## DEPARTMENT OF MATHEMATICS, CCNY PRACTICE MIDTERM 2- MATH 201 FALL 2022

Student's Last Name, First Name: \_

Instruction: You must write very clear and neat, and show all your work to receive credit. This midterm and your upcoming midterm 2 cover only chapter 3. Specifically on sections: 3,1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8 and 3.9

- 1. Let  $f(x) = \frac{1}{\sqrt{x}}$ . Find
  - (a) f'(x) use the limit definition of the derivative; then check your answer by direct differentiation.
  - (b) The equation of the tangent line to the curve y = f(x) at x = 4 using part (a).
- 2. Let f(x) = |x+2|. Is f continuous at x = -2? Is f differentiable at x = 2

3. Let 
$$f(x) = \begin{cases} x^7 \sin(1/x) & ; \quad x \neq 0. \\ 0 & ; \quad x = 0 \end{cases}$$

Is f differentiable at x = 0? If so, find f'(0). Is f continuous at x = 0?

4. Let 
$$f(x) = \begin{cases} 4x+1 & ; & x \ge 1 \\ x^2+2x+7 & ; & x < 1. \end{cases}$$

Is f differentiable at x = 0? If so, find f'(0). Is f continuous at x = 0?

- 5. Is there a value of  $\alpha$  and  $\beta$  that make  $f(x) = \begin{cases} x^2 + 2\alpha + \beta & ; & x < 0\\ 1 + 4\sin(\pi x/6) & ; & 0 \le x \le 1\\ 3\beta + \alpha\cos(\pi x) & ; & x > 1 \end{cases}$  continuous for all x?
- 6. Find the derivative of the following:

(a) 
$$y = \frac{x^4 - 8x^9 + \sqrt[5]{x} - \pi}{x^{3/2}}$$
.  
(b)  $g(x) = (2x - 3)^4 (x^2 + x + 1)^5$   
(c)  $f(x) = \left(\frac{2\sqrt{x}}{2\sqrt{x} + 1}\right)$   
(d)  $y = \sin(4\tan(x^3 + x))$   
(e)  $e^{1+x+\sec(3x)}$   
(f)  $(2x - 3)^4 (x^2 + x + 1)^5$ 

(g) 
$$y = \sqrt{\frac{s^2 + 1}{s^2 + 4}}$$
  
(h)  $y = x \cos^{-1}(x) - \sqrt{1 - x^2}$   
(i)  $y = \sqrt[3]{\frac{2x - 5}{x + 7}}$   
(j)  $y = \sqrt{3t + \sqrt{2 + \sqrt{1 - t}}}$   
(k)  $y = \cos^4(\sec^2(3x))$ 

7. Find 
$$\frac{dy}{dx}$$
 by implicit differentiation.  
(a)  $2x^3 + x^2y - xy^3 = 2$   
(b)  $y^5 + x^2y^3 = 1 + x^4y$   
(c)  $\cos(xy) = 1 + \sin(y)$   
(d)  $\tan(x/y) = x + y$ 

- 8. Find y'' by implicit differentiation.
  - (a)  $9x^2 + y^2 = 9$ (b)  $\sqrt{x} + \sqrt{y} = 1$ (c)  $x^3 + y^3 = 1$ (d)  $x^4 + y^4 = a^4$  where *a* is a constant
- 9. Find the derivative of the following; that is find  $f^n(x)$ .
  - (a) A formula for the  $n^{th}$  derivative of  $f(x) = xe^x$ .
  - (b) A formula for the  $n^{th}$  derivative of f(x) = 1/x. (c)  $\frac{d^{13673}y}{dx^{13673}}$  [sin(3x)].
- 10. Find the points on the curves  $y = x^4 6x^2 + 4$  where the tangent line is horizontal.
- 11. Let  $s(t) = t^3 6t^2 + 9t$  meter be the portion of a body along the horizontal s-axis
  - (a) Find the body's acceleration , a(t), each time the velocity is zero.
  - (b) Find the body's speed each time the acceleration is zero.
  - (c) find the total distance traveled by the body from t = 0 to t = 2.
- 12. At time  $t \ge 0$ , the velocity of a body moving along the horizontal s-axis is  $v(t) = t^2 4t + 3$ .
  - (a) Find the body's acceleration each time the velocity is zero.
  - (b) When is the body moving forward? Backward?
  - (c) When is the body's velocity increasing? Decreasing?

13. Suppose that the functions f and g and their first derivatives f'(x) and g'(x) have the following values at x = 0 and x = 1.

x	f(x)	g(x)	f'(x)	g'(x)
0	1	1	-3	1/2
1	3	5	1/2	-4

Find the derivative of the following at the given value of x.

(a)  $6f(x) - g(x), \quad x = 1$ 

(b) 
$$\frac{f(x)}{g(x)+1}$$
,  $x = 1$   
(c)  $\sqrt[3]{[x+f(x)]^2}$ ,  $x = 0$ 

14. Find all points (x, y) on the graph of  $y = f(x) = (x - 3)^2 + 2$  with tangent lines passing the point (6,10).