Series Tests (for convergence)

Test	*When to Use*	*Conclusions*	*Section*
Geometric Series	$\sum_{k=0}^{\infty} ar^k$	Converges to $\frac{a}{1-r}$ if $\lvert r\rvert < 1$; diverges if $\lvert r\rvert \geq 1$.	8.2
***k*th-Term Test**	All series	If $\lim_{k\to\infty} a_k \neq 0$, the series diverges.	8.2
Integral Test	$\sum_{k=1}^{\infty} a_k$ where $f(k) = a_k$, f is continuous and decreasing and $f(x) \geq 0$	$\sum_{k=1}^{\infty} a_k$ and $\int_1^{\infty} f(x)\,dx$ *both* converge or *both* diverge.	8.3
***p*-series**	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	Converges for $p > 1$; diverges for $p \leq 1$.	8.3
Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $0 \leq a_k \leq b_k$	If $\sum_{k=1}^{\infty} b_k$ converges, then $\sum_{k=1}^{\infty} a_k$ converges. If $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} b_k$ diverges.	8.3
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$, where $a_k, b_k > 0$ and $\lim_{k\to\infty} \frac{a_k}{b_k} = L > 0$	$\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ *both* converge or *both* diverge.	8.3
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^{k+1} a_k$ where $a_k > 0$ for all k	If $\lim_{k\to\infty} a_k = 0$ and $a_{k+1} \leq a_k$ for all k, then the series converges.	8.4
Absolute Convergence	Series with some positive and some negative terms (including alternating series)	If $\sum_{k=1}^{\infty} \lvert a_k\rvert$ converges, then $\sum_{k=1}^{\infty} a_k$ converges absolutely.	8.4
Ratio Test	Any series (especially those involving exponentials and/or factorials)	For $\lim_{k\to\infty} \left\lvert \frac{a_{k+1}}{a_k} \right\rvert = L$, if $L < 1$, $\sum_{k=1}^{\infty} a_k$ converges absolutely if $L > 1$, $\sum_{k=1}^{\infty} a_k$ diverges, if $L = 1$, no conclusion.	8.4
Root Test	Any series (especially those involving exponentials)	For $\lim_{k\to\infty} \sqrt[k]{\lvert a_k\rvert} = L$, if $L < 1$, $\sum_{k=1}^{\infty} a_k$ converges absolutely if $L > 1$, $\sum_{k=1}^{\infty} a_k$ diverges, if $L = 1$, no conclusion.	8.4

There is also a method that works for "telescoping series". We will describe what "telescoping series" are, and how to determine their convergence in class.