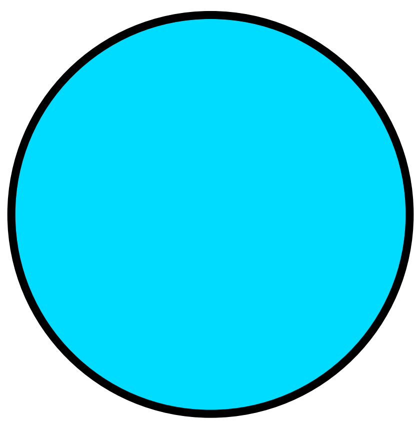
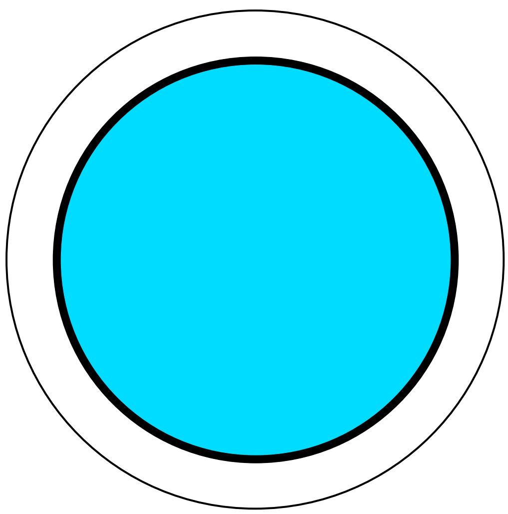
On the
Geometry
of
Orbits

