Sample Final 1, Fall 2018
Math 201

Instructions: Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly. **Important: No books, calculators, cell phones, computers, laptops or notes are allowed.** You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation for each question. You have 135 minutes to complete this exam. Good luck!

[1] (16 pts, 4 pts each) Compute the derivative $f'(x)$ for each of the functions below. You do not need to simplify your answer.

(a) $f(x) = \cos(x + \frac{1}{x})$

(b) $f(x) = (\ln(2x))\left(3\sqrt{x} - \frac{1}{x^2}\right)$

(c) $f(x) = \dfrac{x^2 - 5x^5 + x^7}{e^{2x}}$

(d) $f(x) = x^{x+1}$

[**1**] (16 pts)

Please leave blank!

[2] **(16 pts, 4 pts each)** Find each integral and simplify your answer.

(a) $\displaystyle\int_1^e \frac{\ln(x)}{x}dx$

(b) $\int(\cos^3(3x) - \cos(3x))dx$

(c) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}dx$, for $x \in (-1,1)$.

(d) $\displaystyle\int x3^{x^2}\,dx$

[**2**] (16 pts)

Please leave blank!

[3] (9 pts, 3 pts each) Find the limits, or state that the limit does not exist (you must justify your answer):

(a) $\lim_{x \to +\infty} \left(\sqrt{x^2 + 3x} - \sqrt{x^2 - 2x} \right)$

(b) $\lim_{x \to 0^+} (\ln x + \ln(\sin x))$

(c) $\lim_{x \to +\infty} (1 + 2x)^{1/(2 \ln x)}$

[3] (9 pts)

Please leave blank!

[4] **(6 pts, 3 pts each)**

(a) Using the limit definition of the derivative, compute $f'(x)$ if $f(x) = x^2 - 2x$ (no credit will be given for any other method).

(b) Find an equation of the tangent line to the graph $y = x^2 - 2x$ at the point $(1, -1)$.

[4] (6 pts)

Please leave blank!

[5] (8 pts, 4 pts each)

(a) Let $F(x) = \int_{x^2}^{\sin(x)} \sqrt{1 - t^3}\, dt$. Find $F'(x)$.

(b) Find an equation for the tangent line to the curve $e^{x^2 y} = 2x + 2y$ at the point $(0, \frac{1}{2})$.

[5] (8 pts)

Please leave blank!

[6] **(4 pts)** When a circular plate is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plate's area increasing when the radius is 50cm? Be sure to include units in your answer.

[6] (4 pts)

Please leave blank!

[7] (6 pts, 3 pts each)

(a) Let f be a differentiable function at $x = a$. Write down the expression for the linearization $L(x)$ of the function f at $x = a$.

(b) Find an approximation for $\tan(0.01)$ using calculus.

[7] (6 pts)

Please leave blank!

[8] **(8 pts, 4 pts each)**

(a) State the Mean Value Theorem, including the hypotheses.

(b) Suppose $f(x)$ is a differentiable function such that $f(1) = 2$ and $f'(x) \leq 5$ for all x. What is the largest possible value for $f(4)$? You must justify your answer.

[8] (8 pts)

Please leave blank!

[9] **(8 pts)**

(a) (4 pts, 2 pts each) Let $f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } x = 1, \\ x^2 & \text{if } x > 1. \end{cases}$

(i) Sketch the graph of $y = f(x)$ for $x \in [0, 4]$.

(ii) Is $f(x)$ continuous at the point $x = 1$? Please justify your answer.

(b) (4 pts) For the function f above, use a Riemann Sum to estimate $\int_0^4 f(x)dx$ by using the Midpoint Rule with 4 subdivisions. You may leave your answer as a sum of unsimplified fractions.

[**9**] (8 pts)

Please leave blank!

[10] (9 pts) A cylindrical can is made of two different materials: the side is made of a material that costs 1 dollar per square foot, while the top and bottom are made of a material that costs 2 dollars per square foot.

If the total volume of the can must be 1 cubic feet, find the dimensions of the can that minimize cost.

Note: Justify your answer using calculus.

[10] (9 pts)

Please leave blank!

[11] (8 pts) For the function $f(x) = \dfrac{x^2}{x^2 - 4}$, you are given (do not compute!) that

$$f'(x) = \frac{-8x}{(x^2-4)^2} \quad \text{and} \quad f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find the coordinates of all intercepts, and the equations of all asymptotes, of the graph of $y = f(x)$.

(c) In what intervals is the function f increasing? decreasing?

(d) In what intervals is the graph of f concave up? concave down?

(e) Find the coordinates of all local maxima, local minima, and points of inflection of the function f.

(f) Sketch the graph of $y = f(x)$. Label the features you found in items b and e.

[11] (8 pts)

Please leave blank!