+7)^3)^2} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - 3(x^2+1)^4((2x+7)^2))}{4(2x+7)^6} \\ &= \frac{2(8x(x^2+1)^3(2x+7)^3 - (x^2+1)^4(3(2x+7)^2))}{4(2x+7)^6} \\ &= \frac{(x^2+1)^3(8x(2x+7)^3 - (x^2+1)(3(2x+7)^2))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(2x+7)^2(8x(2x+7) - (x^2+1)(3))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(2x+7)^2(16x^2+56x - (3x^2+3))}{2(2x+7)^6} \\ &= \frac{(x^2+1)^3(16x^2+56x-3x^2-3)}{2(2x+7)^{6-2}} \\ &= \boxed{\frac{(x^2+1)^3(13x^2+56x-3)}{2(2x+7)^4}} \end{aligned}$$

Note: the Quadratic polynomial factoring criterion assures us that $13x^2 + 56x - 3$ does not factor because $b^2 - 4ac = 56^2 - 4(13)(-3) = 3292$ is not a perfect square.

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$.

Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$.

Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

$$\text{Substitute into } \frac{f'g - fg'}{g^2} \qquad \frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$.

Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

$$\text{Substitute into } \frac{f'g - fg'}{g^2} \quad \frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

$$\text{Recognize as sum of two terms} \quad = \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$ $\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$

Recognize as sum of two terms $= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$

Rewrite each term with constant at left $= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$

Pull out common factor 2 from numerator

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

$$\text{Substitute into } \frac{f'g - fg'}{g^2} \quad \frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

$$\text{Recognize as sum of two terms} \quad = \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

$$\text{Rewrite each term with constant at left} \quad = \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

$$\text{Pull out common factor 2 from numerator} \quad = \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$.

Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$ $\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$

Recognize as sum of two terms $= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$

Rewrite each term with constant at left $= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$

Pull out common factor 2 from numerator $= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$

Cancel before continuing

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$ $\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$

Recognize as sum of two terms $= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$

Rewrite each term with constant at left $= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$

Pull out common factor 2 from numerator $= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$

Cancel before continuing $= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$
$$\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms
$$= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

Pull out common factor 2 from numerator
$$= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Cancel before continuing
$$= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$$

Pull out lowest common power A^3 of A

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$ $\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$

Recognize as sum of two terms $= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$

Rewrite each term with constant at left $= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$

Pull out common factor 2 from numerator $= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$

Cancel before continuing $= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{2}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$

Pull out lowest common power A^3 of A $= \frac{A^3(8xB^3 - 3A^{-3}B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$
$$\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms
$$= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

Pull out common factor 2 from numerator
$$= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Cancel before continuing
$$= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$$

Pull out lowest common power A^3 of A
$$= \frac{A^3(8xB^3 - 3A^4B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$$

Pull out lowest common power B^2 of B
$$= \frac{A^3B^2(8xB^{3-2} - 3A)}{2B^6} = \frac{A^3B^2(8xB - 3A)}{2B^6}$$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$ $\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$

Recognize as sum of two terms $= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$

Rewrite each term with constant at left $= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$

Pull out common factor 2 from numerator $= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$

Cancel before continuing $= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$

Pull out lowest common power A^3 of A $= \frac{A^3(8xB^3 - 3A^4 \cdot 3B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$

Pull out lowest common power B^2 of B $= \frac{A^3B^2(8xB^{3-2} - 3A)}{2B^6} = \frac{A^3B^2(8xB - 3A)}{2B^6}$

Cancel common power of B $= \frac{A^3\cancel{B^2}(8xB - 3A)}{\cancel{2B^6}} = \frac{A^3(8xB - 3A)}{2B^4}$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$
$$\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms
$$= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

Pull out common factor 2 from numerator
$$= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Cancel before continuing
$$= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$$

Pull out lowest common power A^3 of A
$$= \frac{A^3(8xB^3 - 3A^4B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$$

Pull out lowest common power B^2 of B
$$= \frac{A^3B^2(8xB^{3-2} - 3A)}{2B^6} = \frac{A^3B^2(8xB - 3A)}{2B^6}$$

Cancel common power of B
$$= \frac{A^3\cancel{B^2}(8xB - 3A)}{2\cancel{B^6}} = \frac{A^3(8xB - 3A)}{2B^4}$$

Go back to x
$$= \frac{(x^2+1)^3(8x(2x+7) - 3(x^2+1))}{2(2x+7)^4}$$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$
$$\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms
$$= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