The Possible Orbits

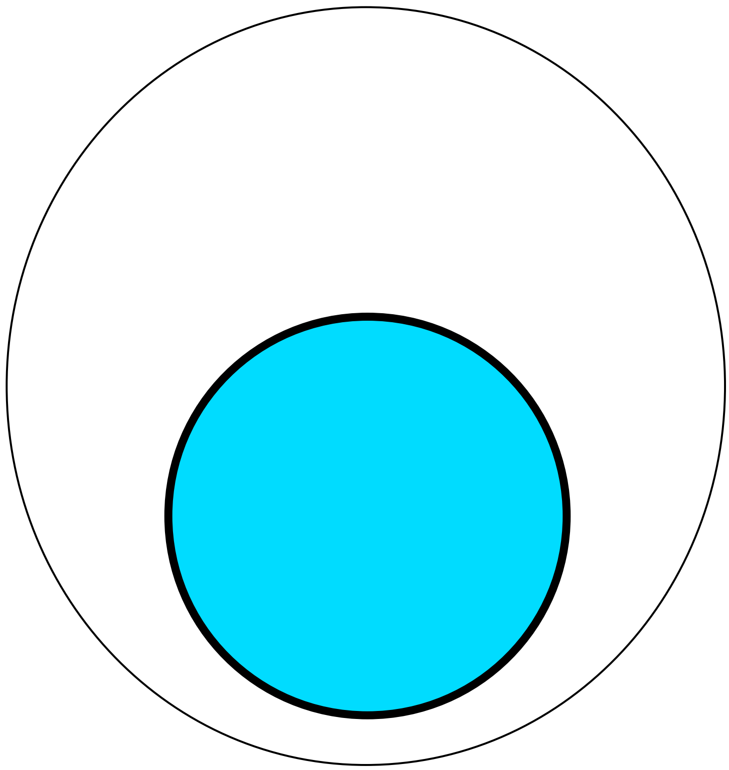


The Possible Orbits



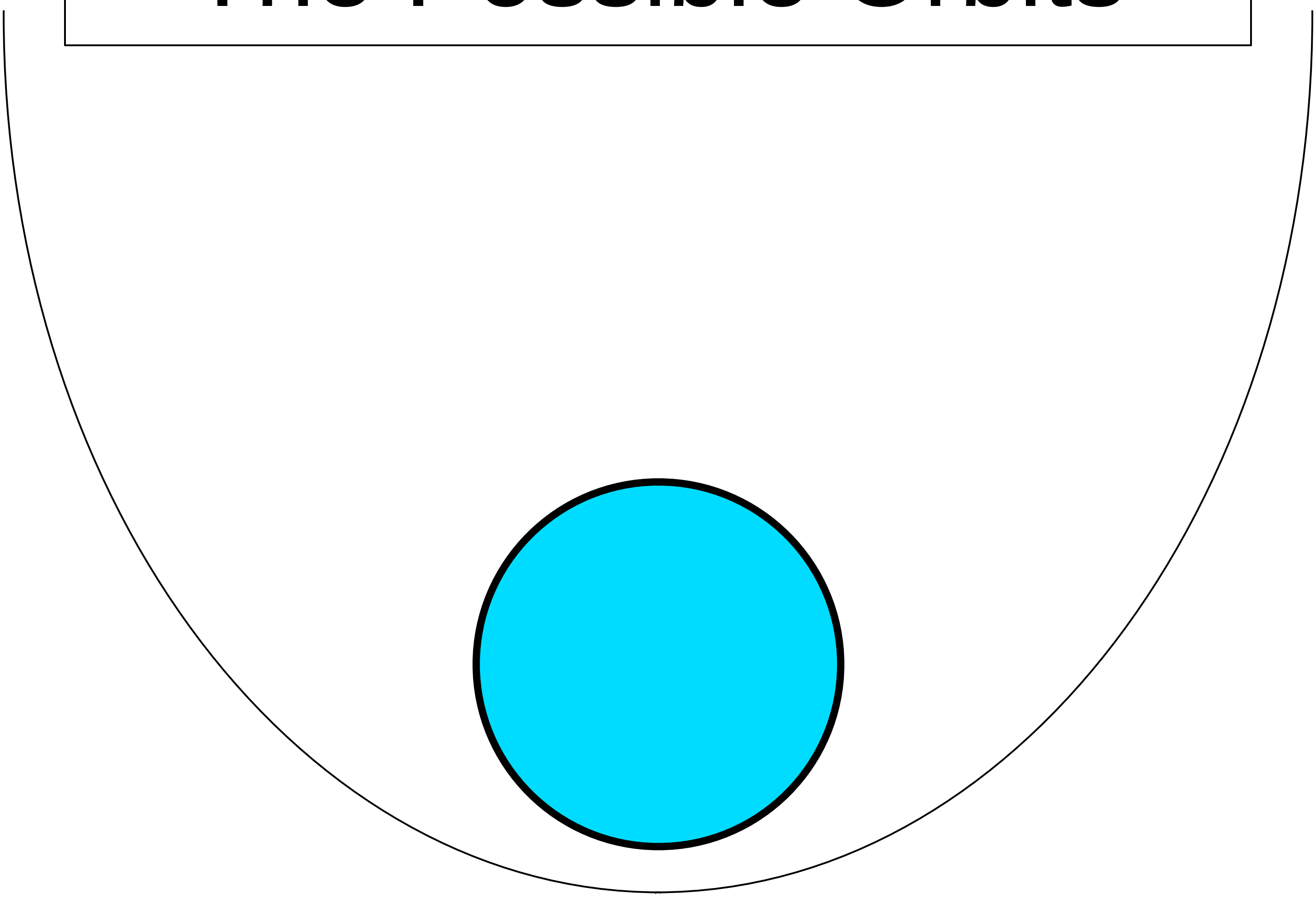
circle

The Possible Orbits



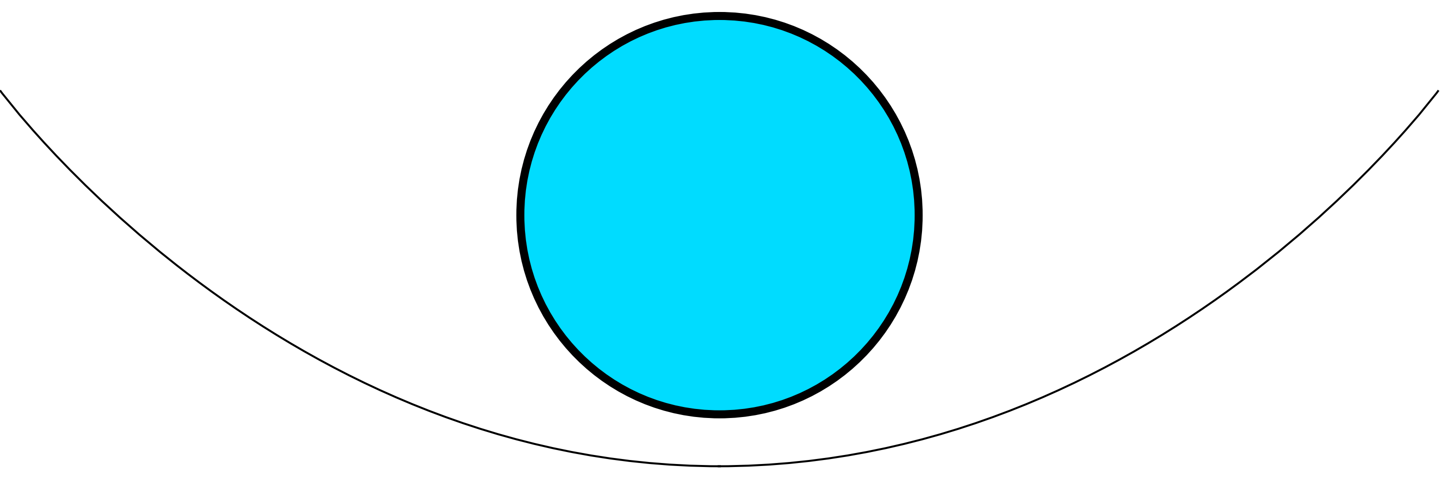
ellipse

The Possible Orbits



parabola

The Possible Orbits



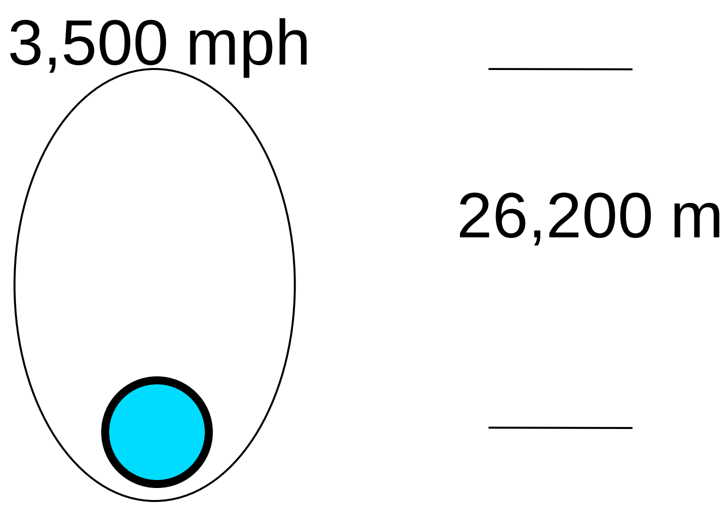
hyperbola

Speed and Distance

17,600 mph

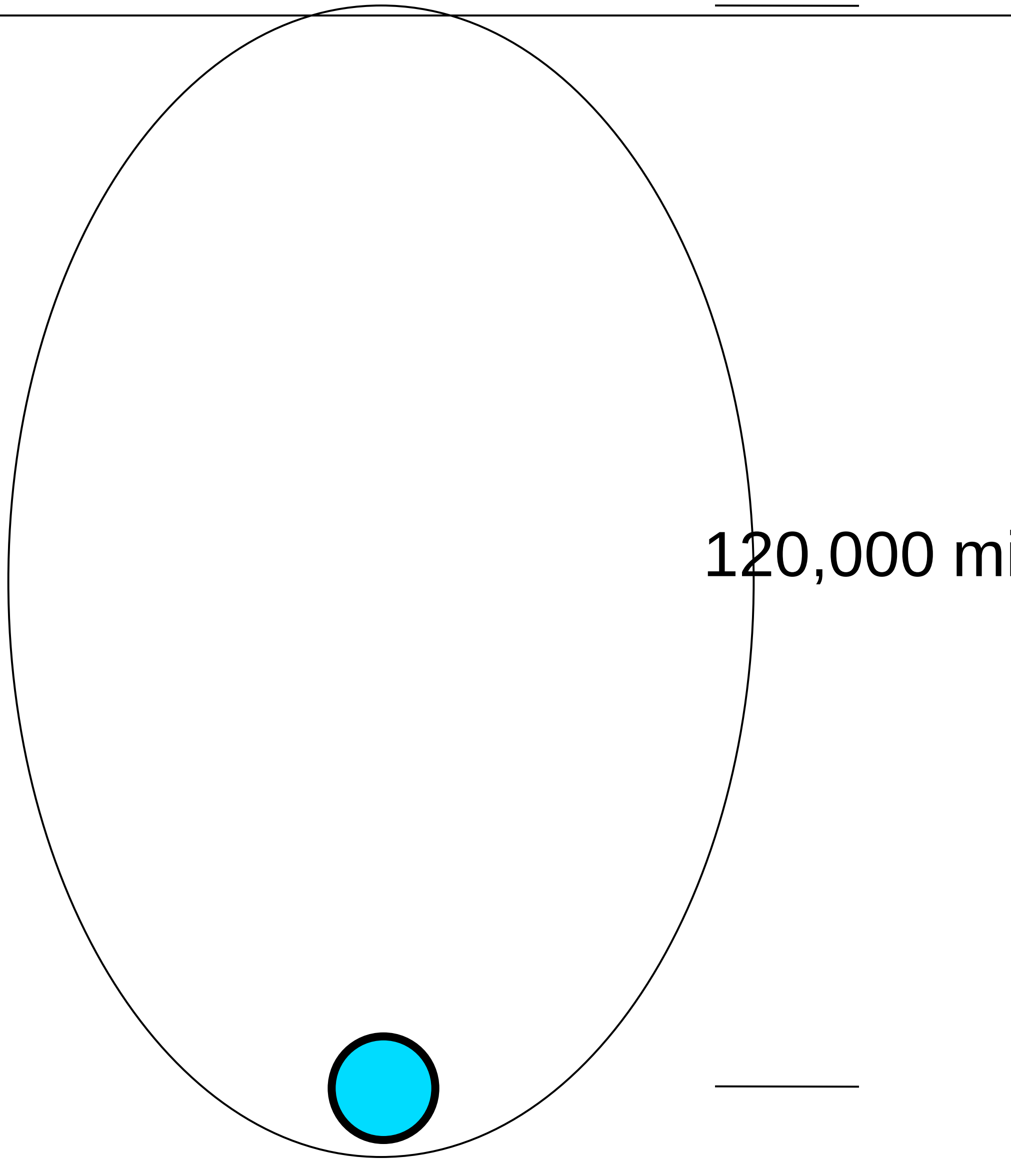
1.4 hr

Speed and Distance



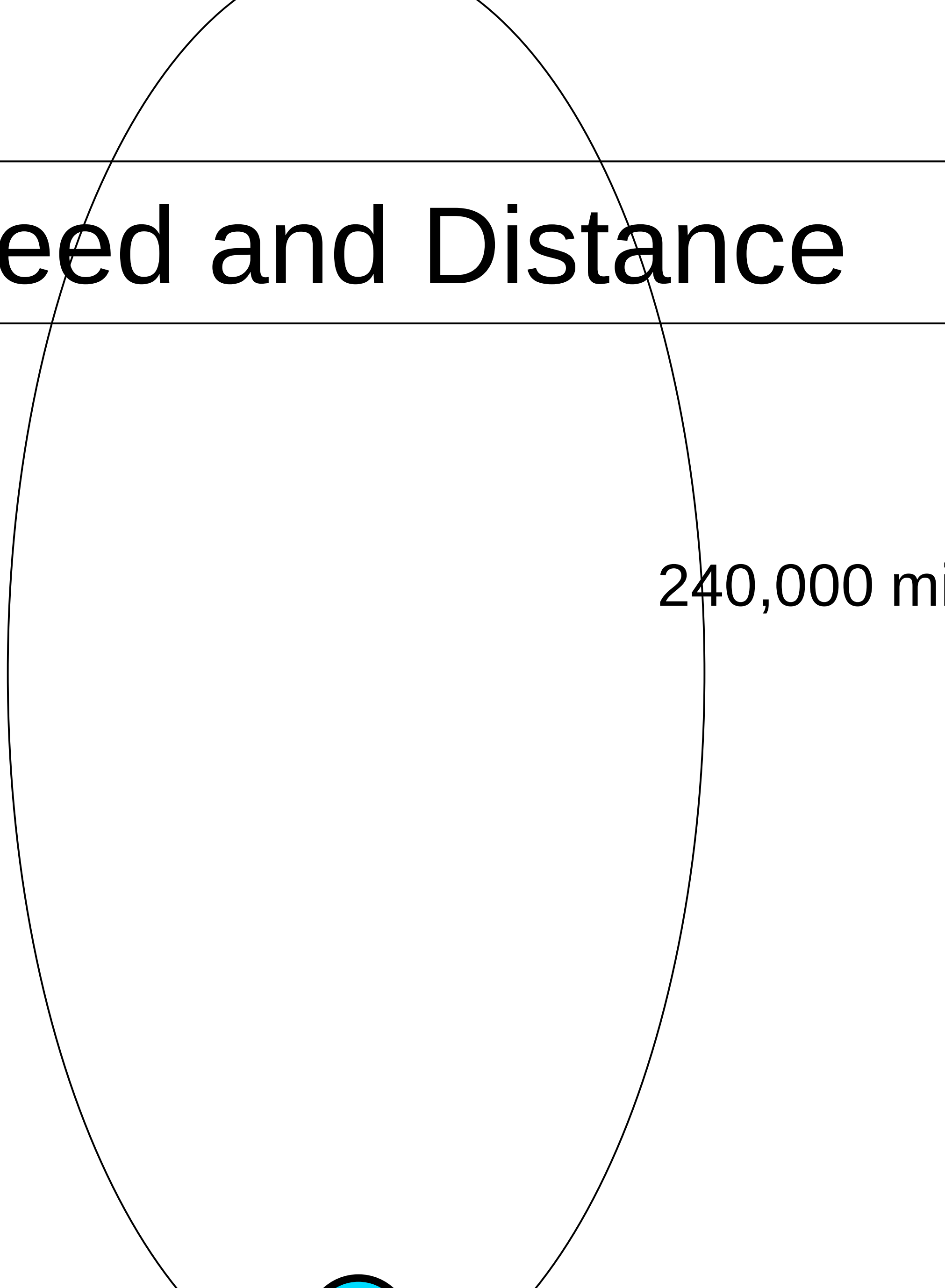
Add 32%
23,200 mph
10.4 hr

Speed and Distance

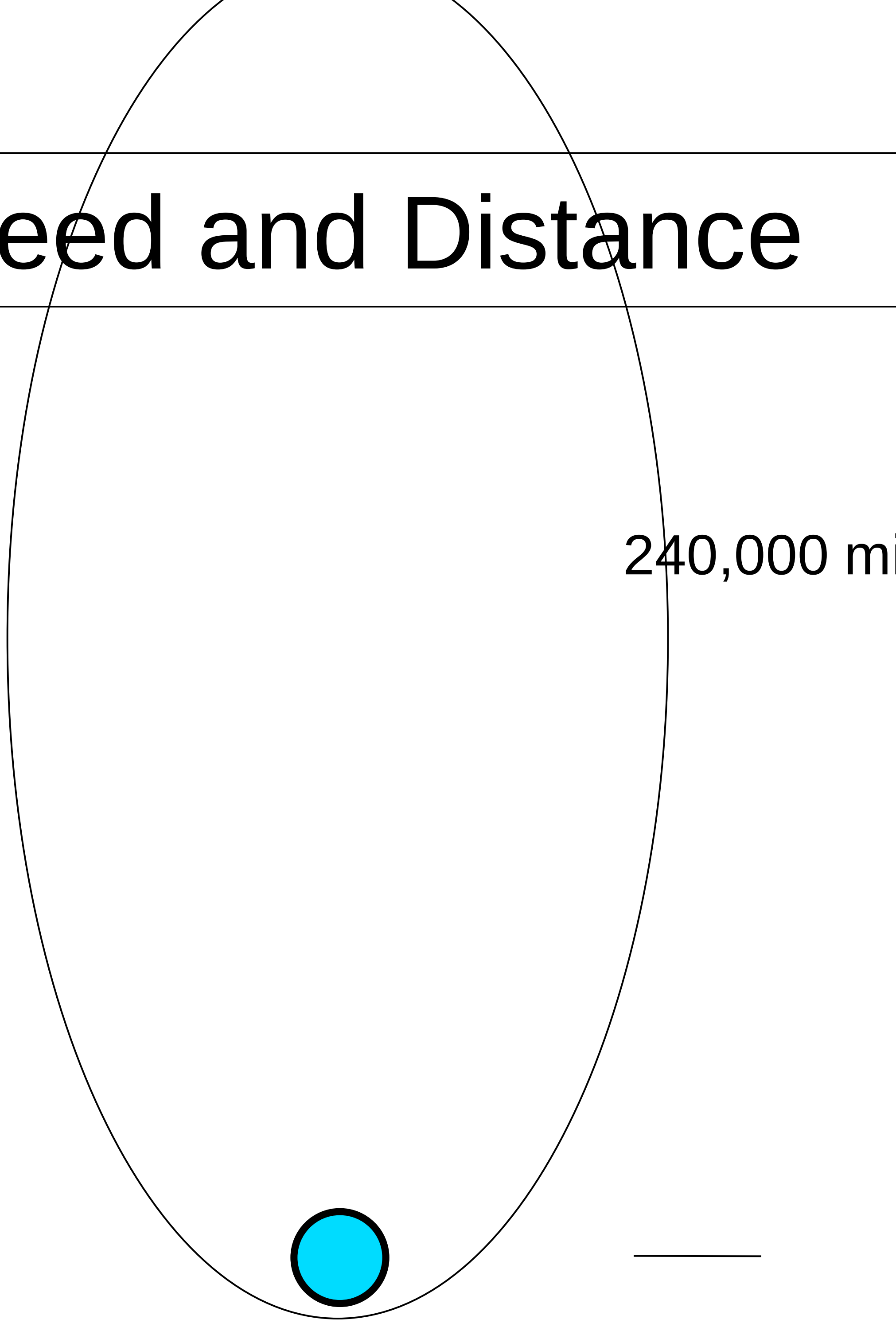


Add 39%
24,500 mph

Speed and Distance



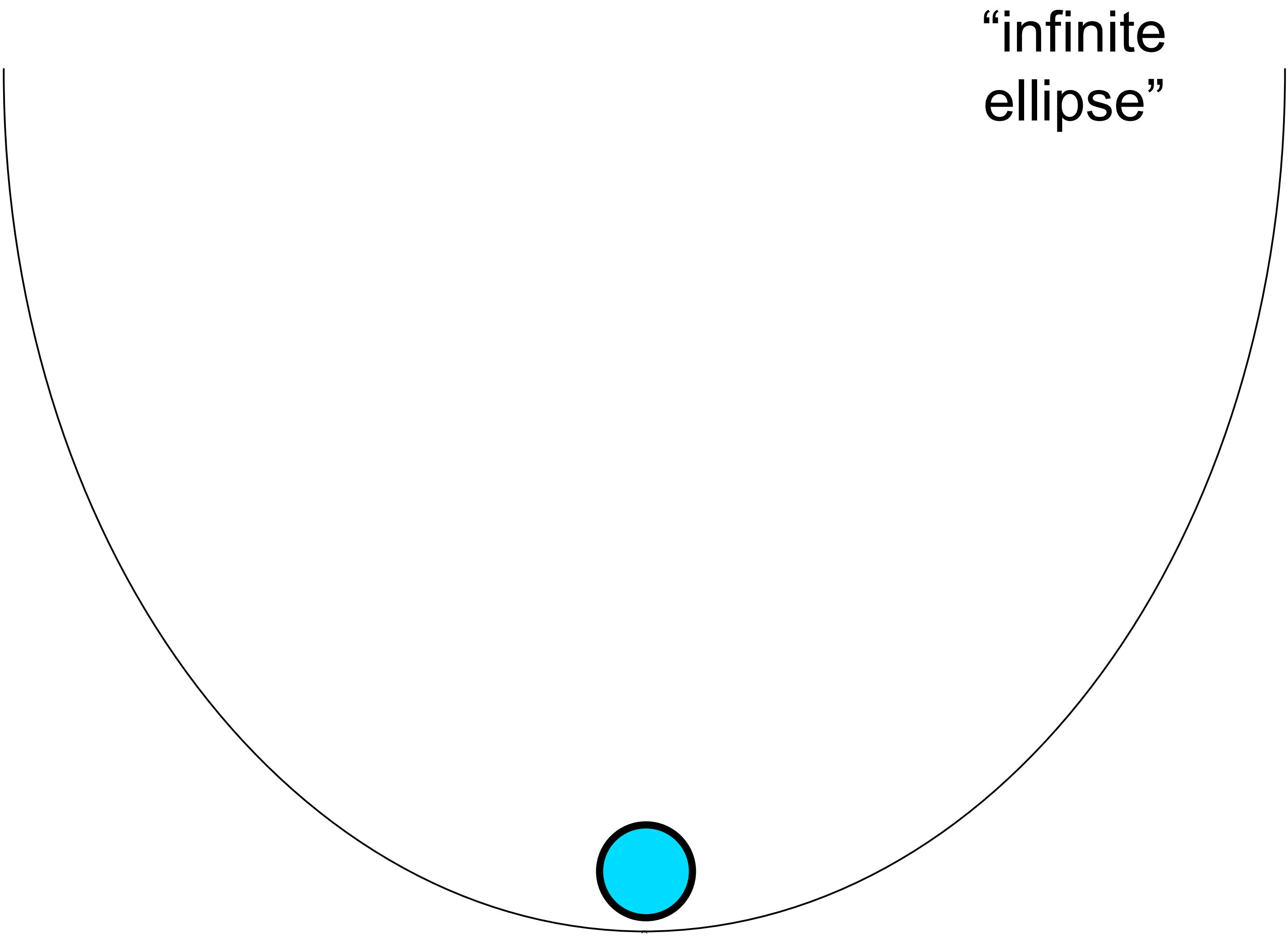
Speed and Distance



Add 40%

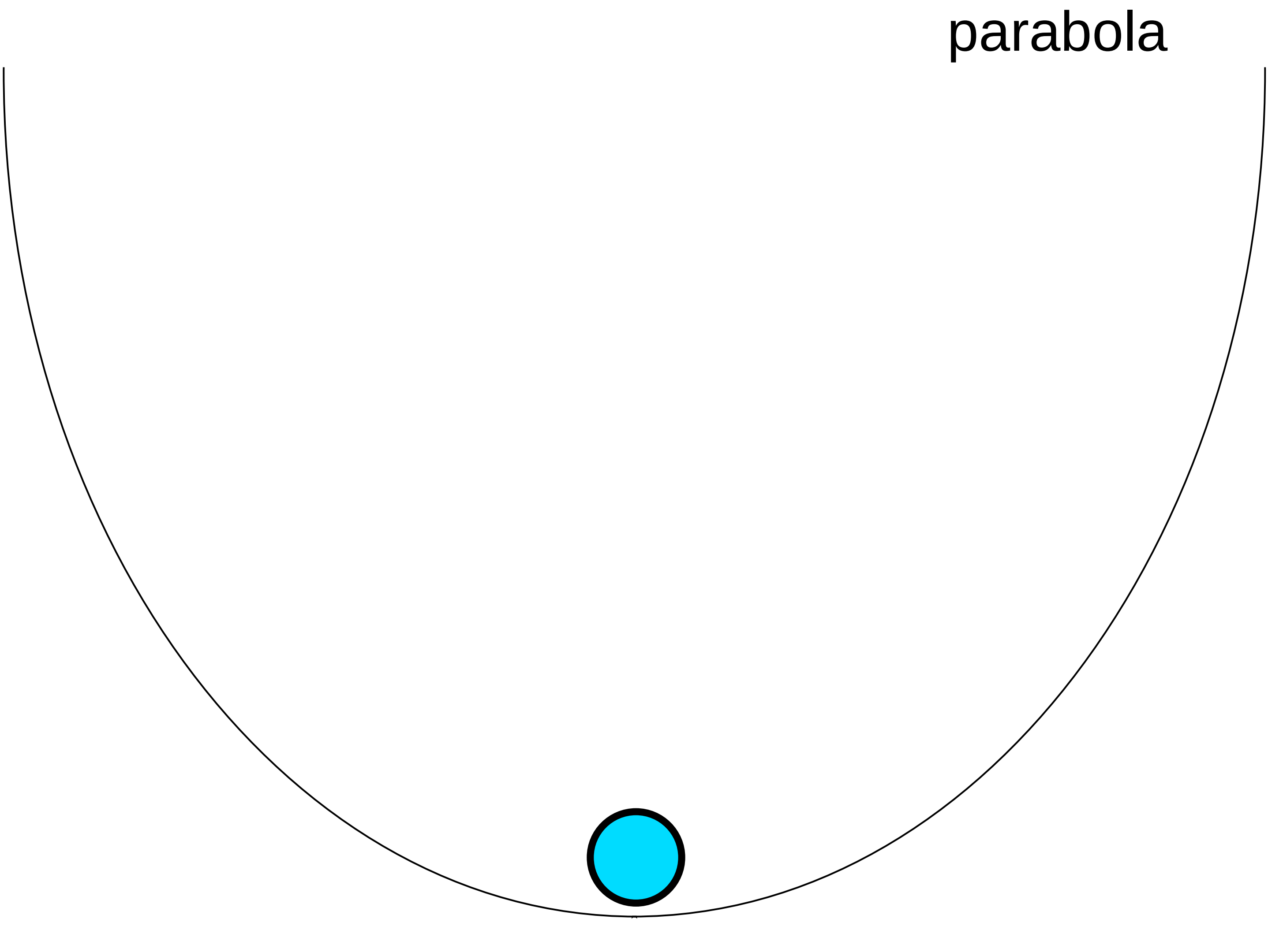
24,640 mph

Speed and Distance



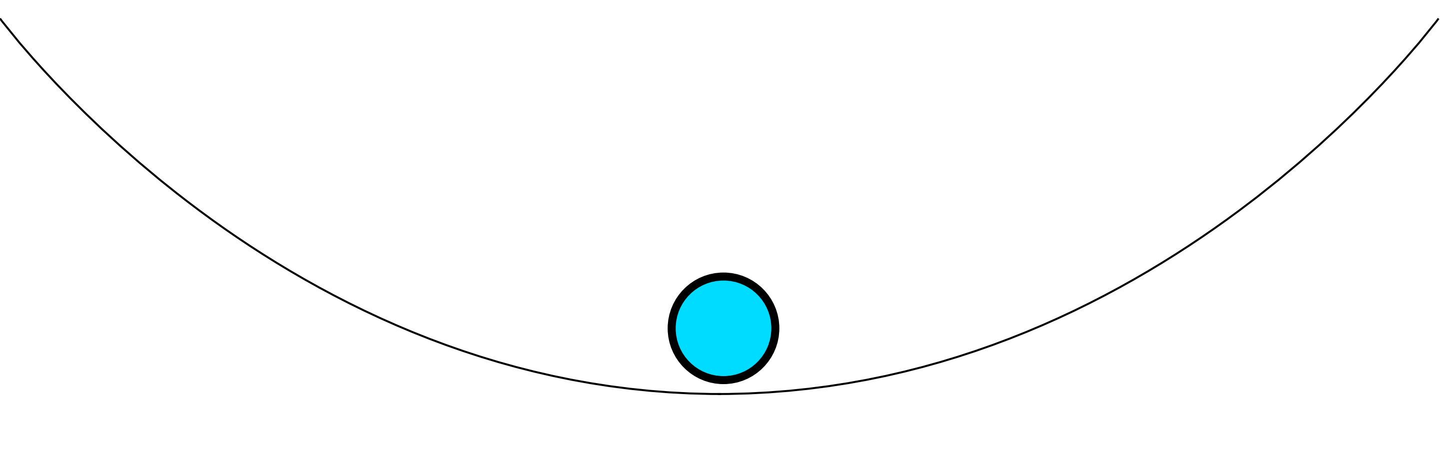
Add 41.4%
24,900 mph

Speed and Distance

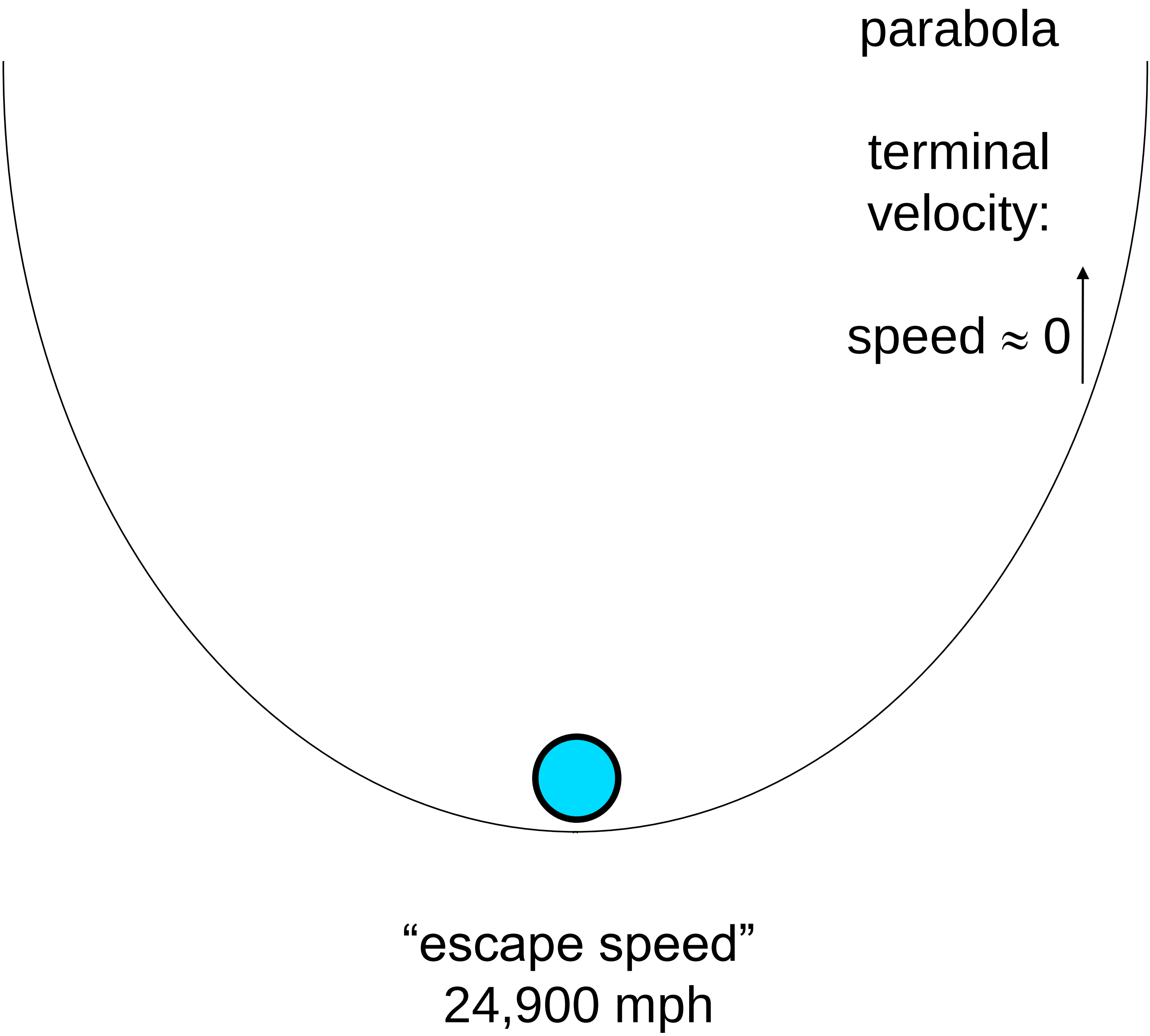


"escape speed"
24,900 mph

