THE CITY COLLEGE OF NEW YORK
DEPARTMENT OF MATHEMATICS
SPRING 2016

MATH 392, FINAL EXAMINATION

YOUR NAME (print and sign):

NAME OF YOUR INSTRUCTOR:

Pr. 1	Pr. 2	Pr. 3	Pr. 4	Pr. 5	Pr. 6	Pr. 7	Pr. 8	Pr. 9	Pr. 10	Pr. 11	Total

INSTRUCTIONS:

- **There are a total of 11 problems.**
- **DO ALL PROBLEMS 1 THROUGH 7 AND THREE OF THE FOUR PROBLEMS 8-11. IN THE TABLE ABOVE, CROSS OUT ONE PROBLEM AMONG PROBLEMS 8-11 THAT YOU OMITTED.**
- **Each problem is worth 10 points.**
- **Notes, books and calculators are not to be used.**
- **All work on this exam is to be your own.**
- **Read each problem carefully. Be sure to show your work. Remember that it is your obligation to answer each question clearly and in a way that convinces the grader that you understand how to solve the problem.**
- **Stop working immediately at the end of the exam when time is called.**

1. Find the arc-length parametrization of the curve that is the intersection of the elliptic cylinder $x^2 + \frac{y^2}{2} = 1$ and the plane $z = x - 2$. Use s as the arc-length parameter with $s = 0$ corresponds to the point $(1, 0, -1)$. Specify the limits for s.

2. Find the potential of the vector field $\vec{F}(x,y,z) = e^{xy/z}\left((1+\frac{xy}{z})\vec{i} + \frac{x^2}{z}\vec{j} - \frac{x^2y}{z^2}\vec{k}\right)$, and use the Fundamental Theorem for Line Integrals to evaluate

$$\int_C \vec{F}\cdot d\vec{r},$$

where C is the straight line segment from $(1,1,1)$ to $(2,2,1)$. Can one choose an arbitrary piece-wise smooth curve C with initial point $(1,1,1)$ and terminal point $(2,2,1)$ instead of the straight line segment?

3. Determine whether the following system of linear equations is consistent, and, if it is, find the set of solutions.

$$\begin{cases} 3x_1 + 4x_2 - x_4 = -5 \\ -x_2 - 6x_3 + 13x_4 = -4 \\ 2x_3 + x_4 = 3 \end{cases} .$$

4. Use Green's theorem to find

$$\int_C xy^2\,dx + x^3y^2\,dy,$$

where C is the boundary of the region $D = \{(x,y)\colon |x|+|y| \le 1,\ x \ge 0\}$, oriented positively (i.e., counterclockwise).

5. Evaluate the surface integral

$$\iint_S \vec{F} \cdot d\vec{S},$$

where $\vec{F}(x, y, z) = x\vec{i}$, and S is the part of the paraboloid $z = -x^2 - y^2 + 1$ above the xy-plane, oriented upward.

6. Use the Divergence Theorem to evaluate the integral

$$\iint_S \vec{F} \cdot d\vec{S},$$

where $\vec{F}(x, y, z) = x^2 yz\vec{k}$, and S is the positively oriented (i.e., outward pointing normal) boundary of the cube

$$E = \{(x, y, z) \colon 0 \leq x \leq 1,\ 0 \leq y \leq 1,\ |z| \leq 1/2\}.$$

7. Solve the following linear system of ODE's.

$$\begin{cases} y_1' = 5y_1 - 2y_2 \\ y_2' = -2y_1 + y_2 \end{cases} .$$

8. Evaluate the triple integral

$$\iiint_E z\, dV,$$

where E is the region in the first octant, contained in $z^2 \geq x^2 + y^2$, and bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the plane $z = 1$.

9. Evaluate the surface integral

$$\iint_S (2x - y + z)dS,$$

where S is the surface that is the intersection of the plane $x + 2y + 2z = 1$ with the first octant.

10. Use Stokes' Theorem to find the line integral

$$\int_C \vec{F} \cdot d\vec{r},$$

where $\vec{F}(x, y, z) = y\vec{i} + z\vec{j} + 2x\vec{k}$, and C is the circle in the plane $x + z = 1$, centered at the point $(1, 0, 0)$, whose radius is 2, and that is oriented clockwise when viewed from the origin.

11. Find the inverse to the following matrix

$$A = \begin{bmatrix} 1 & -2 & 6 \\ -4 & 9 & -23 \\ -1 & 2 & -5 \end{bmatrix}.$$

Use A^{-1} to solve the following system of linear equations

$$\begin{cases} x_1 - 2x_2 + 6x_3 = 2 \\ -4x_1 + 9x_2 - 23x_3 = 0 \\ -x_1 + 2x_2 - 5x_3 = -1 \end{cases}.$$