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$$= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Cancel before continuing
$$= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$$

Pull out lowest common power A^3 of A
$$= \frac{A^3(8xB^3 - 3A^4B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$$

Pull out lowest common power B^2 of B
$$= \frac{A^3B^2(8xB^{3-2} - 3A)}{2B^6} = \frac{A^3B^2(8xB - 3A)}{2B^6}$$

Cancel common power of B
$$= \frac{A^3\cancel{B^2}(8xB - 3A)}{2\cancel{B^6}} = \frac{A^3(8xB - 3A)}{2B^4}$$

Go back to x
$$= \frac{(x^2+1)^3(8x(2x+7) - 3(x^2+1))}{2(2x+7)^4}$$

Expand remaining factor
$$= \frac{(x^2+1)^3(16x^2+56x-3x^2-3)}{2(2x+7)^4} = \boxed{\frac{(x^2 + 1)^3(13x^2 + 56x - 3)}{2(2x + 7)^4}}$$

Working with abbreviations: Let $f = (x^2 + 1)^4$; $g = 2(2x + 7)^3$, $f' = 8x(x^2 + 1)^3$, and $g' = 6(2x + 7)^2$. Find $\frac{f'g - fg'}{g^2}$ and rewrite your answer as a completely reduced fraction.

Solution: Scroll slowly. Try each step in the left column before you click to see the answer.

Substitute A for $x^2 + 1$ and B for $2x + 7$ to get $f = A^4$; $g = B^3$, $f' = 8xA^3$, and $g' = 6B^2$.

Substitute into $\frac{f'g - fg'}{g^2}$
$$\frac{(8xA^3 \cdot 2B^3) - A^4(6B^2)}{(2B^3)^2}$$

Recognize as sum of two terms
$$= \frac{8xA^3 \cdot 2B^3 - A^4(6B^2)}{(2B^3)^2}$$

Rewrite each term with constant at left
$$= \frac{16xA^3B^3 - 6A^4B^2}{2^2(B^3)^2}$$

Pull out common factor 2 from numerator
$$= \frac{2(8xA^3B^3 - 3A^4B^2)}{4B^6}$$

Cancel before continuing
$$= \frac{\cancel{2}(8xA^3B^3 - 3A^4B^2)}{\cancel{4}B^6} = \frac{(8xA^3B^3 - 3A^4B^2)}{2B^6}$$

Pull out lowest common power A^3 of A
$$= \frac{A^3(8xB^3 - 3A^4B^2)}{2B^6} = \frac{A^3(8xB^3 - 3AB^2)}{2B^6}$$

Pull out lowest common power B^2 of B
$$= \frac{A^3B^2(8xB^{3-2} - 3A)}{2B^6} = \frac{A^3B^2(8xB - 3A)}{2B^6}$$

Cancel common power of B
$$= \frac{A^3\cancel{B^2}(8xB - 3A)}{\cancel{2B^6}} = \frac{A^3(8xB - 3A)}{2B^4}$$

Go back to x
$$= \frac{(x^2+1)^3(8x(2x+7) - 3(x^2+1))}{2(2x+7)^4}$$

Expand remaining factor
$$= \frac{(x^2+1)^3(16x^2+56x-3x^2-3)}{2(2x+7)^4} = \boxed{\frac{(x^2 + 1)^3(13x^2 + 56x - 3)}{2(2x + 7)^4}}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\frac{f(b)-f(a)}{b-a}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\frac{f(b)-f(a)}{b-a} = \frac{f(x-h)-f(x)}{(x-h)-x}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\frac{f(b)-f(a)}{b-a} = \frac{f(x-h)-f(x)}{(x-h)-x} = \frac{f(x-h)-f(x)}{-h}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\begin{aligned}\frac{f(b)-f(a)}{b-a} &= \frac{f(x-h)-f(x)}{(x-h)-x} = \frac{f(x-h)-f(x)}{-h} \\ &= \frac{-1}{h} ((x-h) - (x-h)^2 - (x-x^2))\end{aligned}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\begin{aligned}\frac{f(b)-f(a)}{b-a} &= \frac{f(x-h)-f(x)}{(x-h)-x} = \frac{f(x-h)-f(x)}{-h} \\ &= \frac{-1}{h} ((x-h) - (x-h)^2 - (x-x^2)) \\ &= -\frac{1}{h} (x-h - (x^2 - 2hx + h^2) - x + x^2)\end{aligned}$$