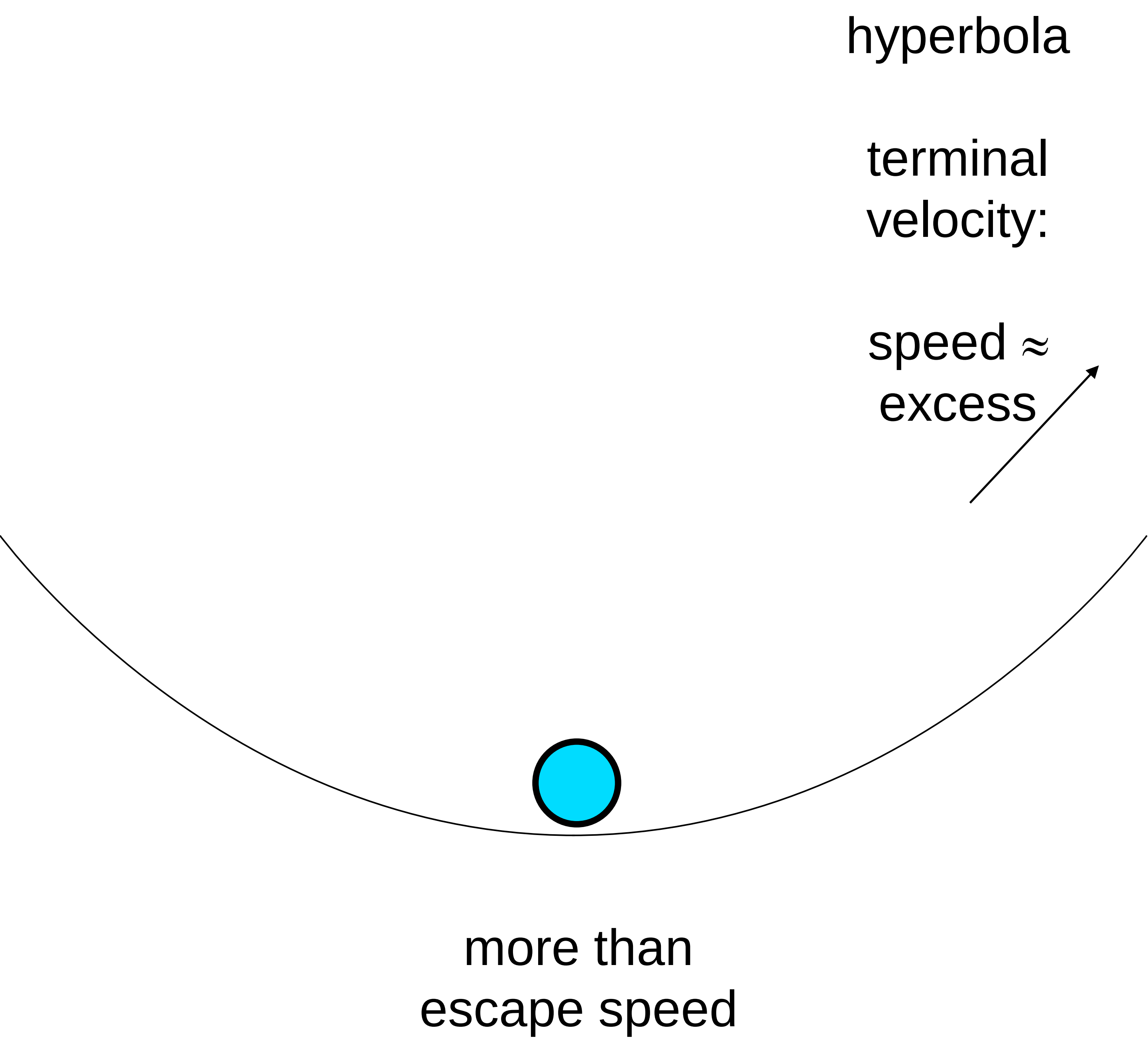
Speed and Distance



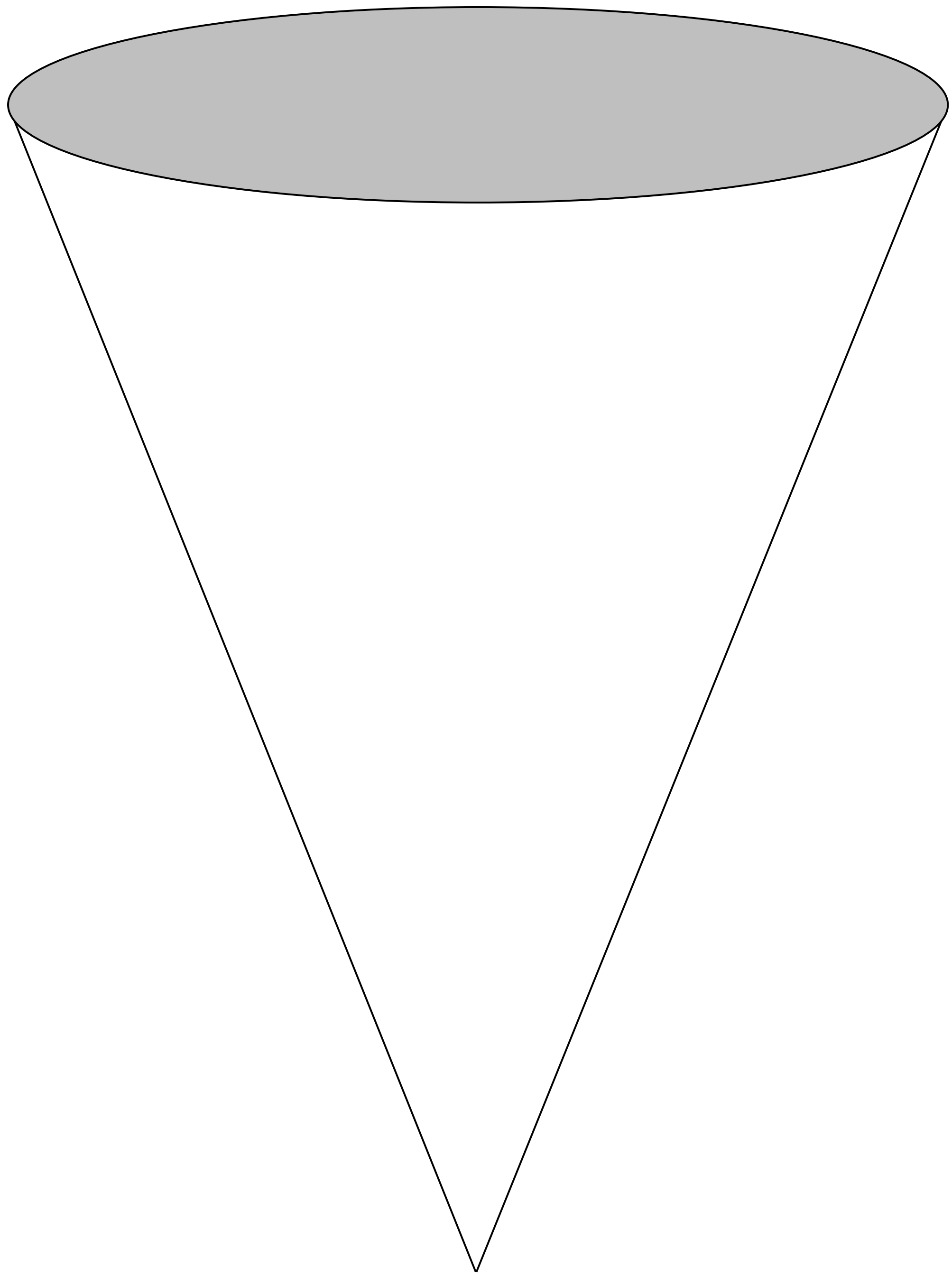
Speed and Distance



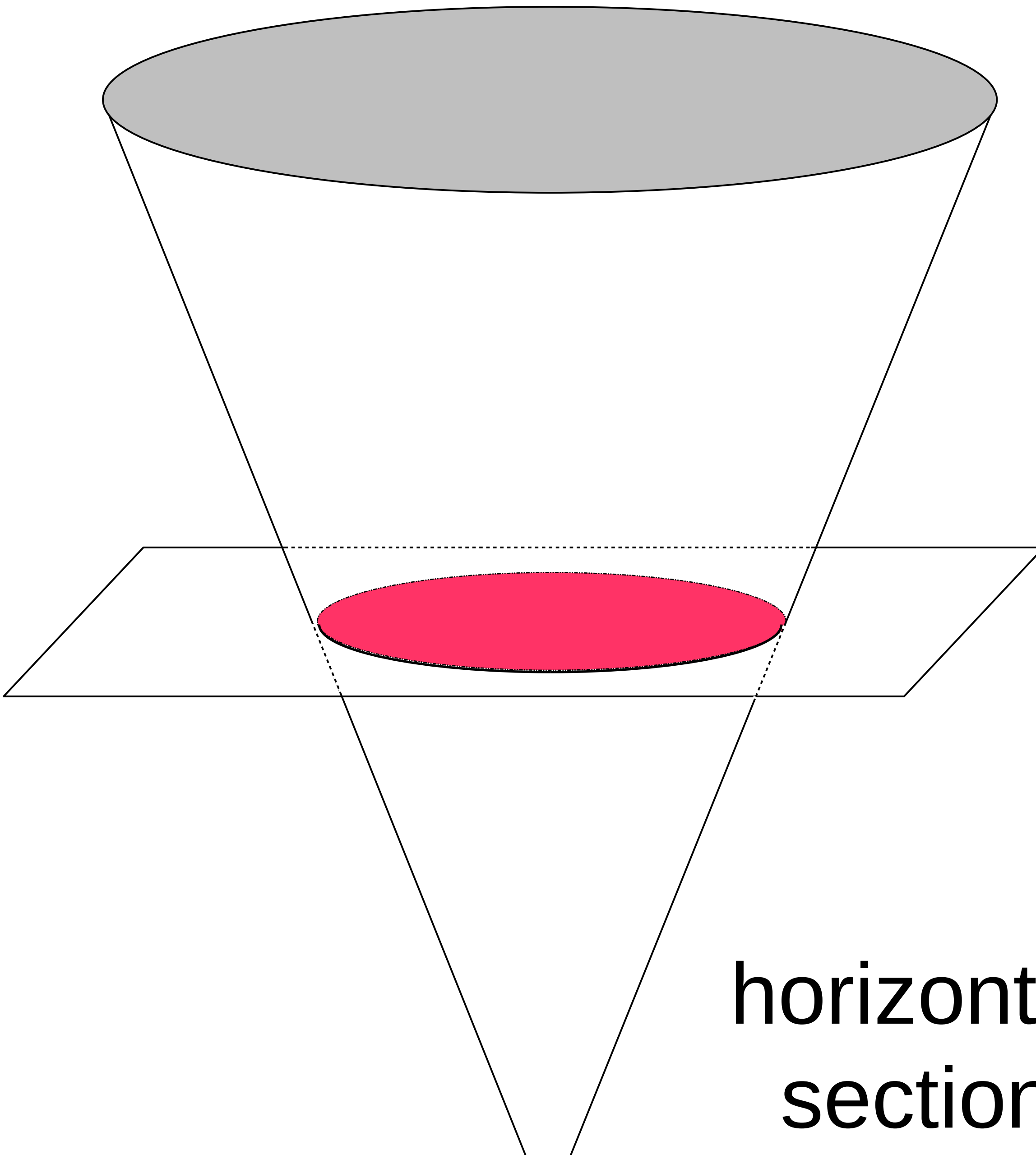
Speed and Distance



The Conic Sections

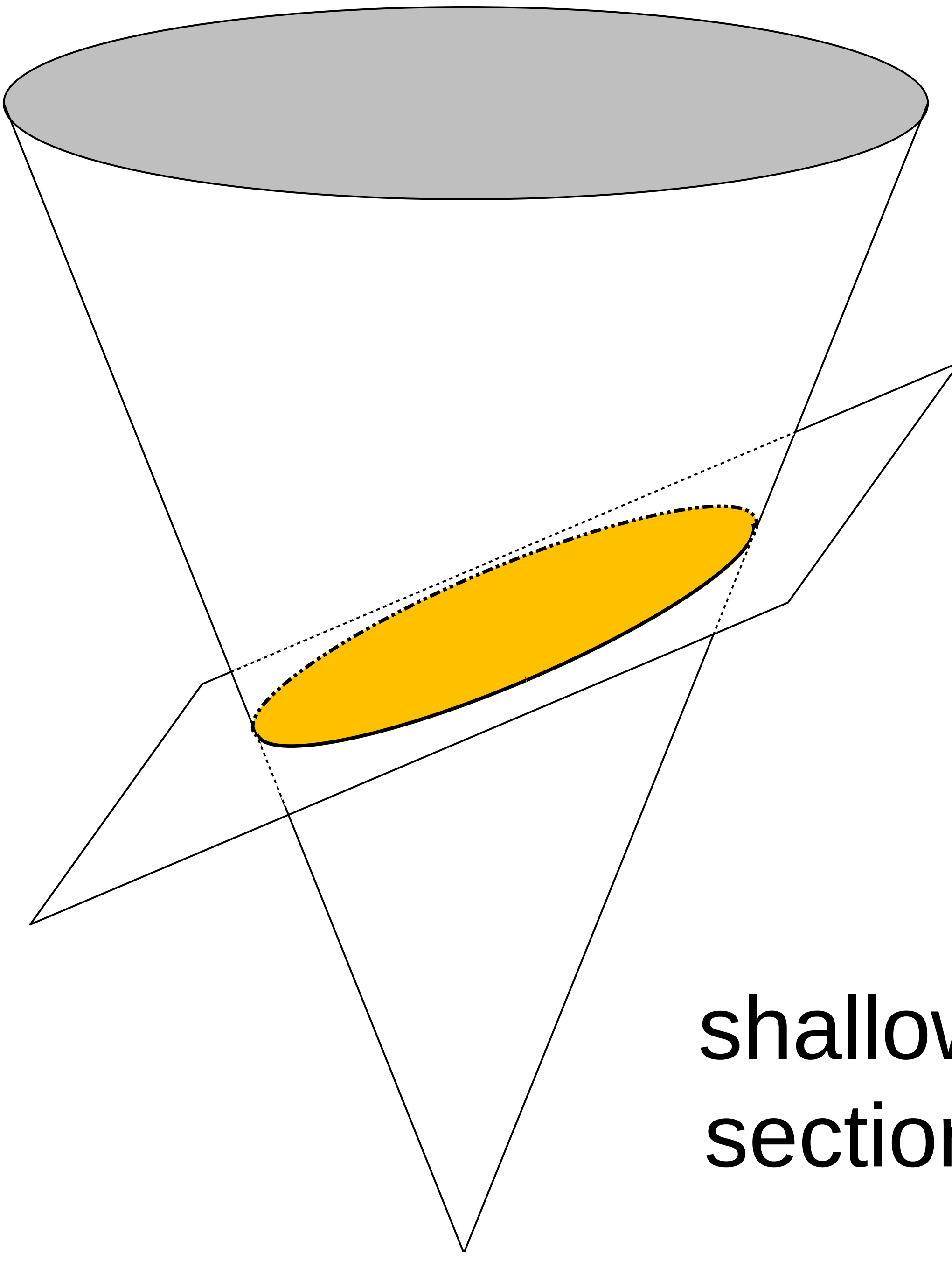


The Conic Sections



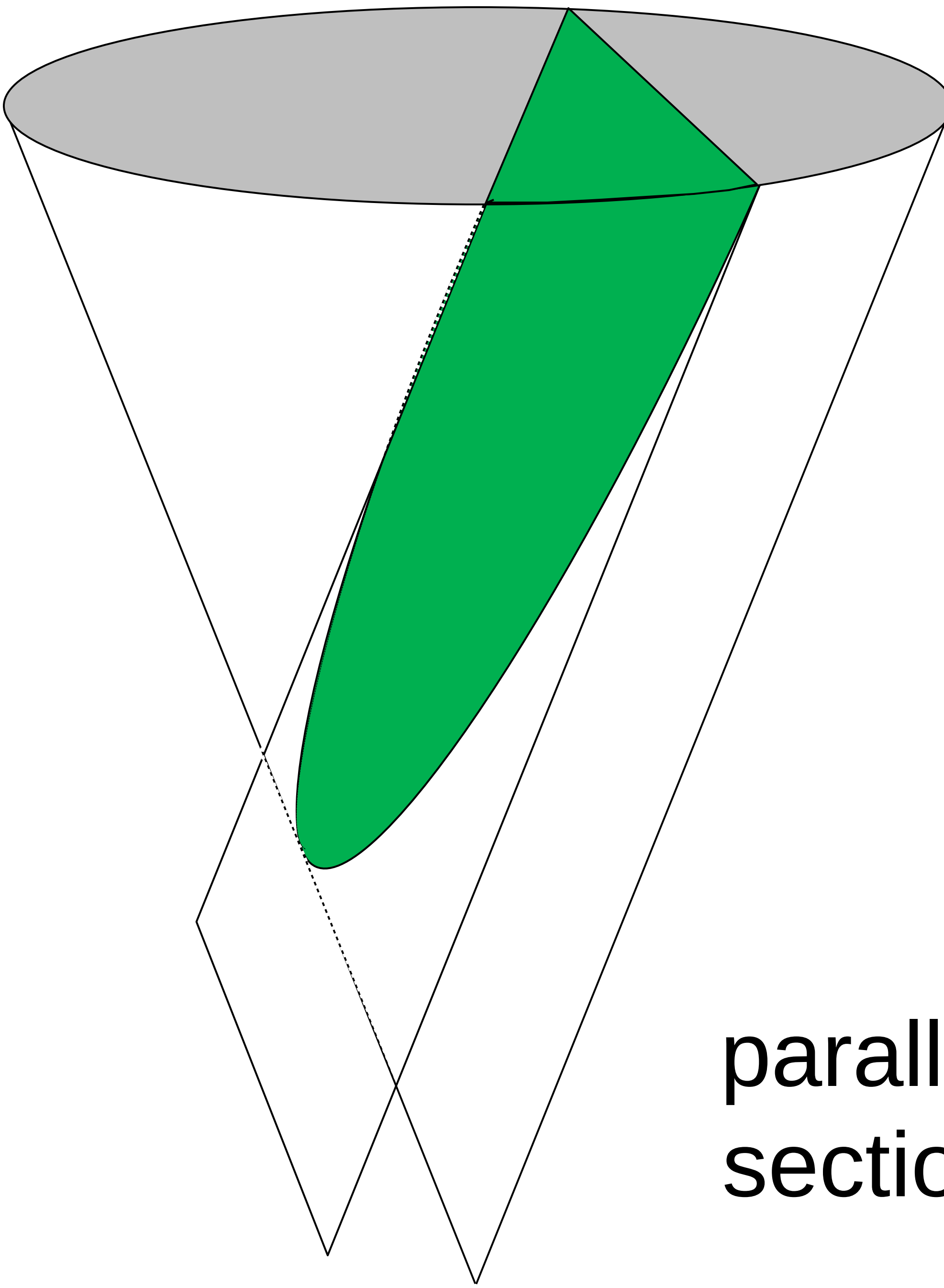
horizontal
section

The Conic Sections



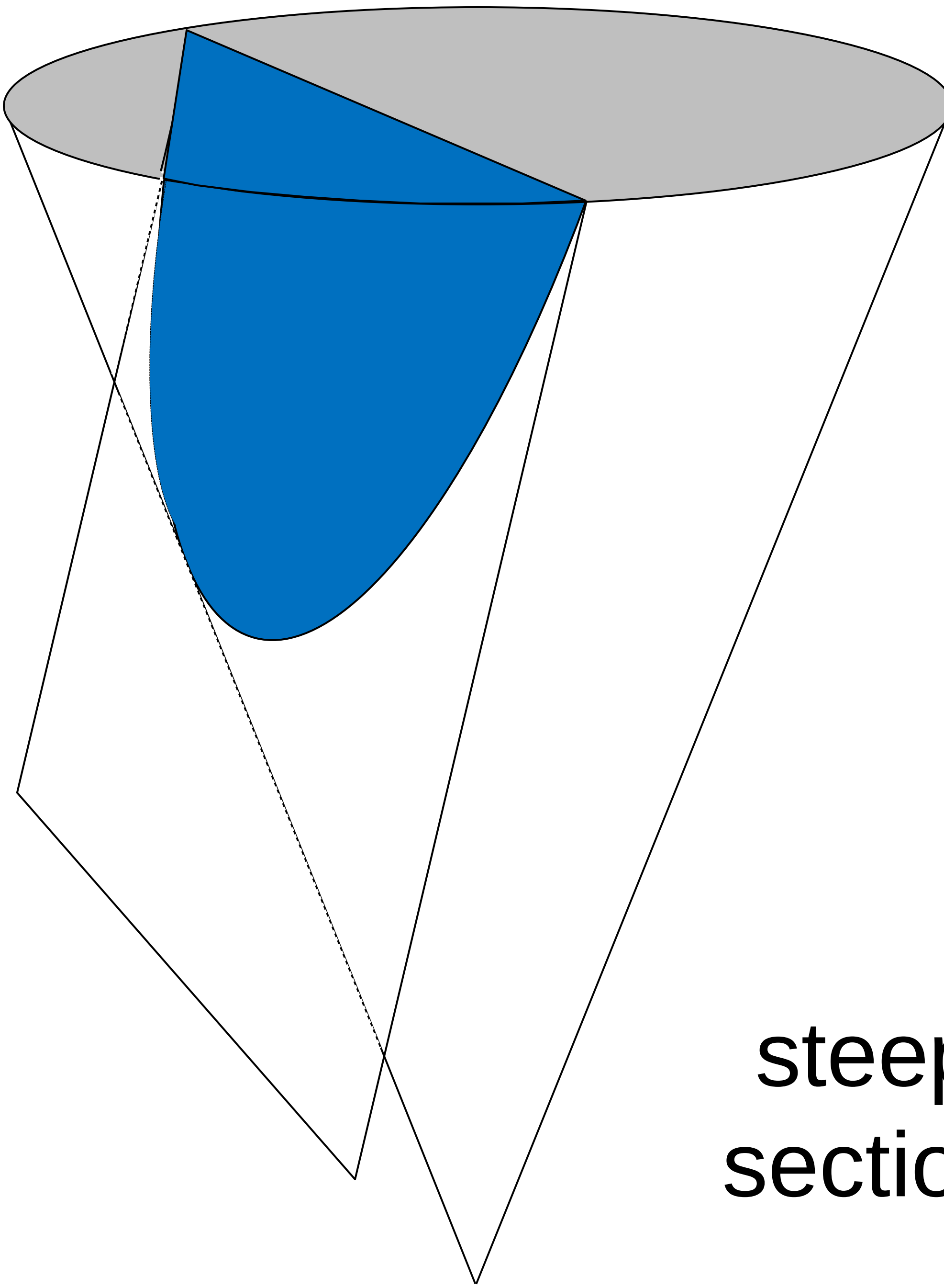
shallow
section

The Conic Sections



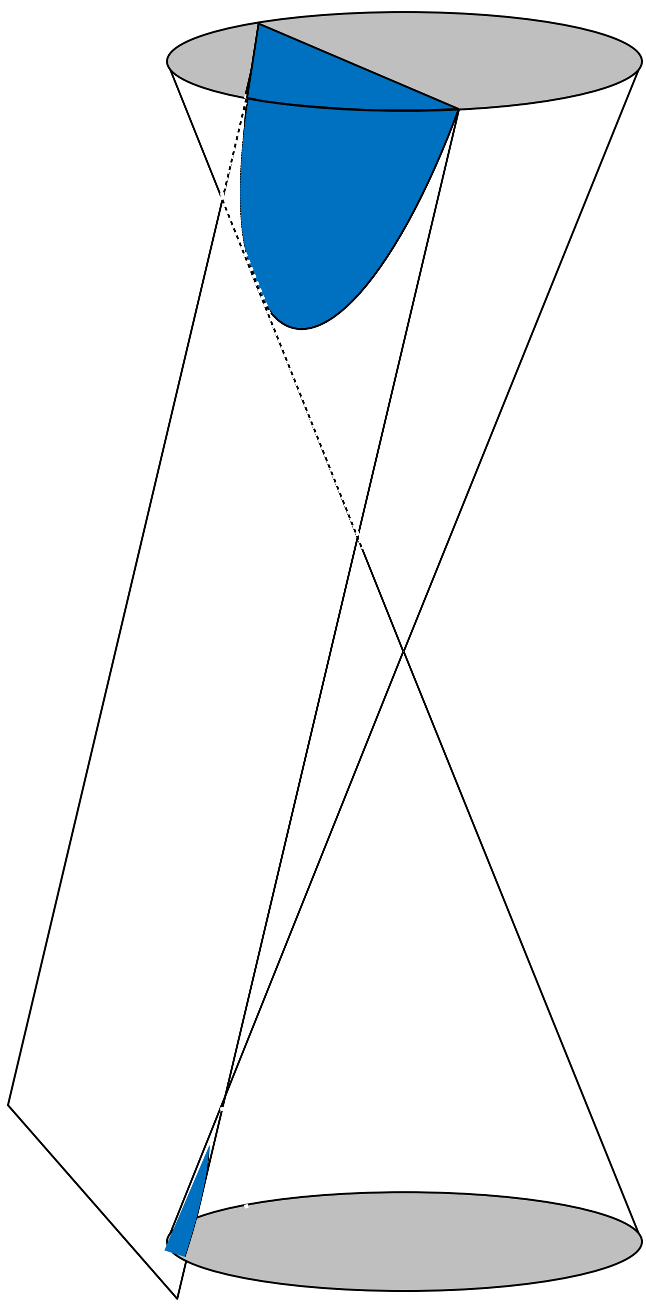
parallel
section

The Conic Sections



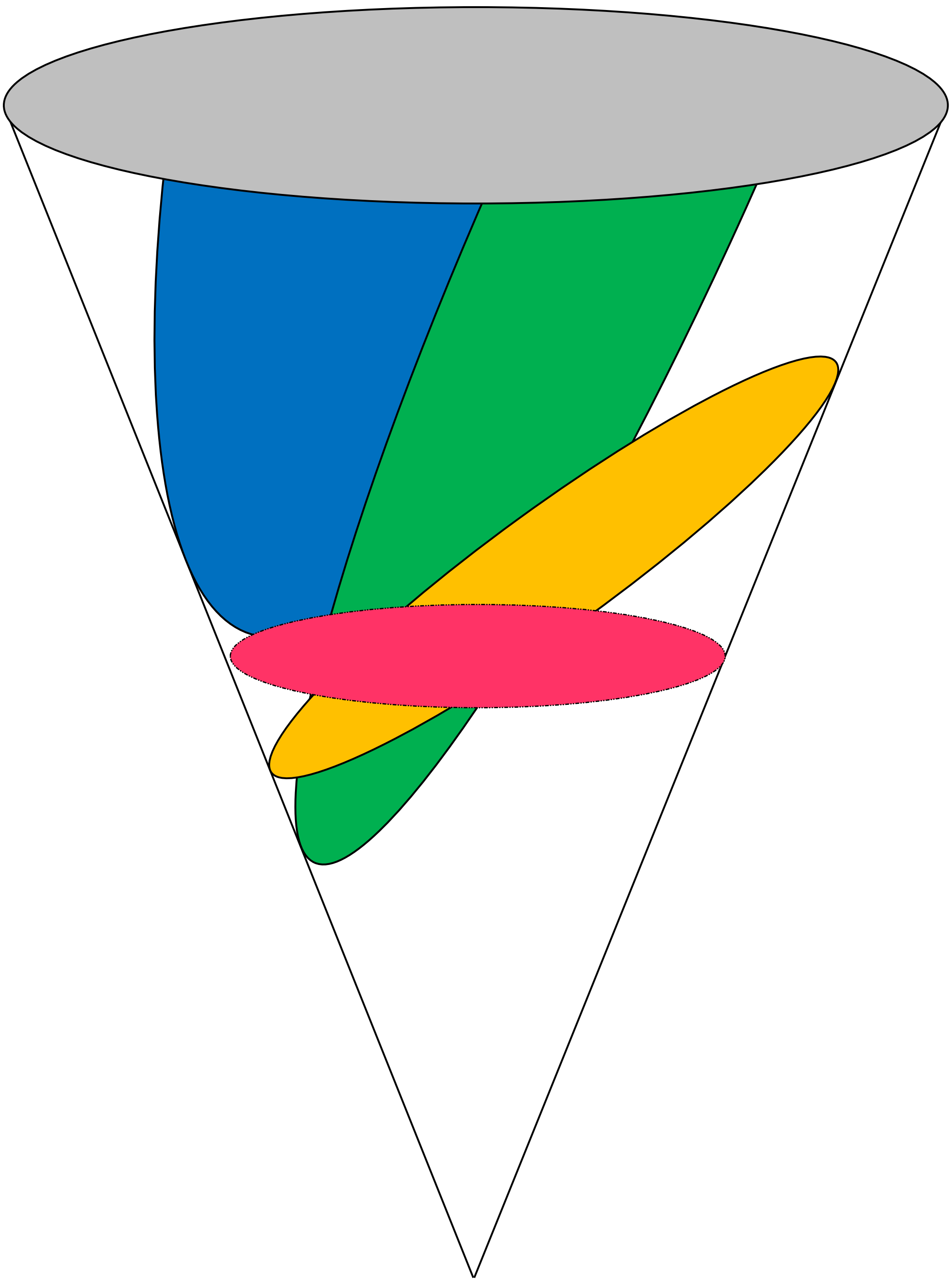
steep
section

The Conic Sections

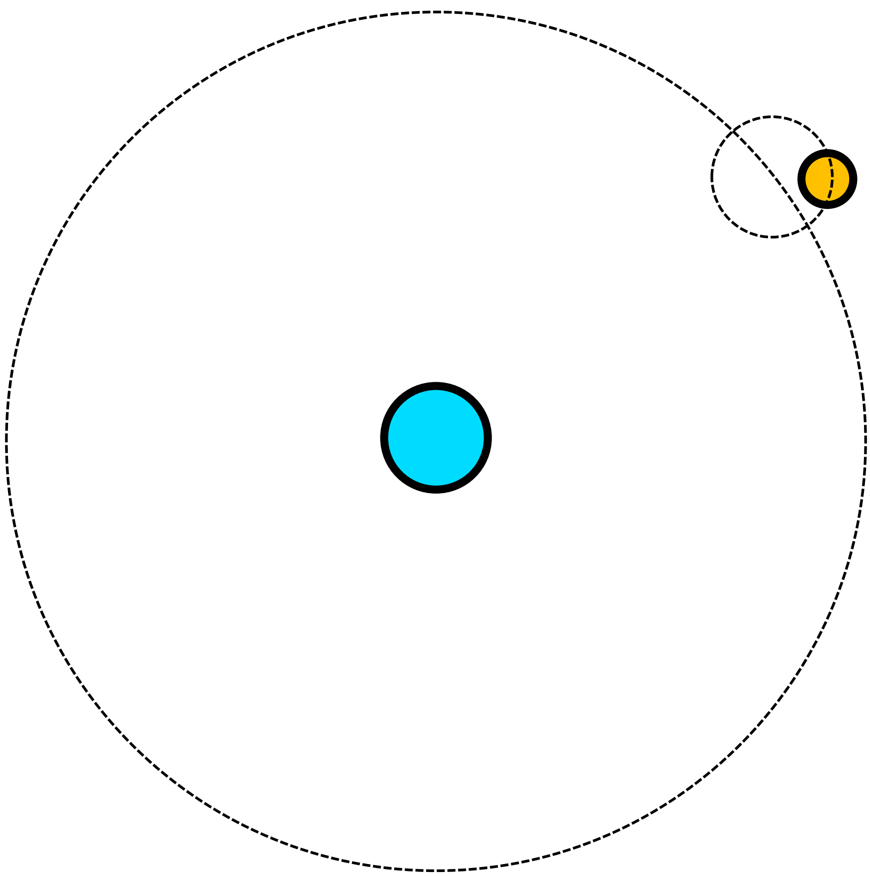


two
branches

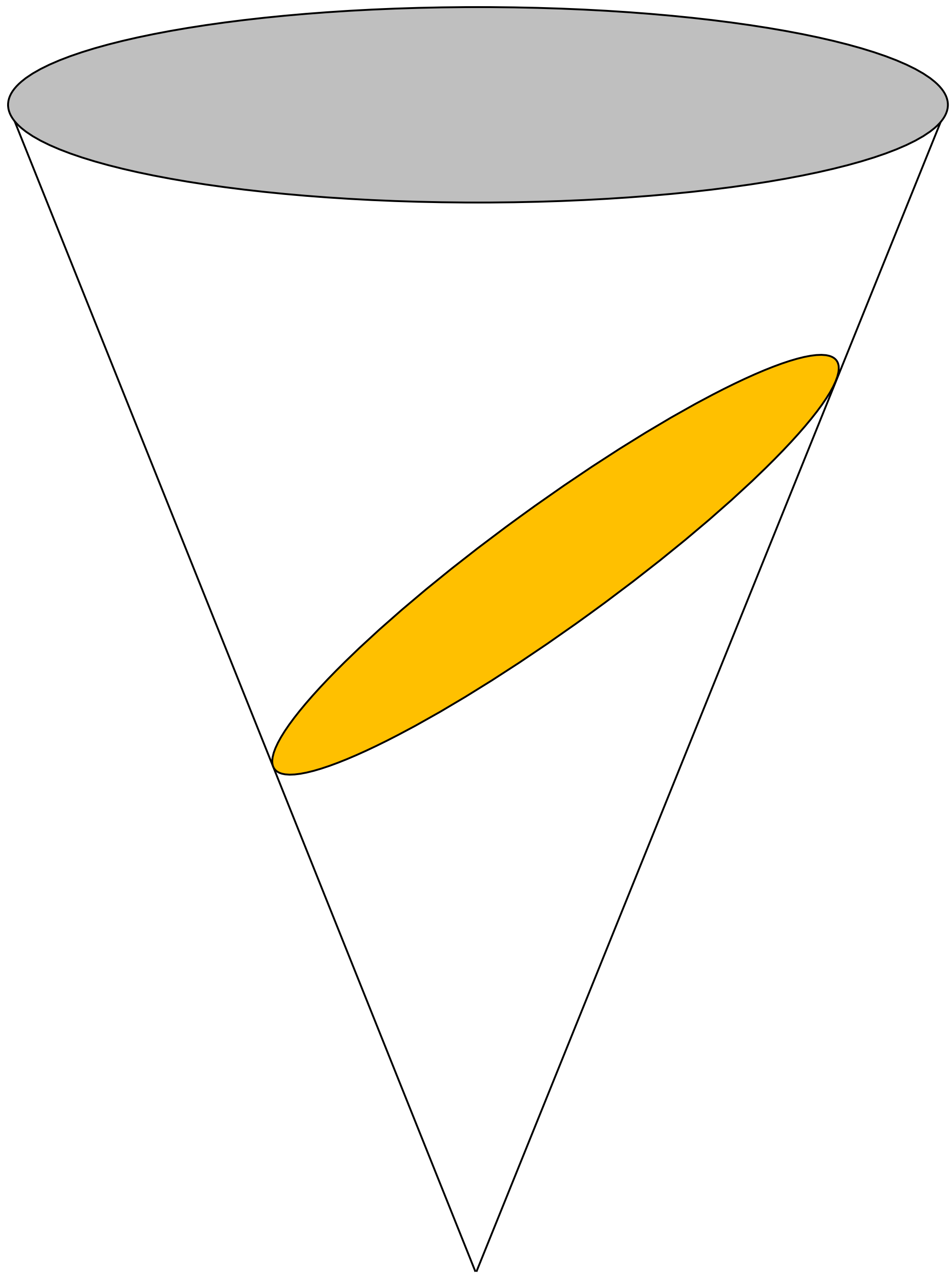
Apollonius's
Sections of One Cone



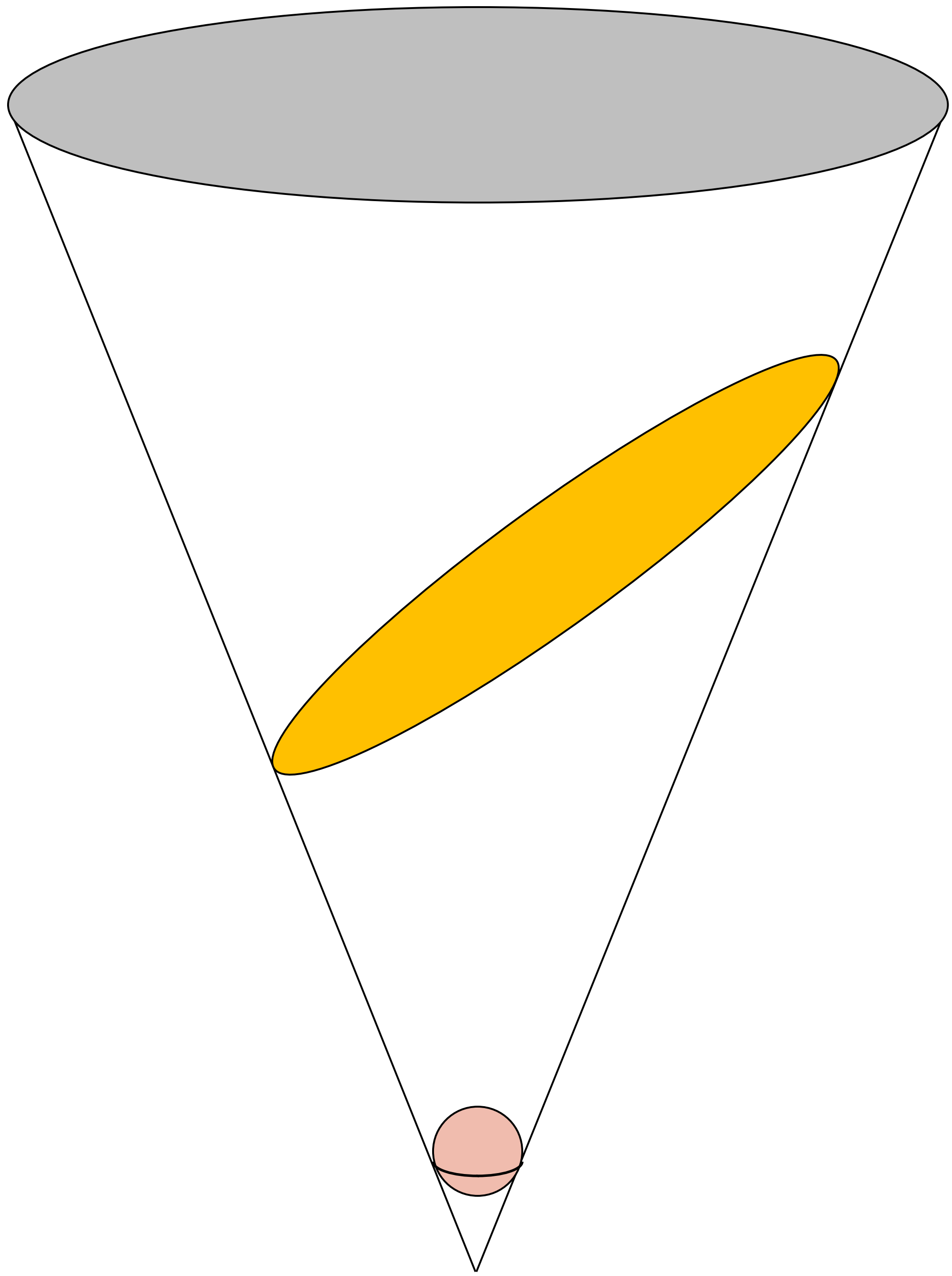
Apollonius's
Epicycle Model



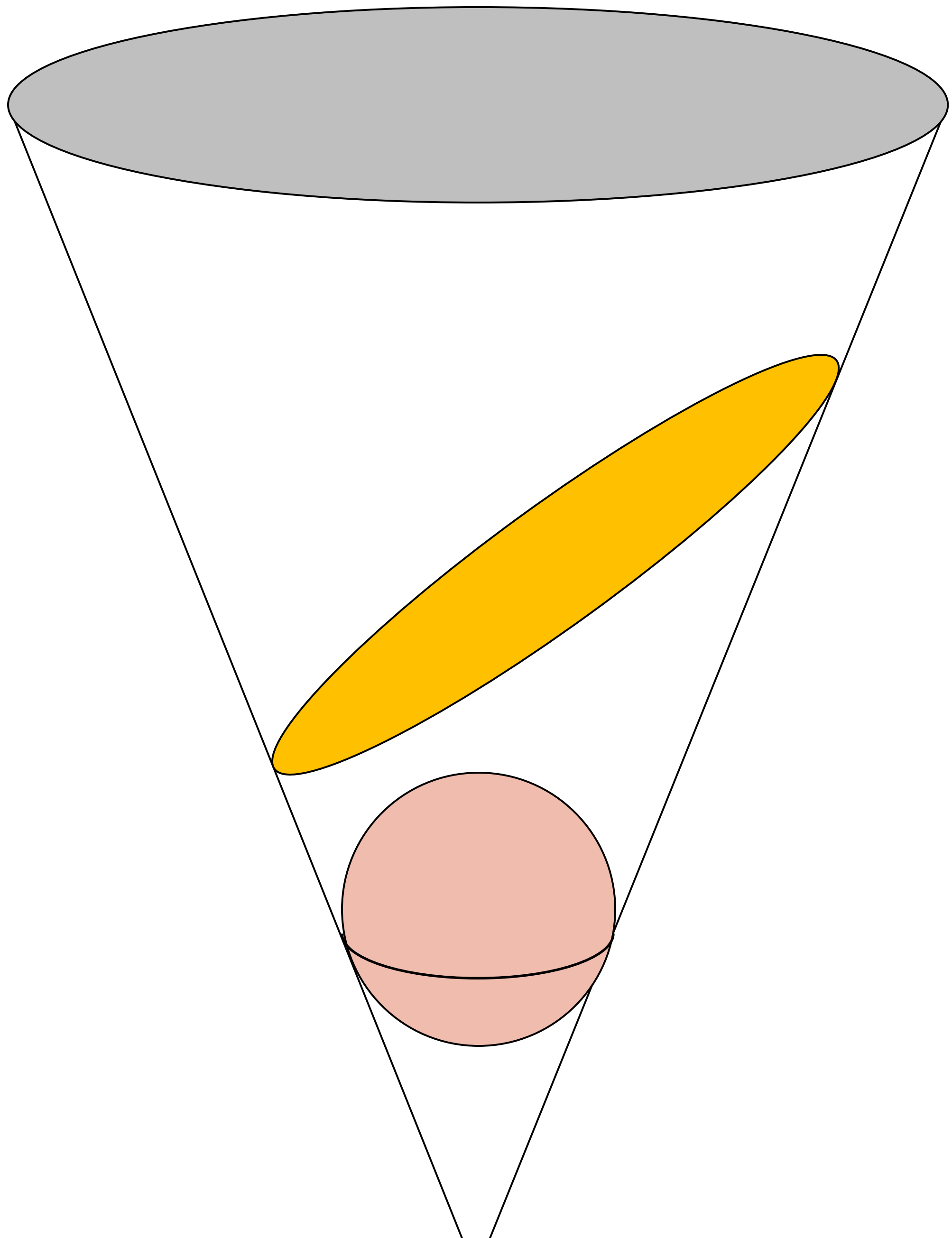
Geometry of the Shallow Section



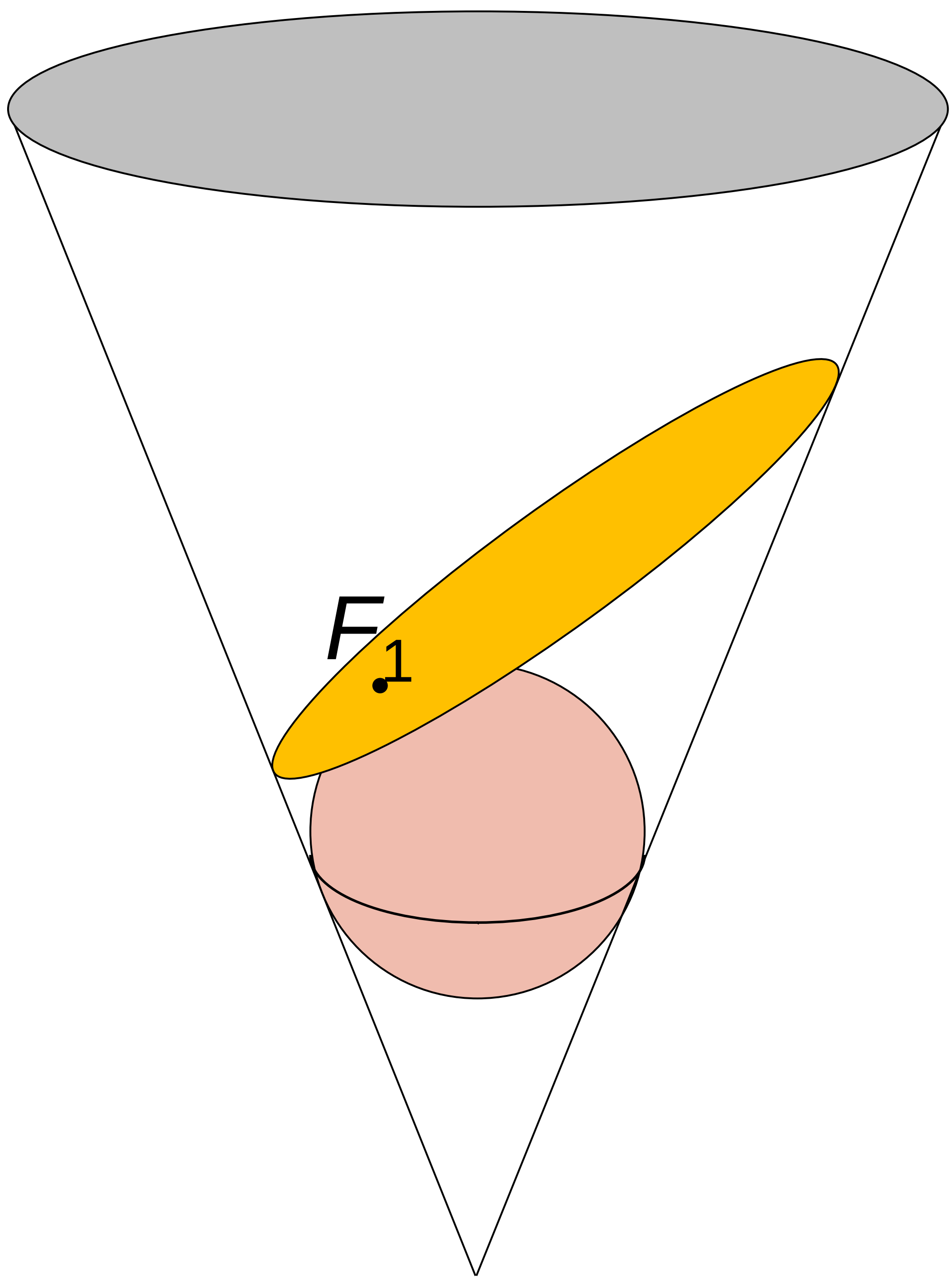
