# Department of Mathematics, CCNY <br> Practice Midterm 2- Math 201 <br> FALL 2022 

## Student's Last Name, First Name:

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Instruction: You must write very clear and neat, and show all your work to receive credit. This midterm and your upcoming midterm 2 cover only chapter 3 . Specifically on sections: $\mathbf{3 , 1}, \mathbf{3 . 2}$,

## $3.3,3.4,3.5,3.6,3.7,3.8$ and 3.9

1. Let $f(x)=\frac{1}{\sqrt{x}}$. Find
(a) $f^{\prime}(x)$ use the limit definition of the derivative; then check your answer by direct differentiation.
(b) The equation of the tangent line to the curve $y=f(x)$ at $x=4$ using part (a).
2. Let $f(x)=|x+2|$. Is $f$ continuous at $x=-2$ ? Is $f$ differentiable at $x=2$
3. Let $f(x)=\left\{\begin{array}{ccc}x^{7} \sin (1 / x) & ; & x \neq 0 . \\ 0 & ; & x=0\end{array}\right.$

Is $f$ differentiable at $x=0$ ? If so, find $f^{\prime}(0)$. Is $f$ continuous at $x=0$ ?
4. Let $f(x)=\left\{\begin{array}{cll}4 x+1 & ; & x \geq 1 \\ x^{2}+2 x+7 & ; & x<1 .\end{array}\right.$

Is $f$ differentiable at $x=0$ ? If so, find $f^{\prime}(0)$. Is $f$ continuous at $x=0$ ?
5. Is there a value of $\alpha$ and $\beta$ that make $f(x)=\left\{\begin{array}{ccc}x^{2}+2 \alpha+\beta & ; & x<0 \\ 1+4 \sin (\pi x / 6) & ; & 0 \leq x \leq 1 \\ 3 \beta+\alpha \cos (\pi x) & ; & x>1\end{array}\right.$ continuous for all $x$ ? Differentiable for all $x$ ?
6. Find the derivative of the following:
(a) $y=\frac{x^{4}-8 x^{9}+\sqrt[5]{x}-\pi}{x^{3 / 2}}$.
(b) $g(x)=(2 x-3)^{4}\left(x^{2}+x+1\right)^{5}$
(c) $f(x)=\left(\frac{2 \sqrt{x}}{2 \sqrt{x}+1}\right)$
(d) $y=\sin \left(4 \tan \left(x^{3}+x\right)\right)$
(e) $e^{1+x+\sec (3 x)}$
(f) $(2 x-3)^{4}\left(x^{2}+x+1\right)^{5}$
(g) $y=\sqrt{\frac{s^{2}+1}{s^{2}+4}}$
(h) $y=x \cos ^{-1}(x)-\sqrt{1-x^{2}}$
(i) $y=\sqrt[3]{\frac{2 x-5}{x+7}}$
(j) $y=\sqrt{3 t+\sqrt{2+\sqrt{1-t}}}$
(k) $y=\cos ^{4}\left(\sec ^{2}(3 x)\right)$
7. Find $\frac{d y}{d x}$ by implicit differentiation.
(a) $2 x^{3}+x^{2} y-x y^{3}=2$
(b) $y^{5}+x^{2} y^{3}=1+x^{4} y$
(c) $\cos (x y)=1+\sin (y)$
(d) $\tan (x / y)=x+y$
8. Find $y^{\prime \prime}$ by implicit differentiation.
(a) $9 x^{2}+y^{2}=9$
(b) $\sqrt{x}+\sqrt{y}=1$
(c) $x^{3}+y^{3}=1$
(d) $x^{4}+y^{4}=a^{4}$ where $a$ is a constant
9. Find the derivative of the following; that is find $f^{n}(x)$.
(a) A formula for the $n^{t h}$ derivative of $f(x)=x e^{x}$.
(b) A formula for the $n^{\text {th }}$ derivative of $f(x)=1 / x$.
(c) $\frac{d^{13673} y}{d x^{13673}}[\sin (3 x)]$.
10. Find the points on the curves $y=x^{4}-6 x^{2}+4$ where the tangent line is horizontal.
11. Let $s(t)=t^{3}-6 t^{2}+9 t$ meter be the portion of a body along the horizontal $s$-axis
(a) Find the body's acceleration , $a(t)$, each time the velocity is zero.
(b) Find the body's speed each time the acceleration is zero.
(c) find the total distance traveled by the body from $t=0$ to $t=2$.
12. At time $t \geq 0$, the velocity of a body moving along the horizontal $s$-axis is $v(t)=t^{2}-4 t+3$.
(a) Find the body's acceleration each time the velocity is zero.
(b) When is the body moving forward? Backward?
(c) When is the body's velocity increasing? Decreasing?
13. Suppose that the functions $f$ and $g$ and their first derivatives $f^{\prime}(x)$ and $g^{\prime}(x)$ have the follwing values at $x=0$ and $x=1$.

| $x$ | $f(x)$ | $g(x)$ | $f^{\prime}(x)$ | $g^{\prime}(x)$ |
| :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 1 | -3 | $1 / 2$ |
| 1 | 3 | 5 | $1 / 2$ | -4 |

Find the derivative of the following at the given value of $x$.
(a) $6 f(x)-g(x), \quad x=1$
(b) $\frac{f(x)}{g(x)+1}, \quad x=1$
(c) $\sqrt[3]{[x+f(x)]^{2}}, \quad x=0$
14. Find all points $(x, y)$ on the graph of $y=f(x)=(x-3)^{2}+2$ with tangent lines passing the point $(6,10)$.

