Sample Final 1, Fall 2018 Math 201

Instructions: Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly. **Important:** No books, calculators, cell phones, computers, laptops or notes are allowed. You must show all your work to receive credit. Any crossed out work will be disregarded (even if correct). Write one clear answer with a coherent derivation for each question. You have 135 minutes to complete this exam. Good luck!

[1] (16 pts, 4 pts each) Compute the derivative f'(x) for each of the functions below. You do not need to simplify your answer.

(a) $f(x) = \cos(x + \frac{1}{x})$

(b)
$$f(x) = (\ln(2x)) \left(3\sqrt{x} - \frac{1}{x^2} \right)$$

(c)
$$f(x) = \frac{x^2 - 5x^5 + x^7}{e^{2x}}$$

(d)
$$f(x) = x^{x+1}$$

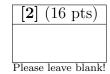
[2] (16 pts, 4 pts each) Find each integral and simplify your answer.

(a)
$$\int_1^e \frac{\ln(x)}{x} dx$$

(b) $\int (\cos^3(3x) - \cos(3x)) dx$

(c)
$$\int \frac{1}{\sqrt{1-x^2}} dx$$
, for $x \in (-1,1)$.

(d)
$$\int x 3^{x^2} dx$$

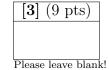


[3] (9 pts, 3 pts each) Find the limits, or state that the limit does not exist (you must justify your answer):

(a)
$$\lim_{x \to +\infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x} \right)$$

(b) $\lim_{x \to 0^+} (\ln x + \ln(\sin x))$

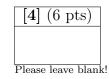
(c)
$$\lim_{x \to +\infty} (1+2x)^{1/(2\ln x)}$$



[4] (6 pts, 3 pts each)

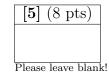
(a) Using the limit definition of the derivative, compute f'(x) if $f(x) = x^2 - 2x$ (no credit will be given for any other method).

(b) Find an equation of the tangent line to the graph $y = x^2 - 2x$ at the point (1, -1).



[5] (8 pts, 4 pts each) (a) Let $F(x) = \int_{x^2}^{\sin(x)} \sqrt{1-t^3} dt$. Find F'(x).

(b) Find an equation for the tangent line to the curve $e^{x^2y} = 2x + 2y$ at the point $(0, \frac{1}{2})$.



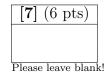
[6] (4 pts) When a circular plate is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50cm? Be sure to include units in your answer.



[7] (6 pts, 3 pts each)

(a) Let f be a differentiable function at x = a. Write down the expression for the linearization L(x) of the function f at x = a.

(b) Find an approximation for tan(0.01) using calculus.



- [8] (8 pts, 4 pts each)
- (a) State the Mean Value Theorem, including the hypotheses.

(b) Suppose f(x) is a differentiable function such that f(1) = 2 and $f'(x) \le 5$ for all x. What is the largest possible value for f(4)? You must justify your answer.



[9] (8 pts)

(a) (4 pts, 2 pts each) Let
$$f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ x^2 & \text{if } x > 1. \end{cases}$$

(i) Sketch the graph of y = f(x) for $x \in [0, 4]$.

(ii) Is f(x) continuous at the point x = 1? Please justify your answer.

(b) (4 pts) For the function f above, use a Riemann Sum to estimate $\int_{0}^{4} f(x)dx$ by using the Midpoint Rule with 4 subdivisions. You may have seen to a function of the function of the subdivision of the subdivisio

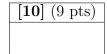
using the Midpoint Rule with 4 subdivisions. You may leave your answer as a sum of unsimplified fractions.



[10] (9 pts) A cylindrical can is made of two different materials: the side is made of a material that costs 1 dollar per square foot, while the top and bottom are made of a material that costs 2 dollars per square foot.

If the total volume of the can must be 1 cubic feet, find the dimensions of the can that minimize cost.

Note: Justify your answer using calculus.



[11] (8 pts) For the function $f(x) = \frac{x^2}{x^2 - 4}$, you are given (do not compute!) that

$$f'(x) = \frac{-8x}{(x^2 - 4)^2}$$
 and $f''(x) = \frac{8(3x^2 + 4)}{(x^2 - 4)^3}$.

(a) Find the domain of f(x).

(b) Find the coordinates of all intercepts, and the equations of all asymptotes, of the graph of y = f(x).

(c) In what intervals is the function f increasing? decreasing?

(d) In what intervals is the graph of f concave up? concave down?

(e) Find the coordinates of all local maxima, local minima, and points of inflection of the function f.

(f) Sketch the graph of y = f(x). Label the features you found in items b and e.