Geometry of the Shallow Section



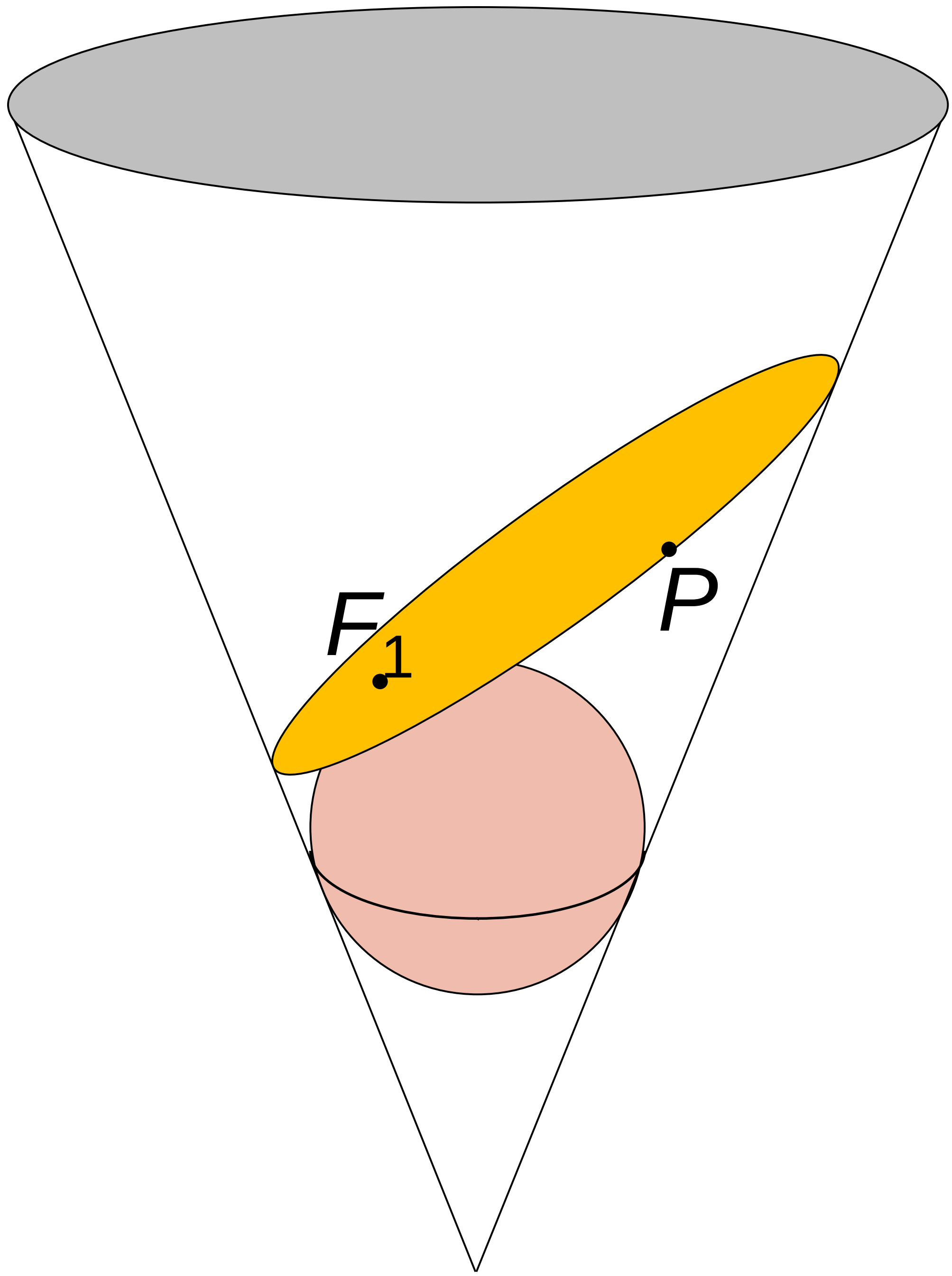
Geometry of the Shallow Section



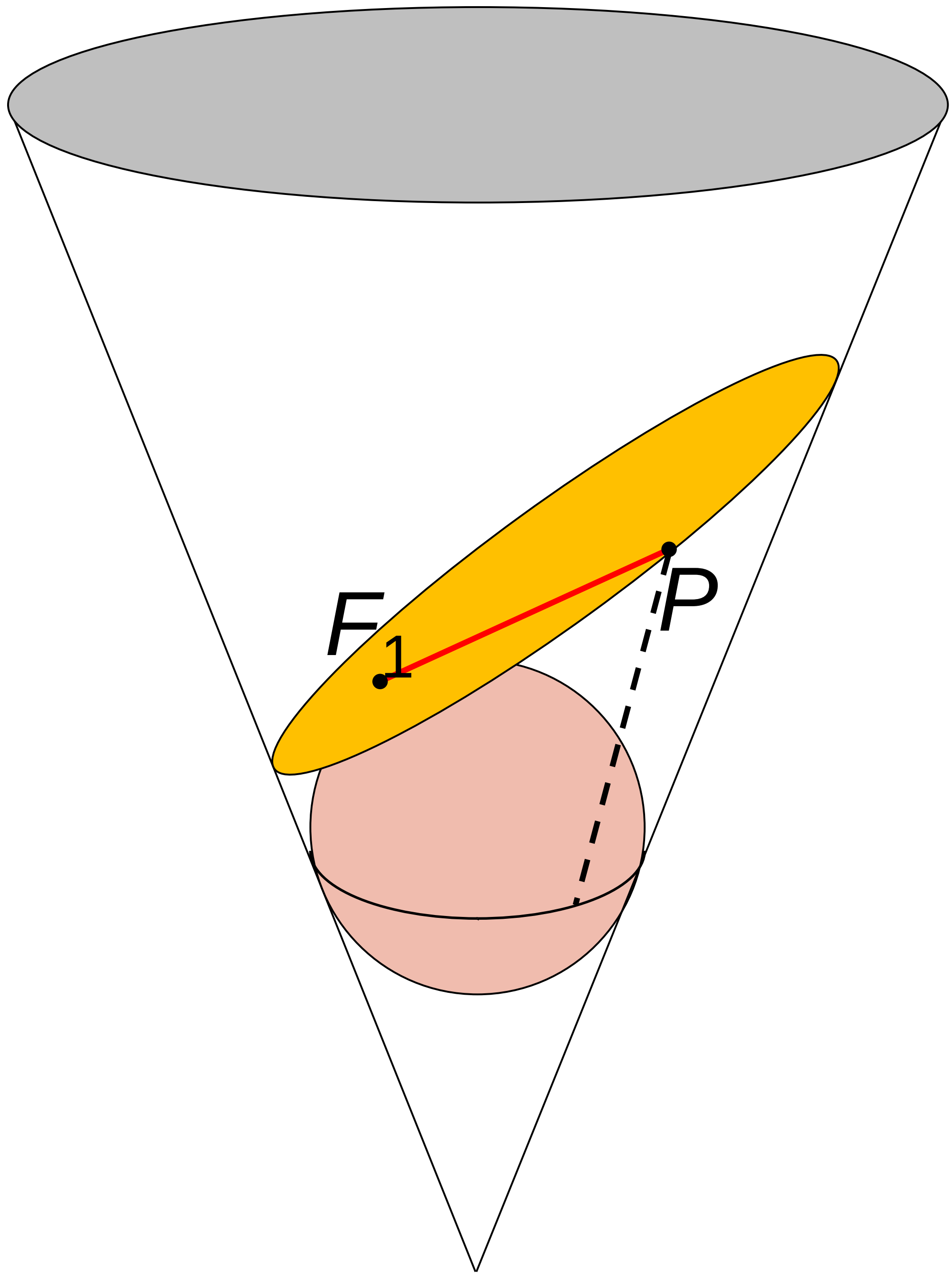
Geometry of the Shallow Section



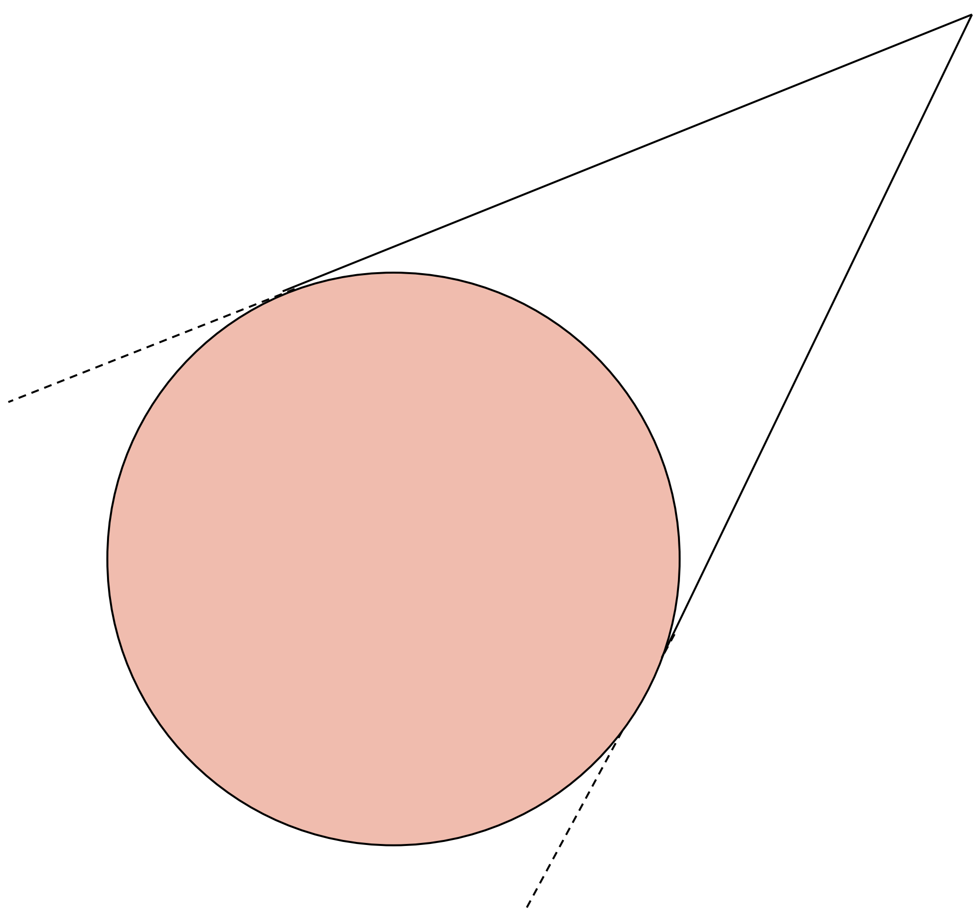
Geometry of the Shallow Section



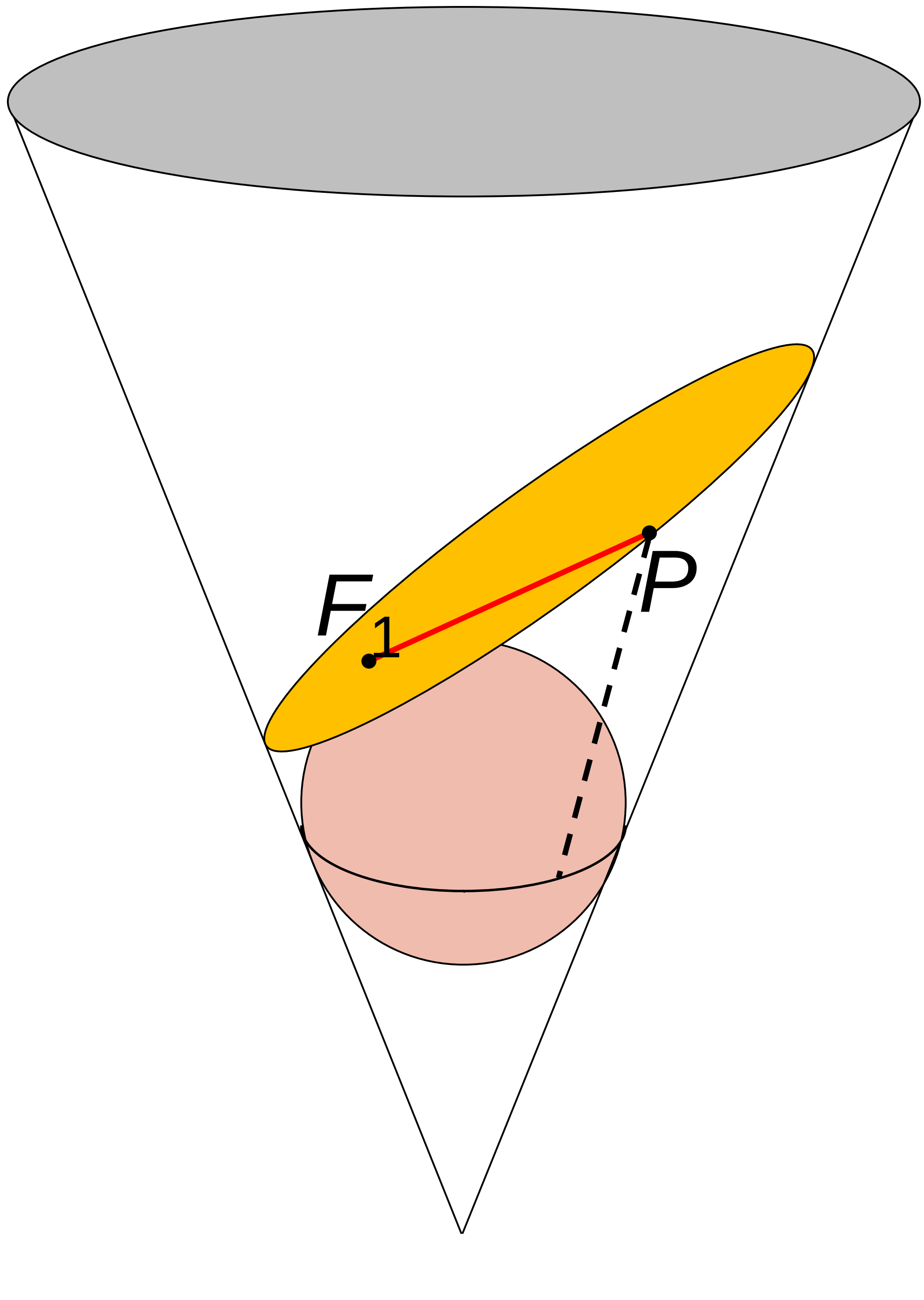
Geometry of the Shallow Section



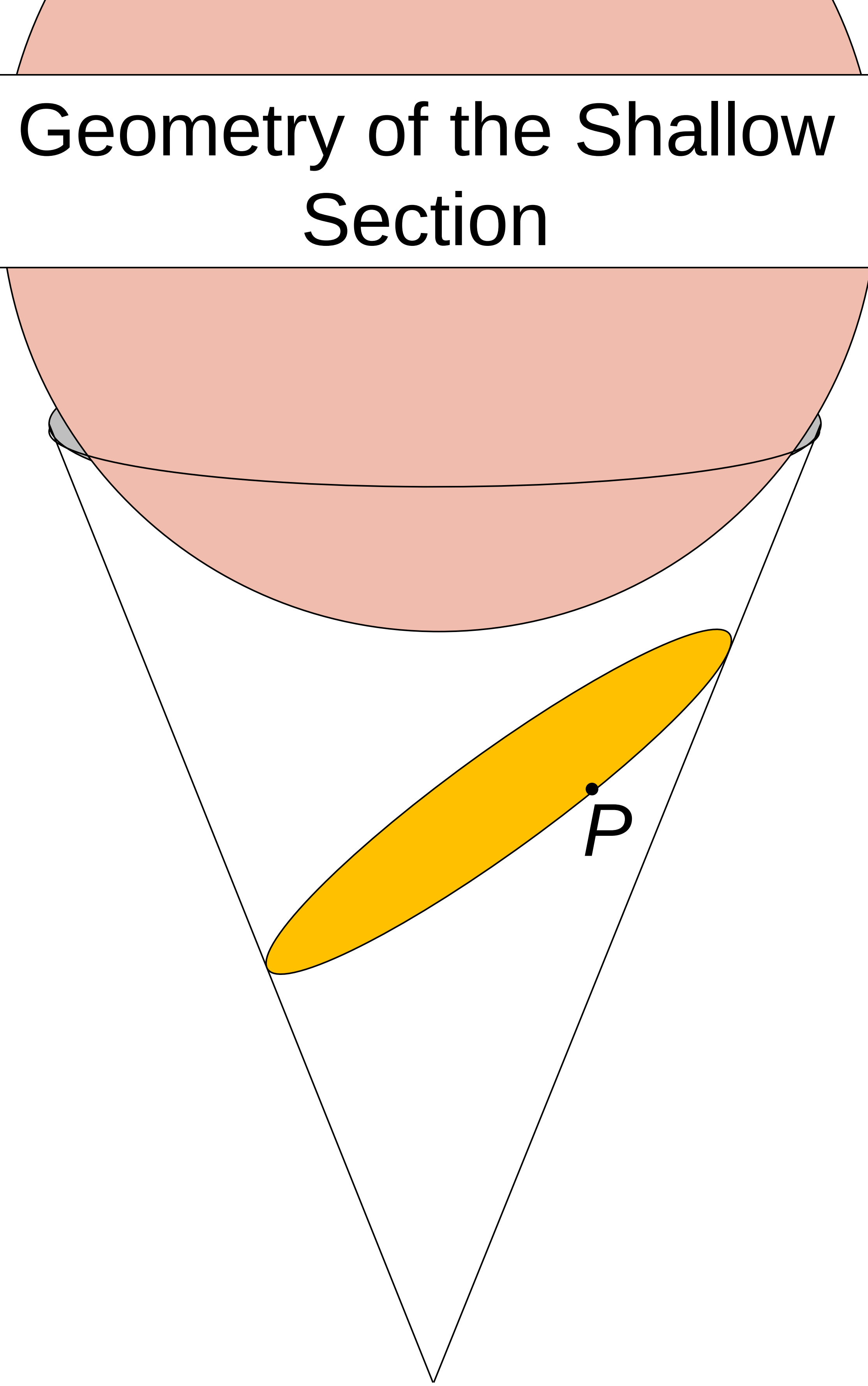
Tangents from a Common Point



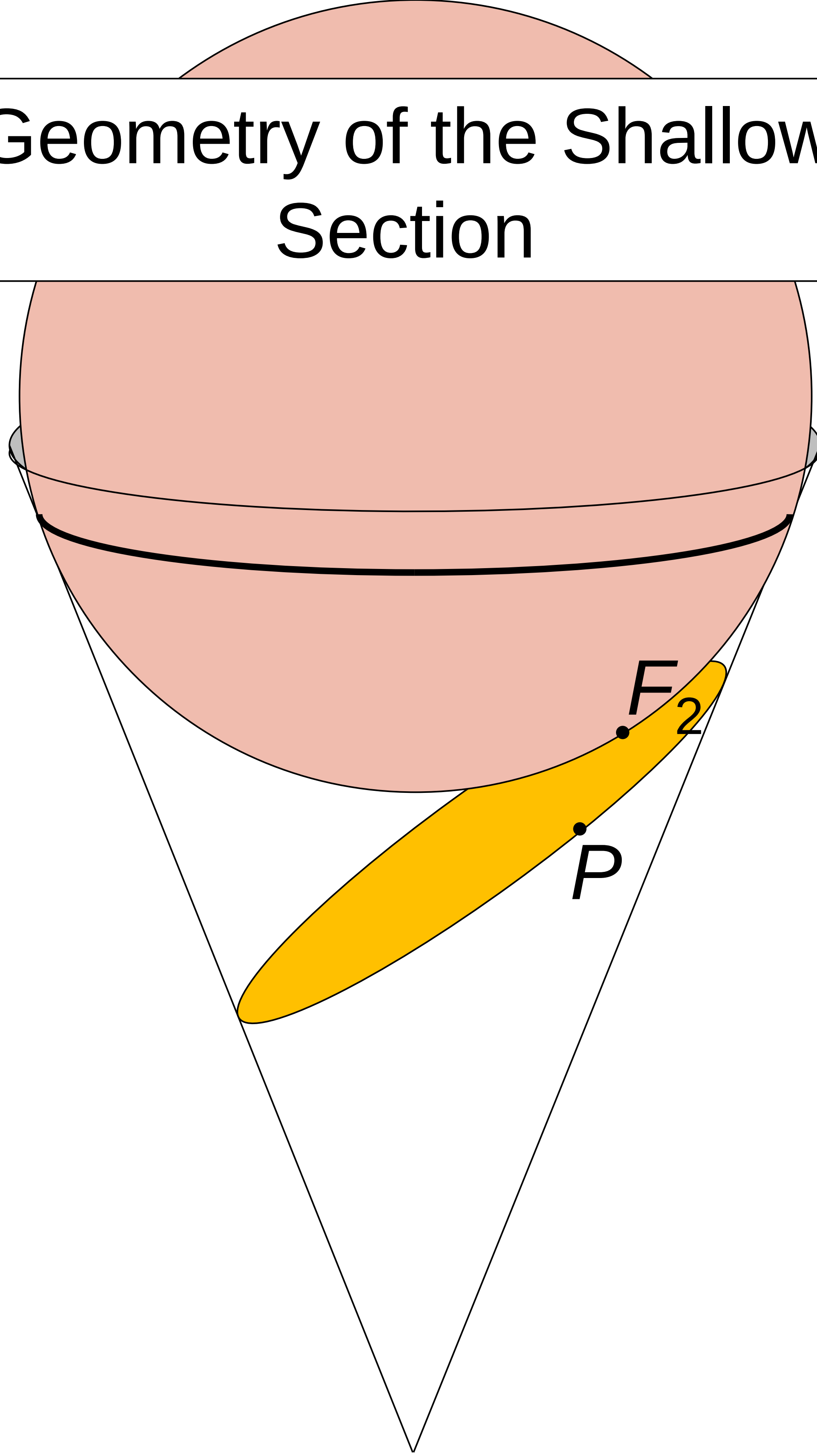
Geometry of the Shallow Section



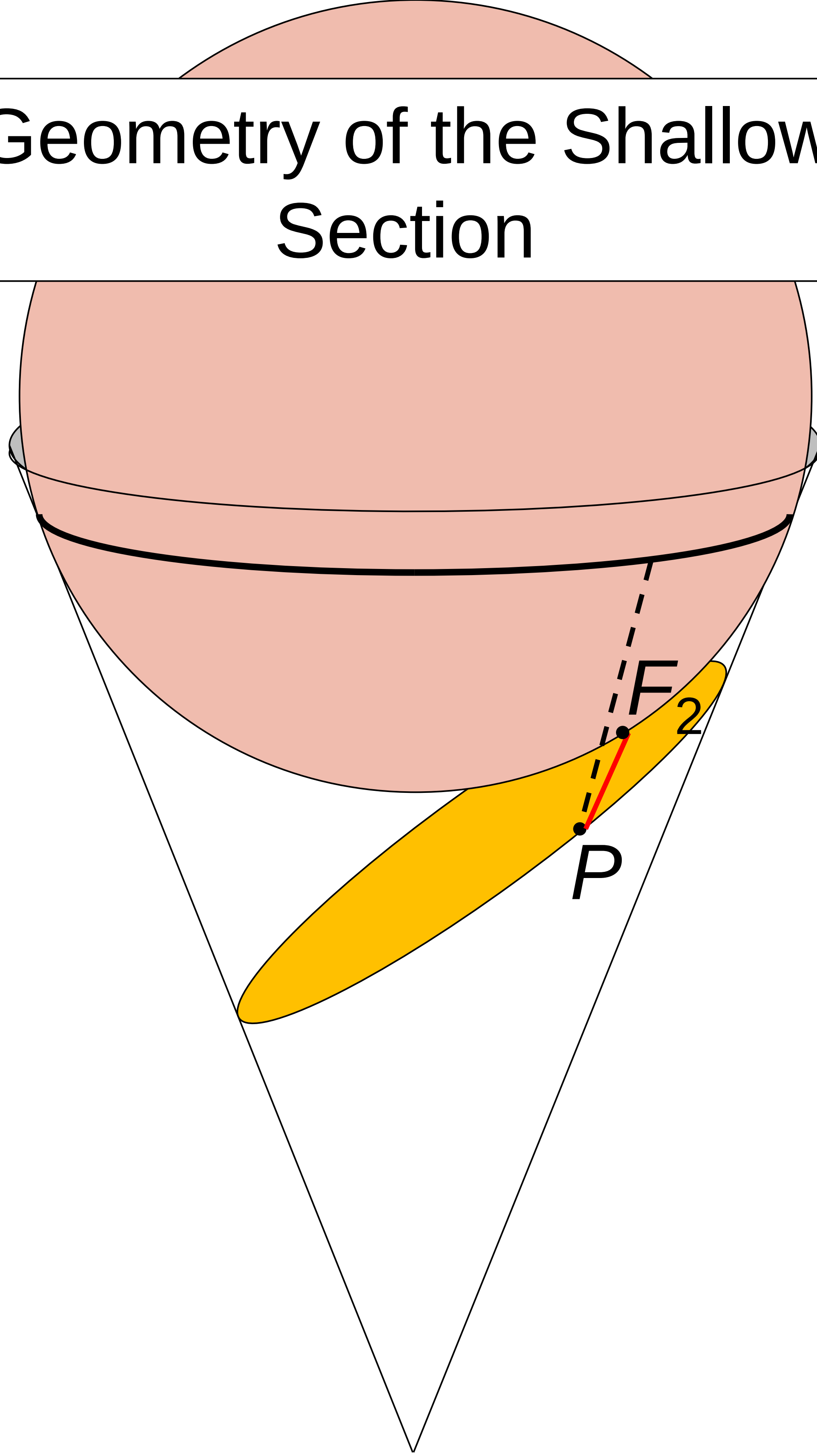
Geometry of the Shallow Section



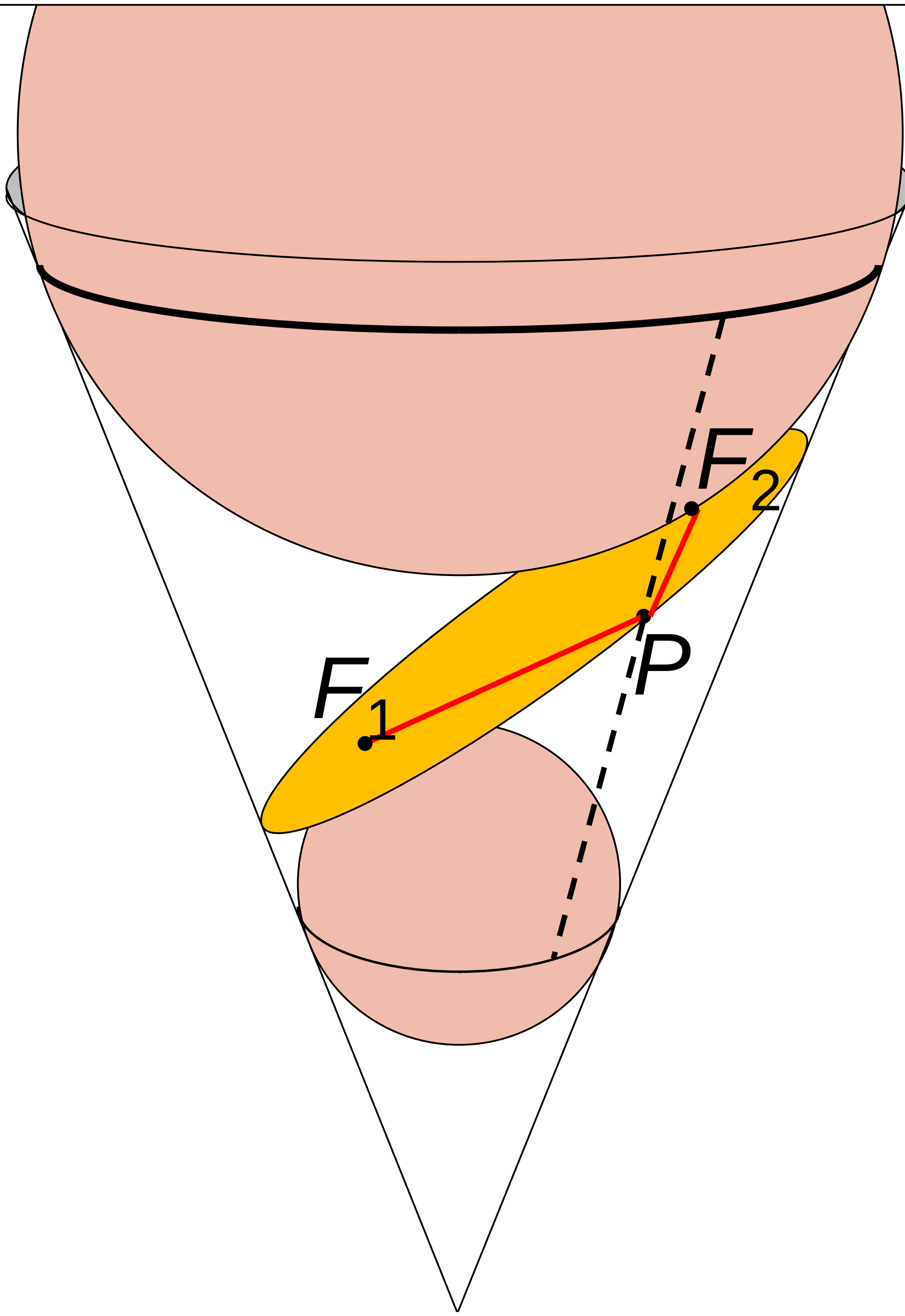
Geometry of the Shallow Section



Geometry of the Shallow Section

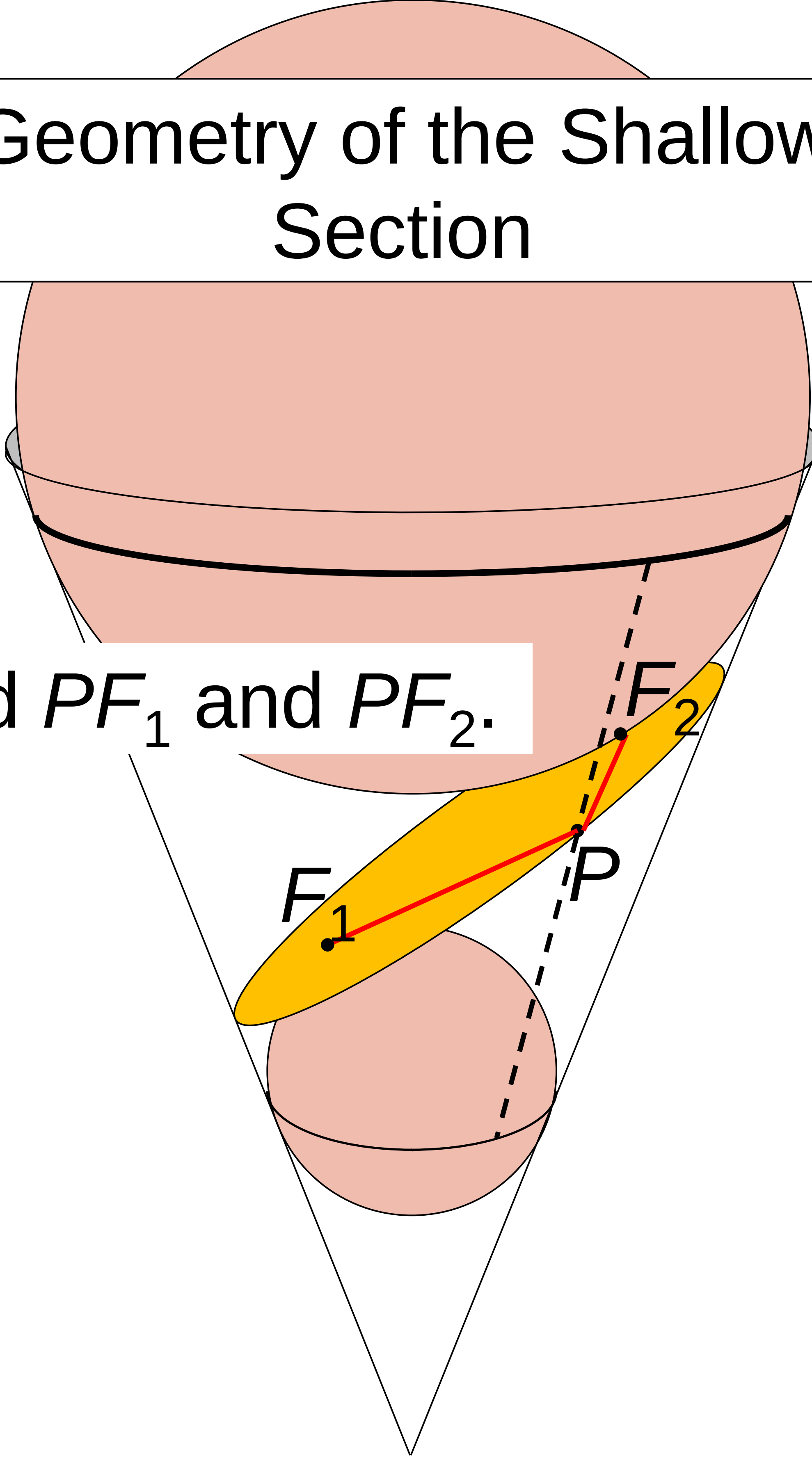


Geometry of the Shallow Section



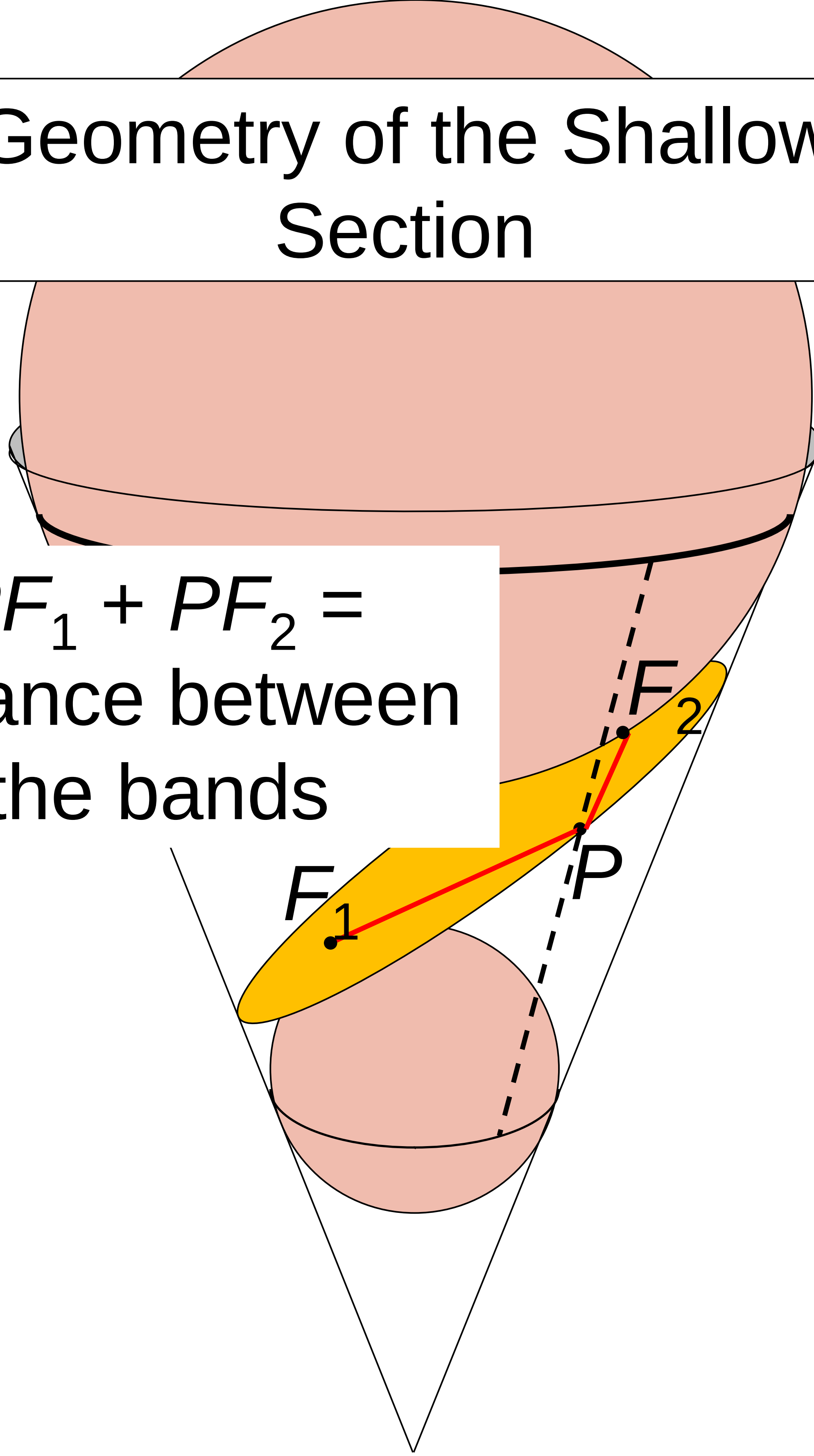
Geometry of the Shallow Section

Add PF_1 and PF_2.



Geometry of the Shallow Section

