THE CITY COLLEGE OF NEW YORK
DEPARTMENT OF MATHEMATICS
SPRING 2017

MATH 392, FINAL EXAMINATION

YOUR NAME (print and sign):

NAME OF YOUR INSTRUCTOR:

Pr. 1	Pr. 2	Pr. 3	Pr. 4	Pr. 5	Pr. 6	Pr. 7	Pr. 8	Pr. 9	Pr. 10	Pr. 11	Total

INSTRUCTIONS:

- **There are a total of 11 problems.**
- **DO ALL PROBLEMS 1 THROUGH 7 AND THREE OF THE FOUR PROBLEMS 8-11. IN THE TABLE ABOVE, CROSS OUT ONE PROBLEM AMONG PROBLEMS 8-11 THAT YOU OMITTED.**
- **Each problem is worth 10 points.**
- **Notes, books and calculators are not to be used.**
- **All work on this exam is to be your own.**
- **Read each problem carefully. Be sure to show your work. Remember that it is your obligation to answer each question clearly and in a way that convinces the grader that you understand how to solve the problem.**
- **Stop working immediately at the end of the exam when time is called.**

1. Find the line integral

$$\int_C y^3 ds,$$

where C is the curve given by

$$\vec{r}(t) = (1, t, t^2/2), \quad 0 \le t \le 1.$$

2. Consider the following system of linear equations.

$$\begin{cases} x_1 - 2x_2 + x_3 - x_4 = 1, \\ x_1 - x_3 + x_4 = 3, \\ -x_2 + x_3 - x_4 = -1. \end{cases}$$

Determine whether this system is consistent, and if it is, find the full set of solutions. Also, find the rank of the matrix of coefficients.

3. Consider the following vector field.

$$\vec{F}(x, y, z) = x\vec{i} + \frac{y}{1 + y^2 + z^2}\vec{j} + \frac{z}{1 + y^2 + z^2}\vec{k}.$$

(a) Find curl$\vec{F}$.
(b) Is $\vec{F}$ conservative? Make sure to justify your claim.

4. Find the work done by the vector field

$$\vec{F}(x,y) = y^2\vec{\imath} - xe^y\vec{\jmath}$$

moving a particle along the curve C given by

$$\vec{r}(t) = (t,t), \quad 0 \leq t \leq 1.$$

5. Use Stokes' Theorem to find

$$\int_C xydx - yzdz,$$

where C is the boundary curve of

$$\{(x, y, z) :\ x + y + z = 1,\ x, y, z \geq 0\},$$

oriented clockwise as seen from the origin.

6. Use the Fundamental Theorem for Line Integrals to find

$$\int_C y\cos(xy)dx + (x\cos(xy) - ze^{yz})dy - ye^{yz}dz,$$

where C is the curve given by

$$\vec{r}(t) = (t, \pi t/2, 1-t), \quad 0 \le t \le 1.$$

7. Consider the following matrix.

$$A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 4 & 1 \\ -3 & 0 & -3 \end{bmatrix}.$$

Find A^{-1} and use it to solve

$$\begin{cases} 2x_2 - x_3 = 0, \\ 2x_1 + 4x_2 + x_3 = 2, \\ -3x_1 - 3x_3 = 3. \end{cases}$$

8. Find the area of the surface S given by

$$\vec{r}(u, v) = (uv, -u, v)\,,$$

for $u^2 + v^2 \leq 1$.

9. (a) Find the eigenvalues and eigenvectors for the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 1 & -1 \end{bmatrix}.$$

(b) Find the general solution to the following system of linear ordinary differential equations.

$$\begin{cases} y_1' = -y_1 + 3y_2, \\ y_2' = y_1 - y_2. \end{cases}$$

10. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 3 \\ -1 & 3 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{bmatrix}.$$

Find $\det(A)$ and $\det\left(\frac{1}{2}AA^TA^{-1}\right)$.

11. Use the Divergence Theorem to find the surface integral

$$\iint_S \vec{F} \cdot d\vec{S},$$

where

$$\vec{F}(x, y, z) = x\vec{i} - y\vec{j} + z^2\vec{k},$$

and S is the boundary surface of

$$E = \{(x, y, z)\colon\ 0 \leq z \leq \sqrt{1 - x^2 - y^2}\},$$

oriented so that the normal is outward pointing.