THE CITY COLLEGE of NEW YORK
Department of Mathematics

MATH 201– Sample Final 3
(135 minutes)

Instructor's name:

Student's Last Name, First Name: ______________________________ Section #:

Instructions: This exam contains **3 pages** (including this cover page) and **10 questions**. Students must solve all **10 questions**. Total of **100 points**. No **calculators** or any other electronic devices are allowed during the examination. Turn off cell phones, alarms, and anything else that makes noises. You must show all your work to receive credit. Good luck!

Distribution of Marks

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total:	100	

1. (10 points) Compute $\dfrac{dy}{dx}$ for each of the functions below. **You do not need to simplify your answer**.
   (a) (3 points) $y = \dfrac{x^3 - 5x^4 + x^8}{e^{3x}}$.
   (b) (3 points) $y = x^{x+2}$.
   (c) (4 points) $xy^3 - x^2 = e^y - 2$.

2. (10 points) Evaluate each integral and **simplify your answer**.
   (a) (3 points) $\displaystyle\int x^5 \sin\left(x^6\right)\ dx$
   (b) (3 points) $\displaystyle\int \frac{2^{1/x^2}}{x^3}\, dx$
   (c) (4 points) $\displaystyle\int_1^e \frac{2+\ln(x)}{x}\, dx$

3. (10 points) Find the limit or state that the limit does not exist.**You must justify your answer**.
   (a) (3 points) $\displaystyle\lim_{x\to-\infty} \frac{x+3}{\sqrt{4x^2+5x-8}}$
   (b) (3 points) $\displaystyle\lim_{x\to\infty} \ln\left(x^{1/x}\right)$
   (c) (4 points) $\displaystyle\lim_{x\to 0} \left[\frac{1}{\ln(x+1)} - \frac{1}{x}\right]$

4. (10 points) This question has **part** (a) and (b).
   (a) (5 points) State the Fundamental Theorem of Calculus, Part 1 including all hypotheses.
   (b) (5 points) Let $F(x) = \displaystyle\int_0^x \frac{t^2}{t^2+t+2}\, dt$ for all real number x. Find $F''(x)$ and determine the concavity of F.

5. (10 points) Water is leaking out of an inverted conical tank at a rate of 100 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4 m. If the water level is rising at a rate of 20 cm/min when the height of the water is 0.3 m, find the rate at which water is being pumped into the tank.

   Note that the volume of the cone is $V = \dfrac{\pi}{3} r^2 h$. Also 1m=100cm

6. (10 points) Let $s(t) = t^3 - 10t^2 + 25t$ be the position function of a moving object. Find the object's acceleration each time the speed is zero.

7. (10 points) An island is $\sqrt{3}$ mi due north of its closest point along a straight shoreline. A guard is staying at a cabin on the shore that is 6 mi west of that point. The guard is planning to go from the cabin to the island. If the guard runs at a rate of 4 mi/h and swims at a rate of

2 mi/h, how far should the guard run before swimming to minimize the time it takes to reach the island?

8. (10 points) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $y = f(x) = \frac{1}{2}x^4 + \frac{8}{3}x^3 - \frac{2}{3}$.
   (a) (2 points) On which intervals is f increasing or decreasing? Explain your reasoning.
   (b) (2 points) At what values of x does f have local maximum or minimum? Explain.
   (c) (2 points) On what interval is f concave up or concave down? Explain your reasoning.
   (d) (2 points) Does f has inflection points? if so, find its x-coordinates.
   (e) (2 points) Sketch the graph of f by using all the information obtained above.

9. (10 points) This Problems has 3 **parts** labeled (a) through (c).
   (a) (4 points) Use the limit definition of derivative to find $f'(x)$ if $f(x) = \sqrt{x+1}$. **No credit will be given for any other method**.
   (b) (3 points) Find an equation of the tangent line to the curve $y = f(x)$ at $x = 3$.
   (c) (3 points) Use differentials (or linear approximations) to estimate $f(3.01)$. **You do not need to simplify your answer**.

10. (10 points) This Problems has 2 **parts** labeled (a) and (b).
   (a) (5 points) Let f be a continuous function. Give the definition for $\int_a^b f(x)\,dx$ in terms of Riemann sums.
   (b) (5 points) Use part (a) with 4 subintervals and right endpoints to estimate the integral $\int_1^9 \sqrt{\ln x}\,dx$. **You do not need to simplify your answer**.

End