$PF_1 + PF_2 =$ distance between the bands



Definition of the Ellipse

- There are two fixed points (“foci”) for which the two distances (“focal radii”) from any point of the curve add up to a fixed number.

Definition of the Ellipse

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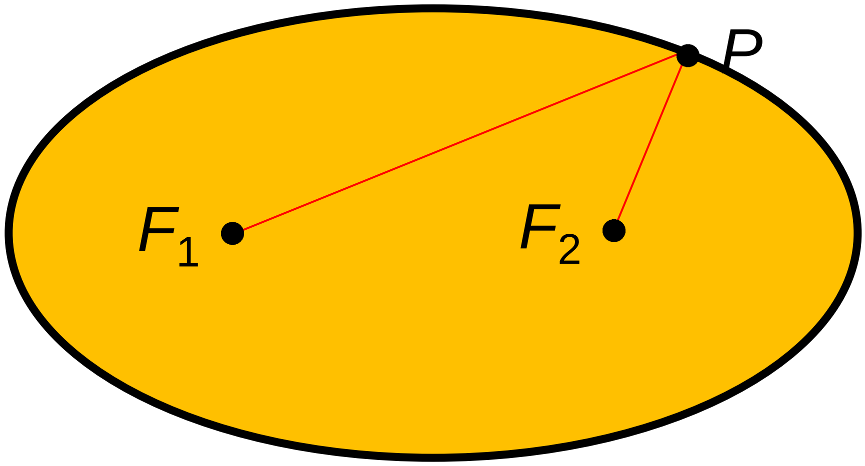
$$PF_1 + PF_2 = \text{constant}$$



Properties of the Ellipse

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- The ellipse is left-right and up-down symmetric.

Properties of the Ellipse

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- The main axis (the one with the foci) is as long as the sum of the focal radii.

Properties of the Ellipse

- There are two fixed points ("foci") for which the two distances ("focal radii") from any point of the curve add up to a fixed number.

- The main axis is longer than the other:

$$M^2 = m^2 + f^2$$

Properties of the Ellipse