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3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

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$$\frac{f(b)-f(a)}{b-a}$$

3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

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3. In each of the following, find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.1 Let $f(x) = x - x^2$; $a = x$, $b = x - h$. Then

$$\begin{aligned} \frac{f(b)-f(a)}{b-a} &= \frac{f(x-h)-f(x)}{(x-h)-x} = \frac{f(x-h)-f(x)}{-h} \\ &= \frac{-1}{h} \left((x-h) - (x-h)^2 - (x-x^2) \right) \\ &= -\frac{1}{h} \left(x-h - (x^2 - 2hx + h^2) - x + x^2 \right) \\ &= -\frac{1}{h} \left(x-h - x^2 + 2hx - h^2 - x + x^2 \right) \\ &= -\frac{1}{h} \left(\cancel{x} - h - \cancel{x^2} + 2hx - h^2 - \cancel{x} + \cancel{x^2} \right) \\ &= -\frac{1}{h} \left(-h + 2hx - h^2 \right) \\ &= -\frac{1}{h} \left(\cancel{h}(-1 + 2x - h) \right) = \boxed{-2x + 1 + h} \end{aligned}$$

3.2 Let $f(x) = x - x^2$; $a = x$; $b = x + h$. Then

$$\begin{aligned} \frac{f(b)-f(a)}{b-a} &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h} \\ &= \frac{1}{h} \left((x+h) - (x+h)^2 - (x-x^2) \right) \\ &= \frac{1}{h} \left(x+h - (x^2 + 2hx + h^2) - x + x^2 \right) \\ &= \frac{1}{h} \left(x+h - x^2 - 2hx - h^2 - x + x^2 \right) \end{aligned}$$

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=

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$$= \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right)$$

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3 Find $\frac{f(b)-f(a)}{b-a}$ and rewrite your answer as a polynomial or as a reduced fraction. Go slowly through the slide and write down the answer to each part before you move ahead.

3.3 Let $f(x) = \frac{1}{x}$; $a = x$, $b = x + h$. Then

$$\begin{aligned} \frac{f(b)-f(a)}{b-a} &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h} \\ &= \frac{1}{h} (f(x+h) - f(x)) \\ &= \frac{1}{h} \left(\frac{1}{x+h} - \frac{1}{x} \right) \\ &= \frac{1}{h} \left(\frac{1}{x+h} \cdot \frac{x}{x} - \frac{1}{x} \cdot \frac{x+h}{x+h} \right) \\ &= \frac{1}{h} \left(\frac{x}{(x+h)x} - \frac{x+h}{x(x+h)} \right) \\ &= \frac{1}{h} \left(\frac{x-(x+h)}{x(x+h)} \right) = \frac{1}{h} \cdot \frac{x-x-h}{x(x+h)} \\ &= \frac{1}{\cancel{h}} \cdot \frac{\cancel{h}}{x(x+h)} = \boxed{\frac{-1}{x(x+h)}} \end{aligned}$$

3.4 Let $f(x) = \frac{x}{2x+3}$; $a = x$, $b = x + h$. Then

$$\begin{aligned} \frac{f(b)-f(a)}{b-a} &= \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h} \\ &= \frac{1}{h} (f(x+h) - f(x)) \\ &= \frac{1}{h} \left(\frac{x+h}{2x+2h+3} - \frac{x}{2x+3} \right) \\ &= \frac{1}{h} \left(\frac{x+h}{2x+2h+3} \cdot \frac{2x+3}{2x+3} - \frac{x}{2x+3} \cdot \frac{2x+2h+3}{2x+2h+3} \right) \\ &= \frac{1}{h} \left(\frac{x(2x)+x(3)+h(2x)+h(3)}{(2x+3+2h)(2x+3)} - \frac{x(2x)+x(2h)+x(3)}{(2x+3+2h)(2x+3)} \right) \\ &= \frac{2x^2+3x+2hx+3h-(2x^2+2xh+3x)}{h(2x+3+2h)(2x+3)} \\ &= \end{aligned}$$

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The following examples use the identity $(A + B)(A - B) = A^2 - B^2$ by substituting $A = \sqrt{x + h}$ and $B = \sqrt{x}$.
Then $(\sqrt{x + h} + \sqrt{x})(\sqrt{x + h} - \sqrt{x}) = (\sqrt{x + h})^2 - (\sqrt{x})^2 = x + h - x = h$.

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