Summer 2026 - Practice Exam 1

MATH 20100 | June 14, 2026

Name: ______________________________

Answer every problem to the best of your abilities. Calculators, notes, or any other outside assistance are not permitted on the real exam. Show all work, as answers without work will not receive credit. This is a little more than 3 times the actual number of problems you will see on the real exam.

Part I (Definitions (10 points))	__________	/ 10
Part II (Limits (45 points))	__________	/ 45
Part III (Derivatives (45 points))	__________	/ 45
Total	__________	/ 100

Part I. Definitions (10 points)

1. State the precise definition of $\lim_{x\to a} f(x) = L$.

2. State the precise definition of $\lim_{x\to a} f(x) = \infty$

3. State the precise definition of $\lim_{x\to a} f(x) = -\infty$

4. State the definition of a function f being continuous at a.

5. List the 3 ways that we discussed a function being discontinuous at a point, and find an example function $f(x)$ that contains each type of discontinuity.

6. State the precise definition of $\lim_{x\to\infty} f(x) = \infty$.

7. State the precise definition of $\lim_{x\to-\infty} f(x) = \infty$.

8. State the precise definition of $\lim_{x\to-\infty} f(x) = -\infty$.

9. State the precise definition of $\lim_{x\to\infty} f(x) = -\infty$.

Part II. Limits (45 points)

Find the limits of the given functions. Show each step of work, and write DNE if the limit does not exist.

1. Find the following limits using limit laws

   (a) $\lim_{x\to\infty} \frac{2x^3-2}{3x^2-x+5}$

   (b) $\lim_{x\to\infty} \frac{5-\sqrt{2x^5+1}}{4x^2+1}$

   (c) $\lim_{x\to\infty} \frac{2x^3+11x^2-1}{\sqrt{49x^6+19}-x^2+2}$

   (d) $\lim_{x\to 3} \frac{x^2-2-15}{(x-3)}$

(e) $\lim_{x\to -2} \frac{(x^2-10x-24)(x^2+5x+6)}{(x+2)^2}$

(f) $\lim_{x\to 4} \frac{(x^2-16)(x^2-x-12)}{(x-4)^2}$

(g) $\lim_{x\to 2} \frac{x-2}{|x-2|}$

(h) $\lim_{x\to -1} \frac{x^2-1}{|x+1|}$

(i) $\lim_{x\to 4} \frac{x^2-2x-8}{|x-4|}$

(j) $\lim_{x\to\infty} \left(\sqrt{9x^2+ax} - \sqrt{9x^2+bx}\right)$

(k) $\lim_{x\to\infty} \left(\sqrt{x^2+ax+1} - \sqrt{x^2+bx+1}\right)$

(l) $\lim_{x\to -\infty} \frac{3x^2+7x-2}{\sqrt[3]{8x^6-x^3+4}}$

(m) $\lim_{x\to-\infty} \frac{\sqrt[4]{x^8+3x^5-1}}{2x^2+x-7}$

(n) $\lim_{x\to-\infty} \frac{3x^3+2x-1}{\sqrt[5]{x^{15}+4x^{10}-2}}$

(o) $\lim_{x\to\infty}(\arctan(x)+\ln(\frac{1+x}{x}))$

(p) $\lim_{x\to 1^-}(\arctan(\ln(1-x)))$

2. Where are the vertical asymptotes of $f(x) = \frac{ln(-2+x^2)}{1-e^{1-\sin(x)}}$? Justify by showing $\lim_{x\to a} f(x) = \pm\infty$ for each a such that $x=a$ is a vertical asymptote.

3. Where are the vertical asymptotes of $f(x) = \frac{\sqrt{x+4}}{1-\cos^2(x)}$? Justify by showing $\lim_{x\to a} f(x) = \pm\infty$ for each a such that $x=a$ is a vertical asymptote.

4. Using the (ε, δ) definition of a limit, prove rigorously that $\lim_{x\to 7}(2x-3) = 11$.

5. Using the (ε, δ) definition of a limit, prove rigorously that $\lim_{x\to a}(19x+100) = 19a+100$ for any real number a.

Part III. Derivatives (45 points)

Compute the derivatives of the following functions. For question 1, you must use the limit definition of the derivative. For question 2 you may use the derivative rules we have to complete the computation. Evaluate at the given value of x if specified by the problem, and otherwise compute the general formula. If the domain is not the same as the original function, state what it changes to.

1. Use the limit definition of a derivative to compute the following derivatives.

   (a) $\frac{d}{dx}(2x^2 + 5x - 3)$ at $x = 2$

   (b) $\frac{d}{dx}(4x^2 - 3x + 7)$ at $x = -1$

   (c) $\frac{d}{dx}(x^2 + 6x - 2)$

   (d) $\frac{d}{dx}\left(\frac{1}{x}\right)$

   (e) $\frac{d}{dx}\left(\frac{-19}{x^3} + 1\right)$

   (f) $\frac{d}{dx}\left(\frac{5}{x^2}\right)$ at $x = 2$

(g) $\frac{d}{dx}\left(\frac{x-1}{x+3}\right)$

(h) $\frac{d}{dx}\left(\frac{2x+1}{x-4}\right)$

(i) $\frac{d}{dx}\left(\frac{3-x}{1+x}\right)$

2. Use the derivative rules we have discussed in class to compute the following derivatives.

   (a) $\frac{d}{dx}\left(\frac{x}{x^{-1}+x^{-2}}\right)$

   (b) $\frac{d}{dx}\left(\frac{x^2}{x^{-1}+x^{-3}}\right)$

   (c) $\frac{d}{dx}\left(e^x(x+1)\right)$

   (d) $\frac{d}{dx}\left(\frac{e^x}{\sqrt[3]{x}+1}\right)$

   (e) $\frac{d}{dx}\left(\frac{1+x^4}{e^x(x-5)}\right)$

(f) $\frac{d}{dx}\left(\frac{\sqrt[3]{x}+2}{x^{4/3}-1}\right)$

(g) $\frac{d}{dx}\left(\frac{x^{3/2}+1}{\sqrt{x}-3}\right)$

(h) $\frac{d}{dx}\left(\frac{\sqrt[4]{x}-1}{x^{5/4}+2}\right)$

3. Find the equation of the tangent line to $f(x) = -5x^2 + 2x - 3$ at $(1, -6)$.

4. Find the equation of the tangent line to $f(x) = 5^x(x-3)^{-1}$ at $(0, \frac{-1}{3})$.

5. Find the equation of the tangent line to $f(x) = 3x^2 - 4x + 1$ at each $a \in \mathbb{R}$. Your answer should provide a formula for the line for each possible value of a.

6. Find the equation of the tangent line to $f(x) = \frac{2+x}{1-x}$ at each $a \in \mathbb{R}$. Your answer should provide a formula for the line for each possible value of a.

7. Find the equation of the tangent line to $f(x) = (x+4)^3(2-x)^4$ at each $a \in \mathbb{R}$. Your answer should provide a formula for the line for each possible value of a.