- There are two fixed points ("foci") for which the two distances ("focal radii") from any point of the curve add up to a fixed number.

- The ratio $\varepsilon = f/M$ (the "eccentricity") determines the shape of the ellipse.

Eccentricity and the Shape of the Ellipse

$M^2 = m^2 + f^2$ and $\varepsilon = f/M$

lead to

$$m = M\sqrt{(1 - \varepsilon^2)}.$$

Eccentricity and the Shape of the Ellipse

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lead to

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- Earth:

  $\varepsilon = .02$ $\quad$ $m = M(.9998)$

Eccentricity and the Shape of the Ellipse

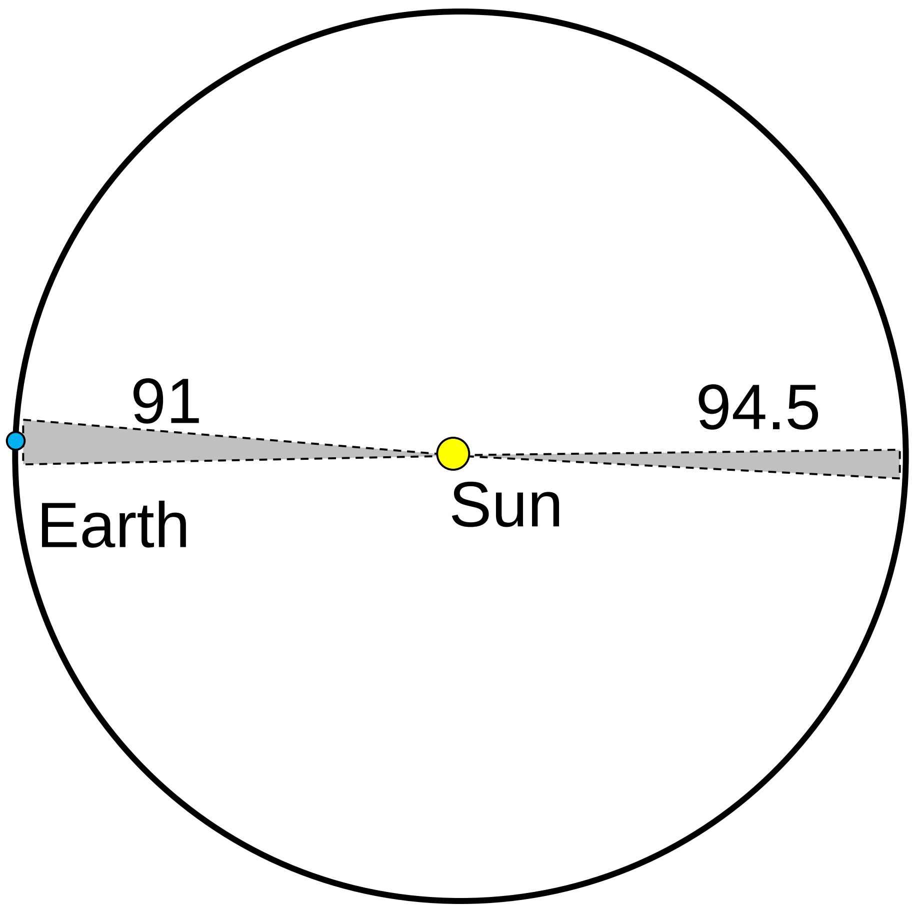
$$M^2 = m^2 + f^2 \text{ and } \varepsilon = f/M$$

lead to

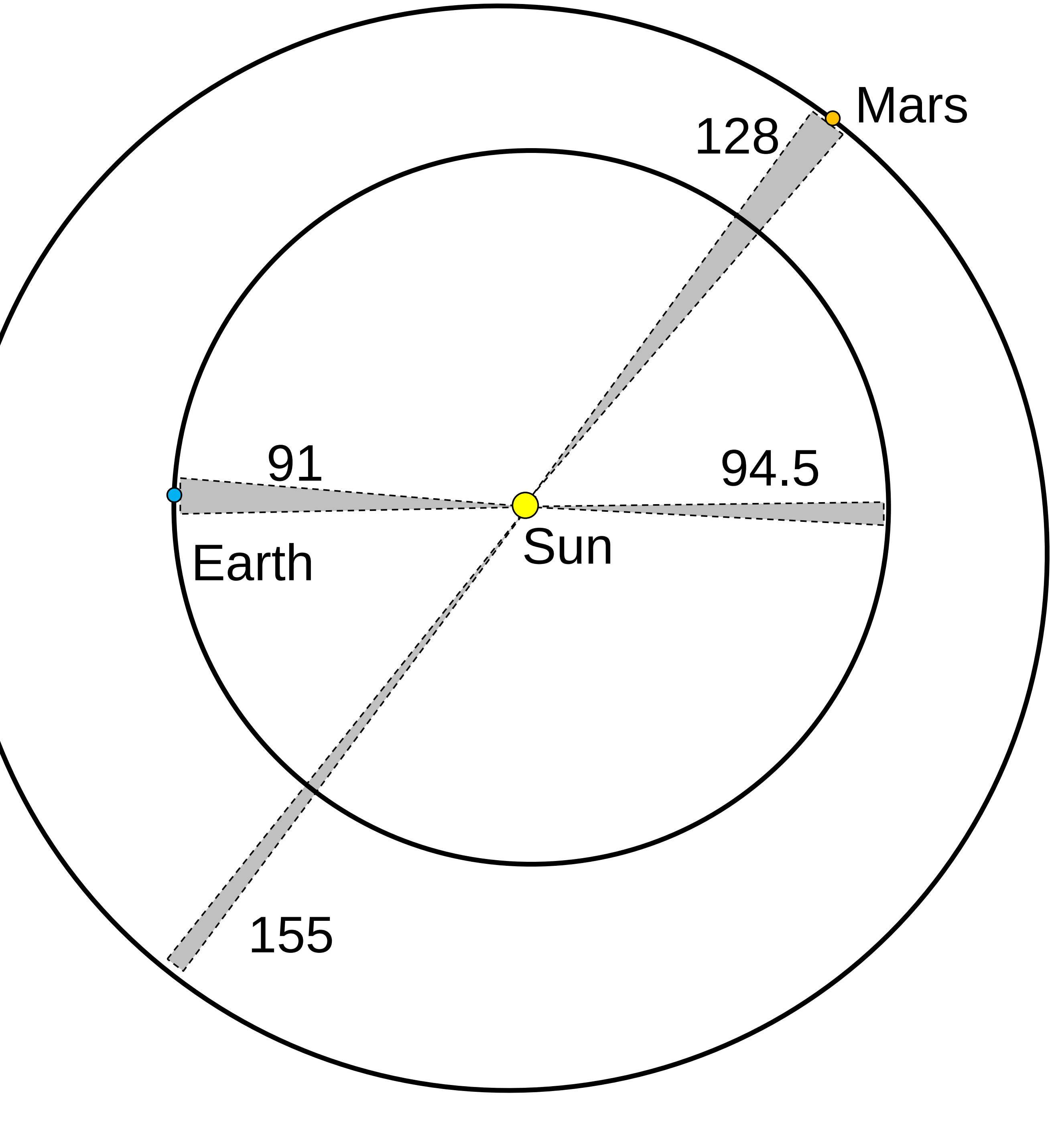
$$m = M\sqrt{(1 - \varepsilon^2)}.$$

- Earth:
  $\varepsilon = .02$ m = M(.9998)
- Mars:
  $\varepsilon = .09$ m = M(.996)

Eccentricity and the Shape of Two Familiar Orbits

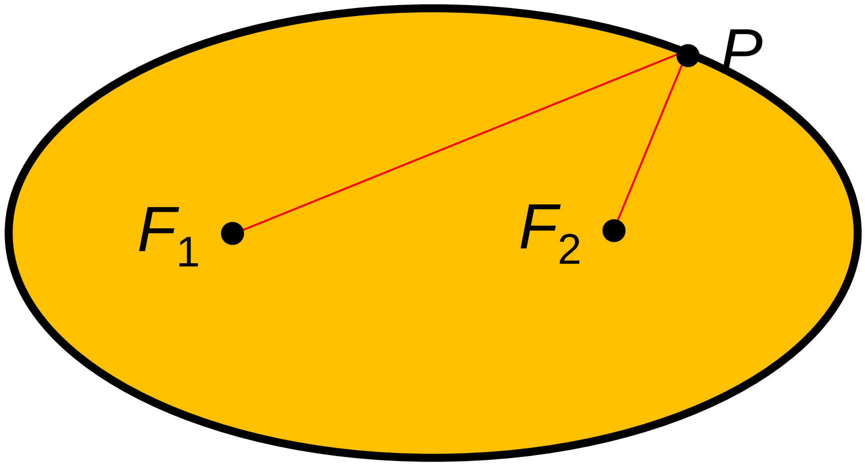


Eccentricity and the Shape of Two Familiar Orbits



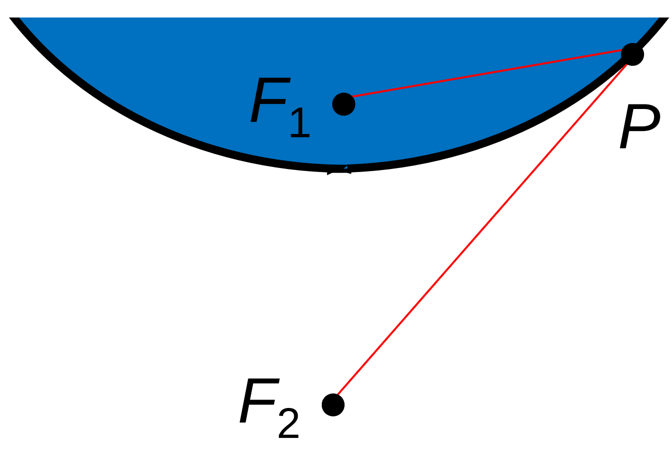
Definition of the Ellipse

$$PF_1 + PF_2 = \text{constant}$$



Definition of the Hyperbola

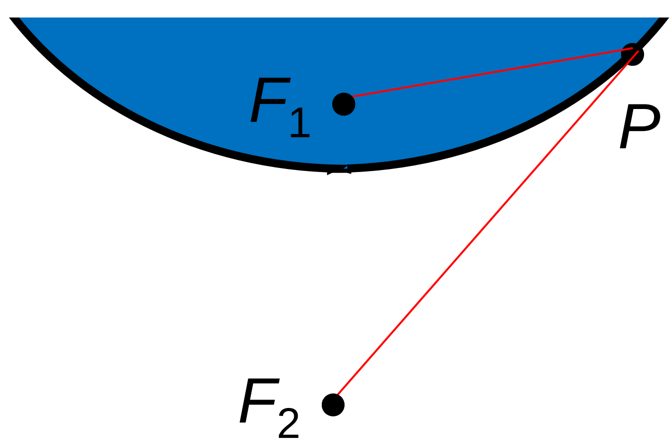
$$PF_2 - PF_1 = \text{constant}$$



Definition of the Hyperbola

- There are two fixed points ("foci") for which the two distances ("focal radii") from any point of the curve *differ by* a fixed number.

$$PF_2 - PF_1 = \text{constant}$$



Definition of the Hyperbola

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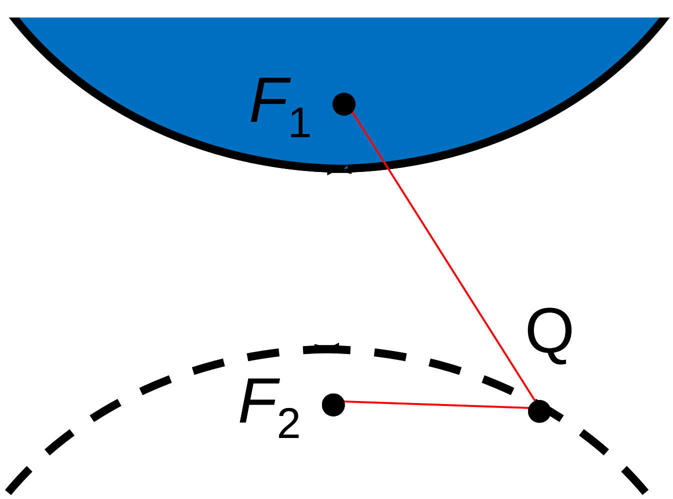


$$QF_1 - QF_2 = \text{constant}$$

Seismography and the Hyperbola

Suppose San Francisco hears an earthquake at 12, New York hears at 5, Miami hears at 5:12.

Seismography and the Hyperbola

distance to New York

- distance to San Francisco

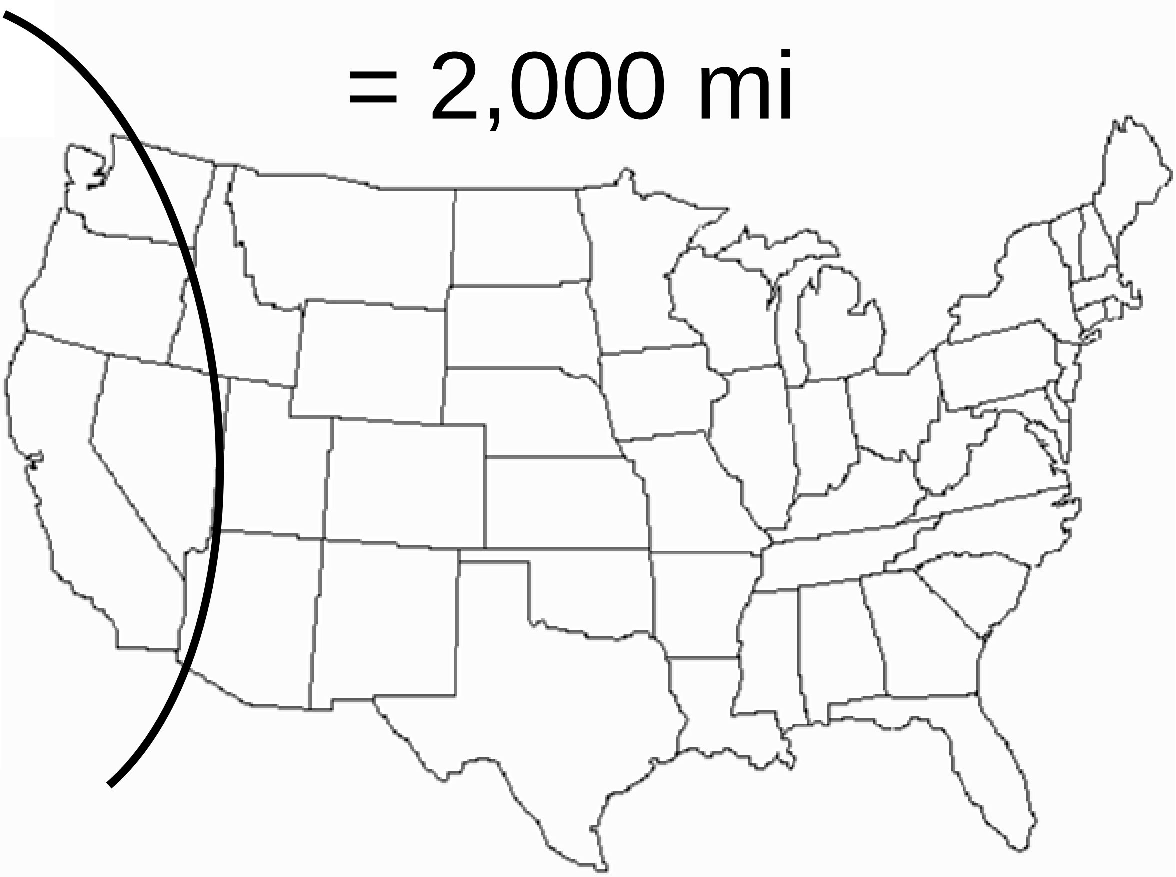
= 2,000 mi

Seismography and the Hyperbola

distance to New York

- distance to San Francisco

= 2,000 mi

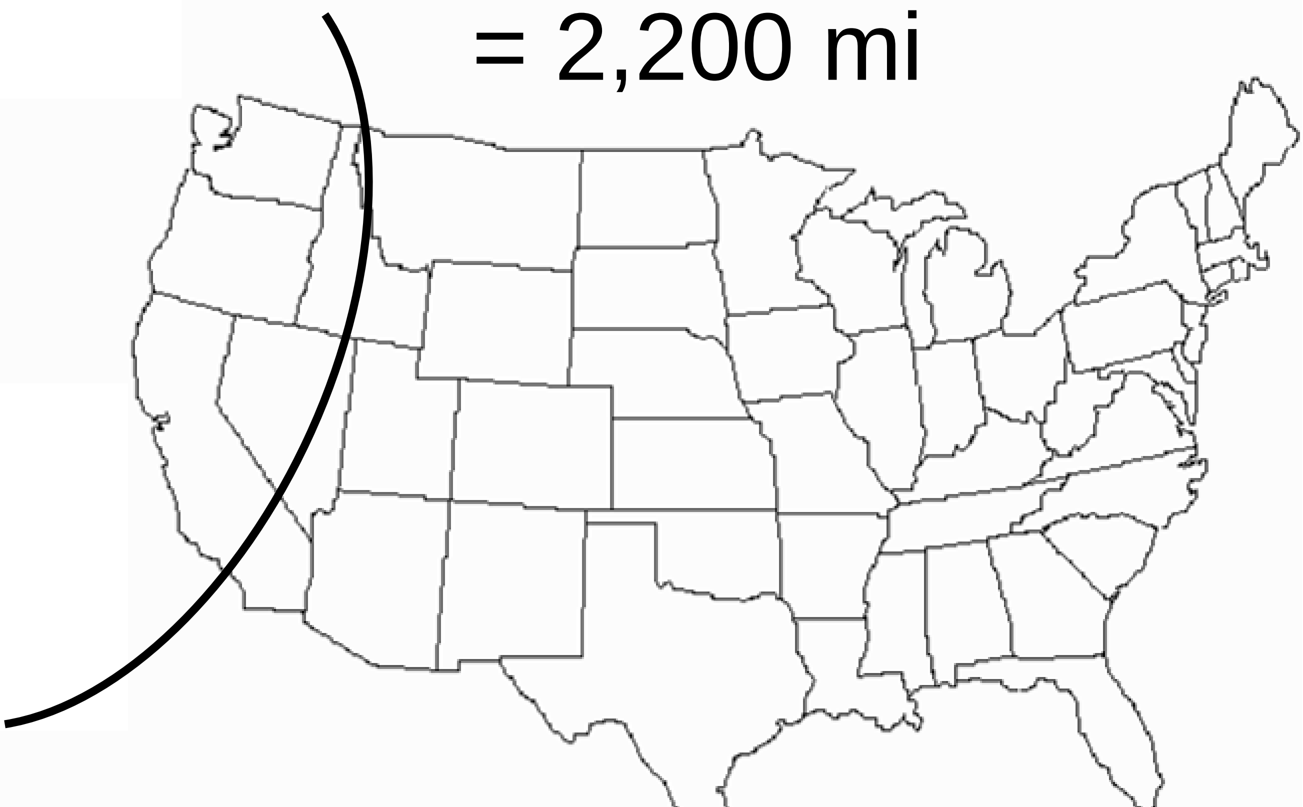


Seismography and the Hyperbola

distance to Miami

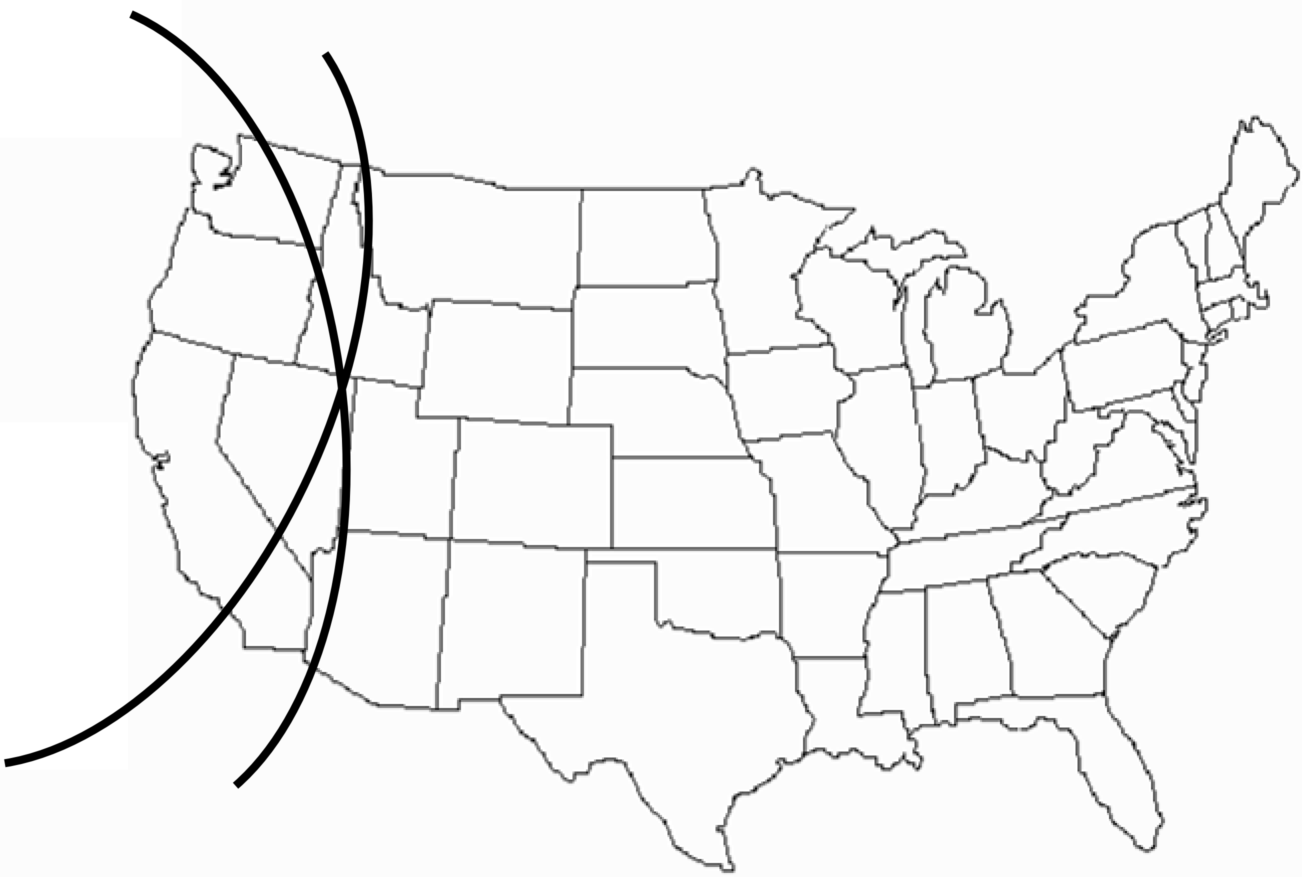
- distance to San Francisco

= 2,200 mi

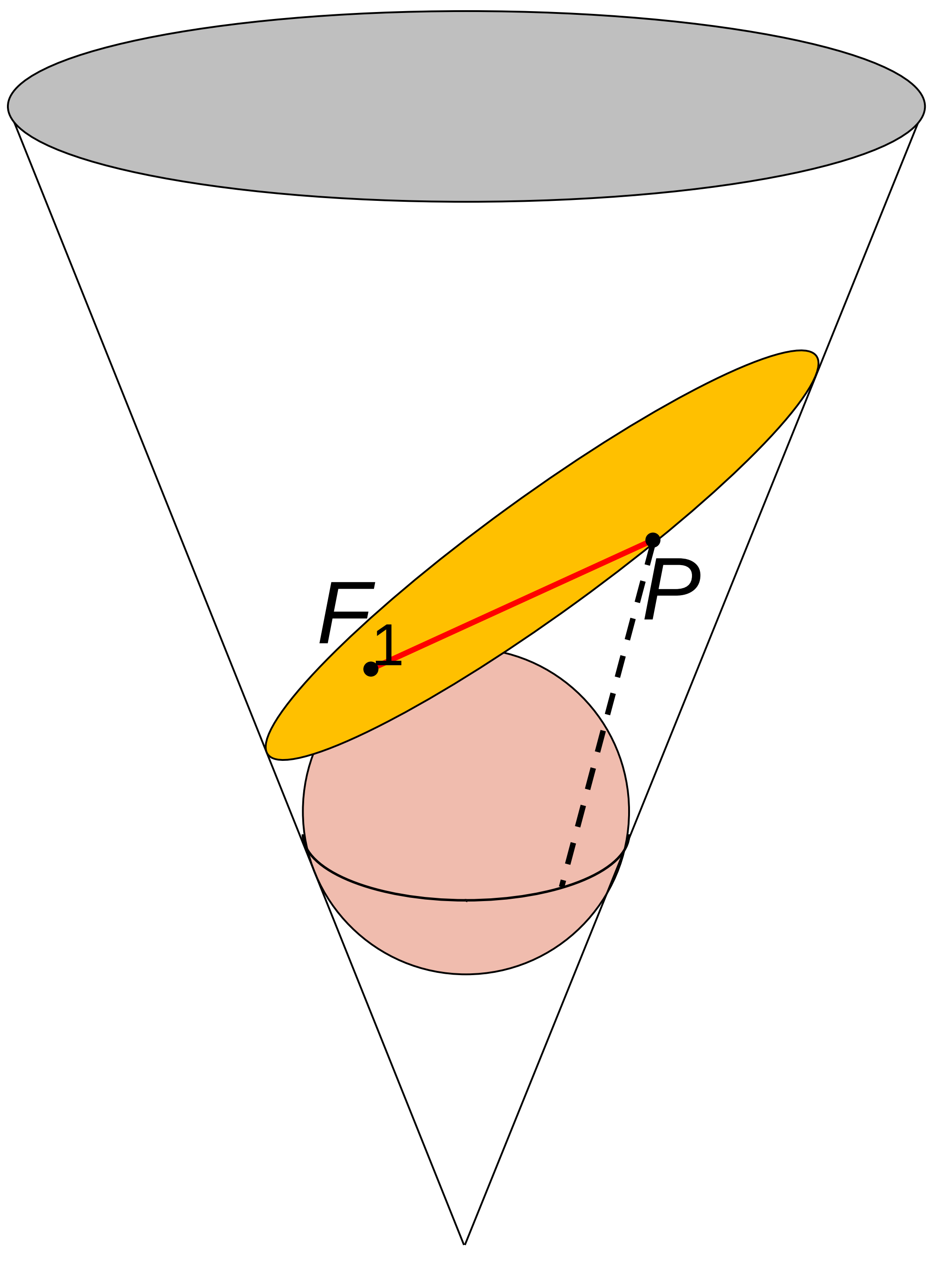


Seismography and the Hyperbola

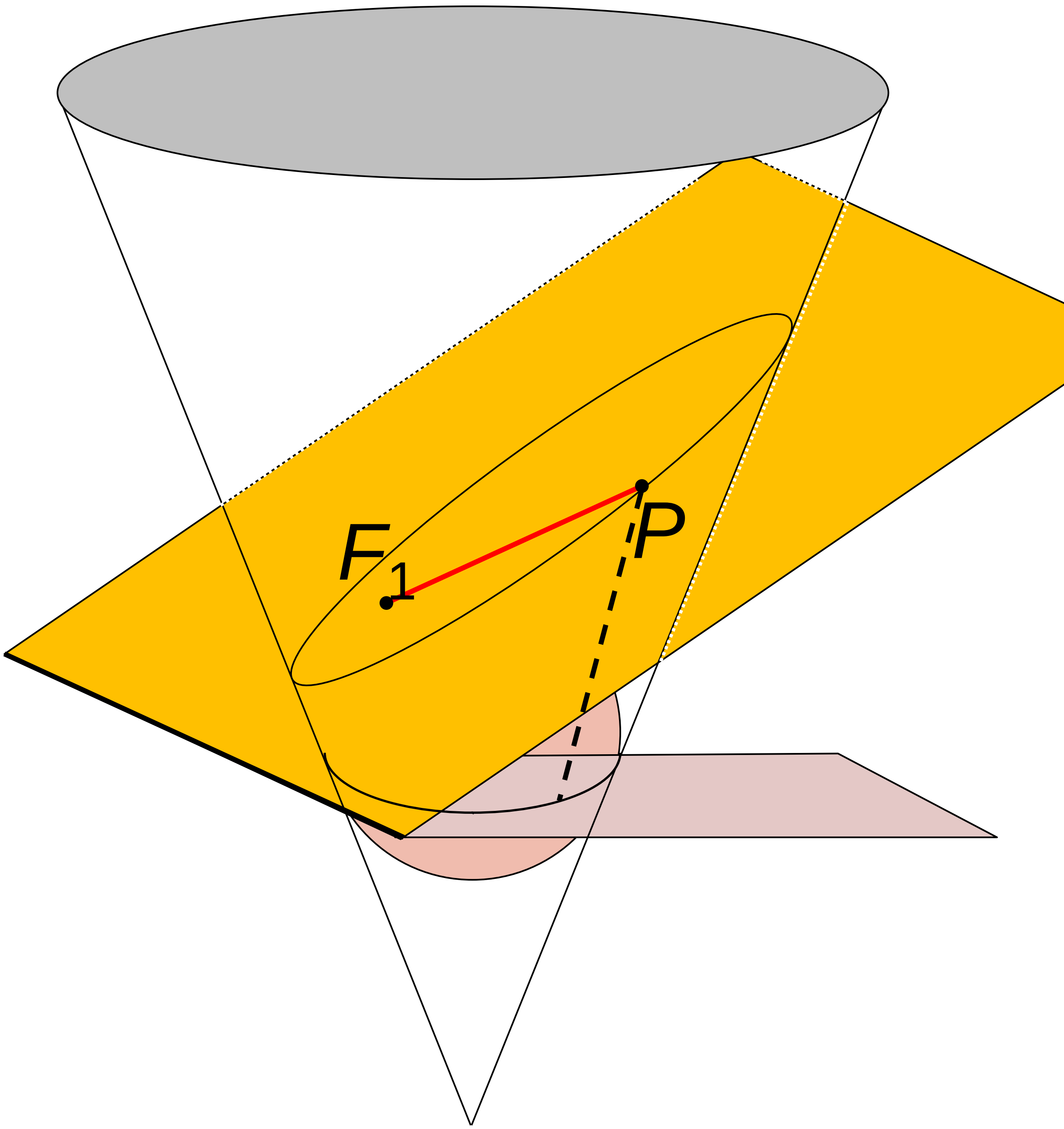
Location:
Elko NV



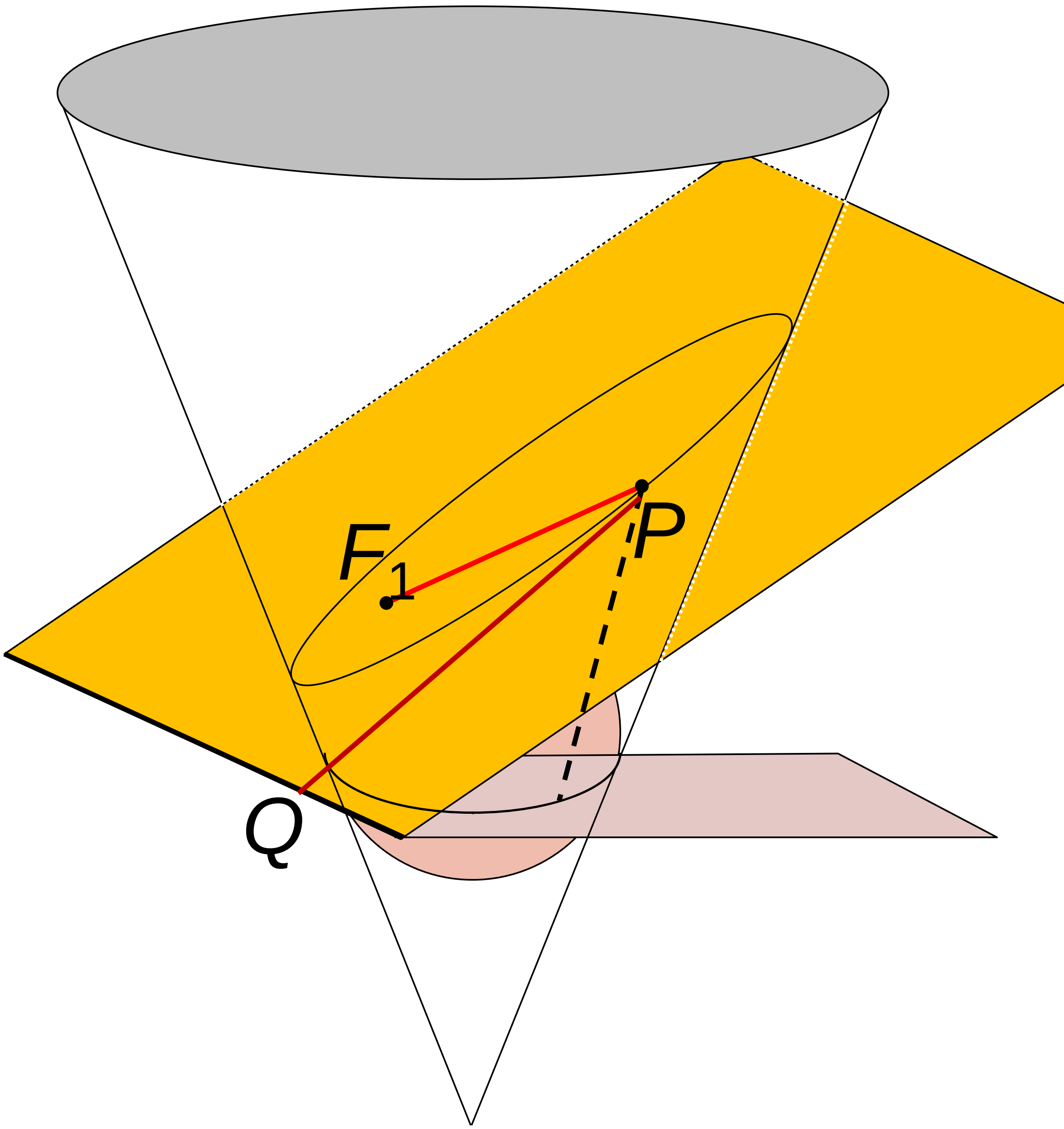
More Geometry of the Sections



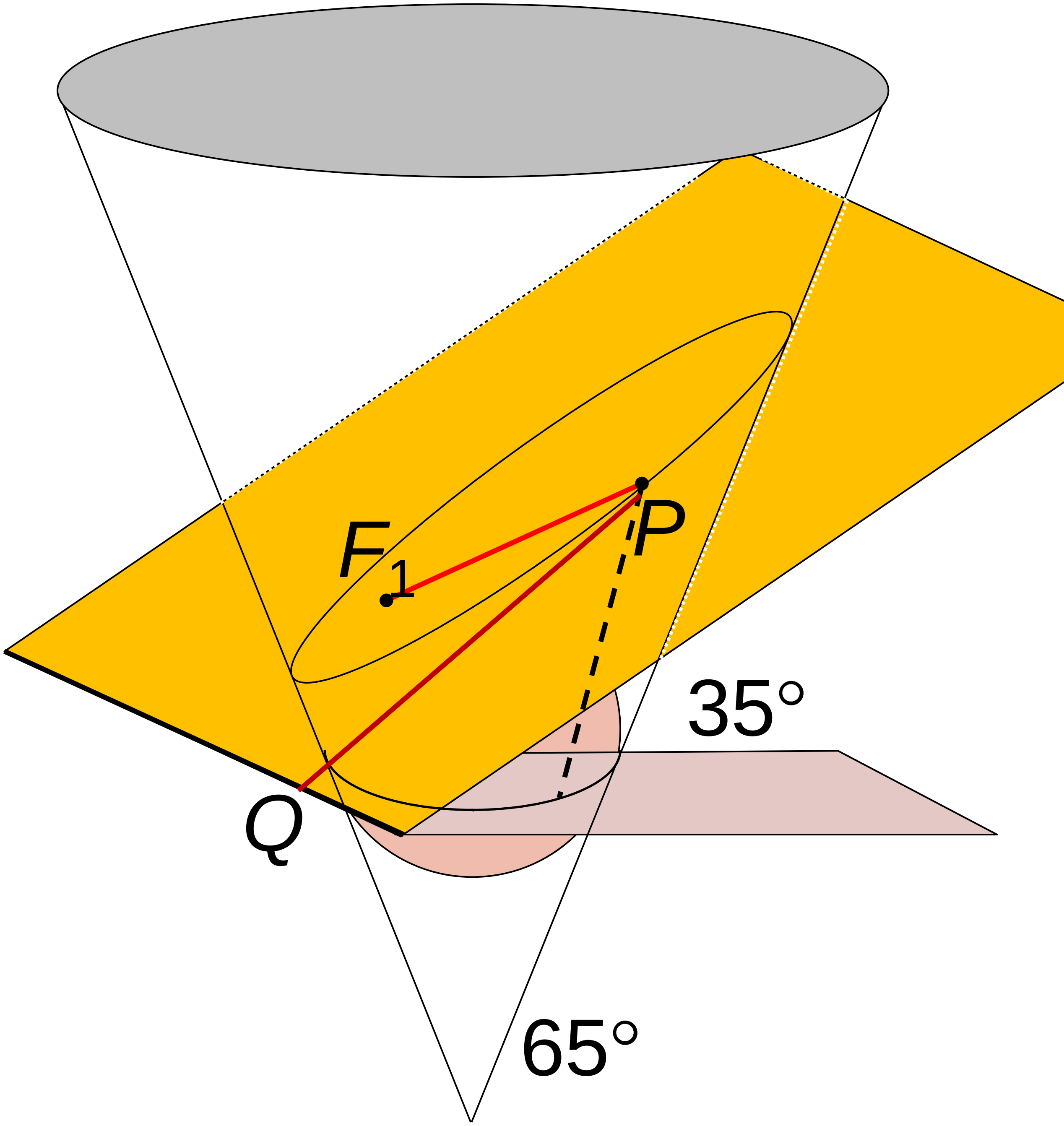
More Geometry of the Sections



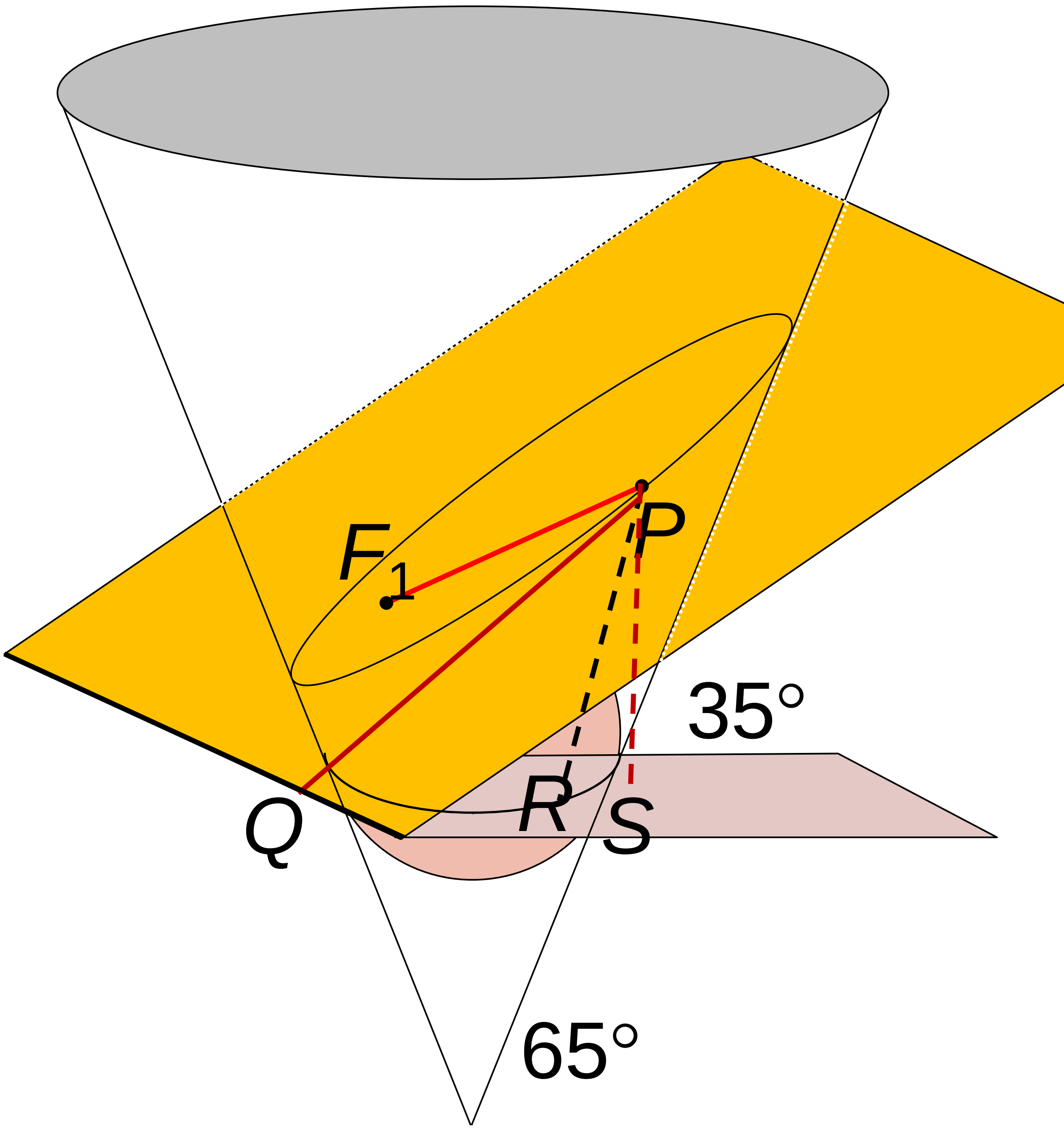
More Geometry of the Sections



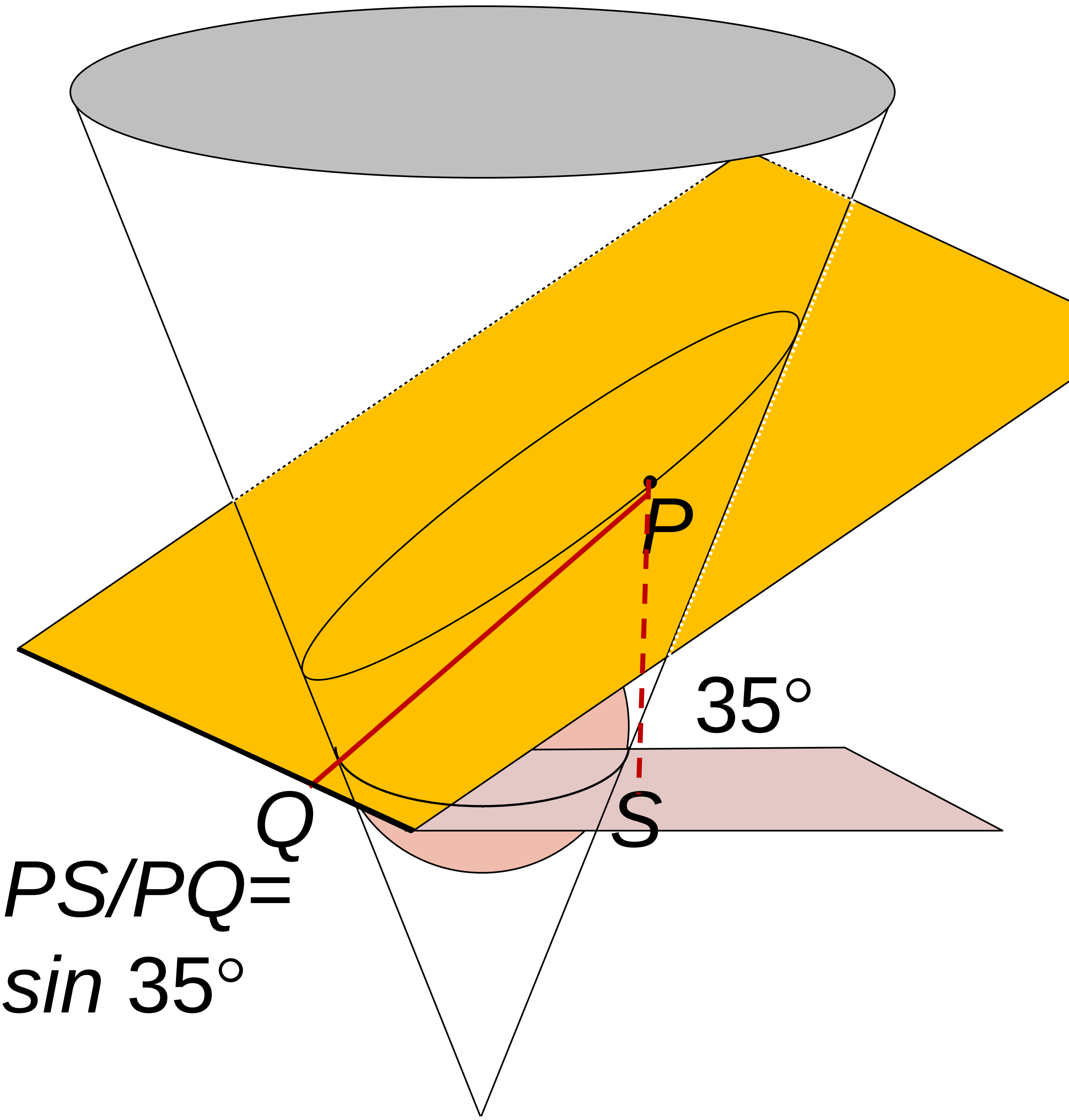
More Geometry of the Sections



More Geometry of the Sections

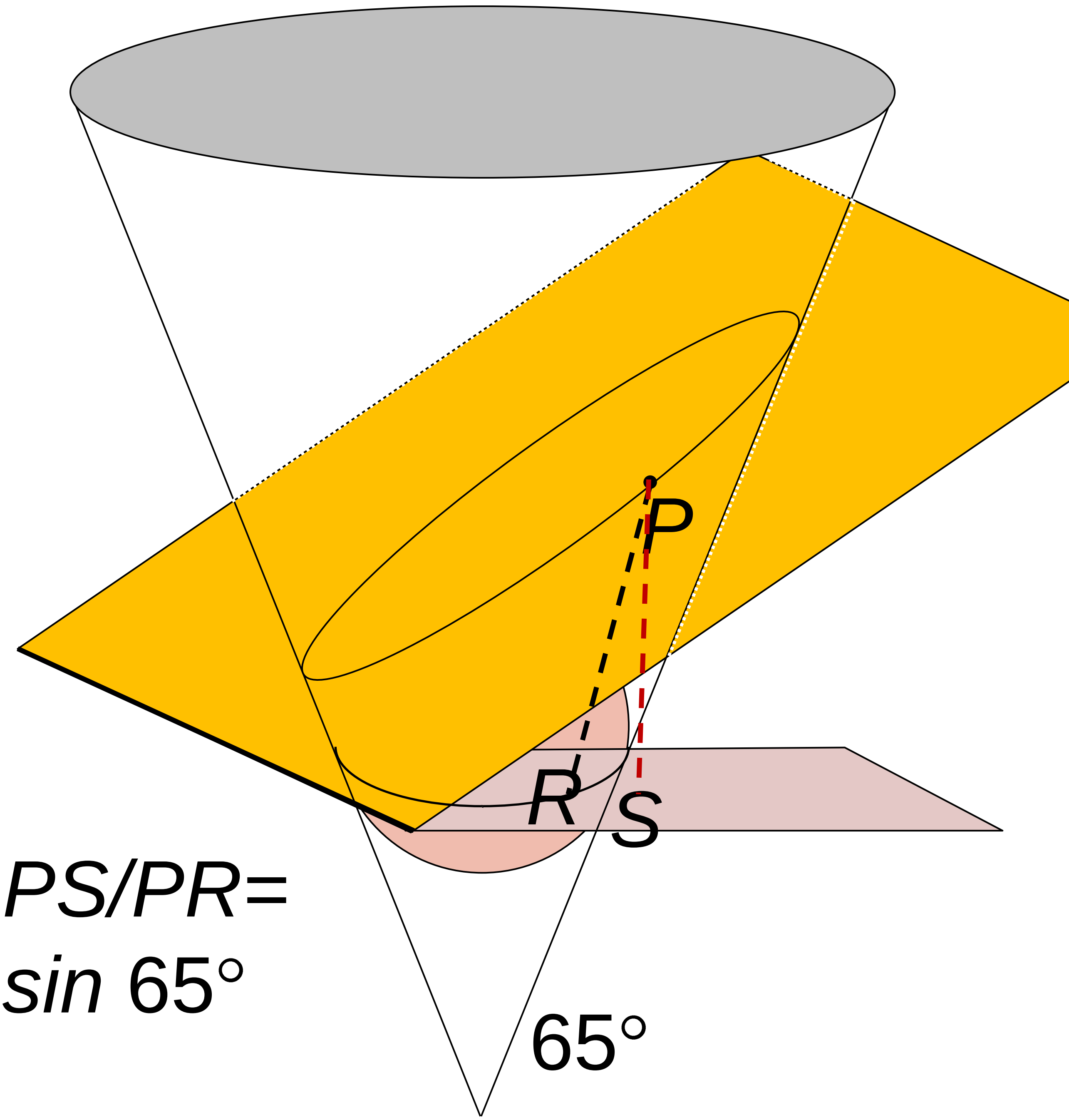


More Geometry of the Sections



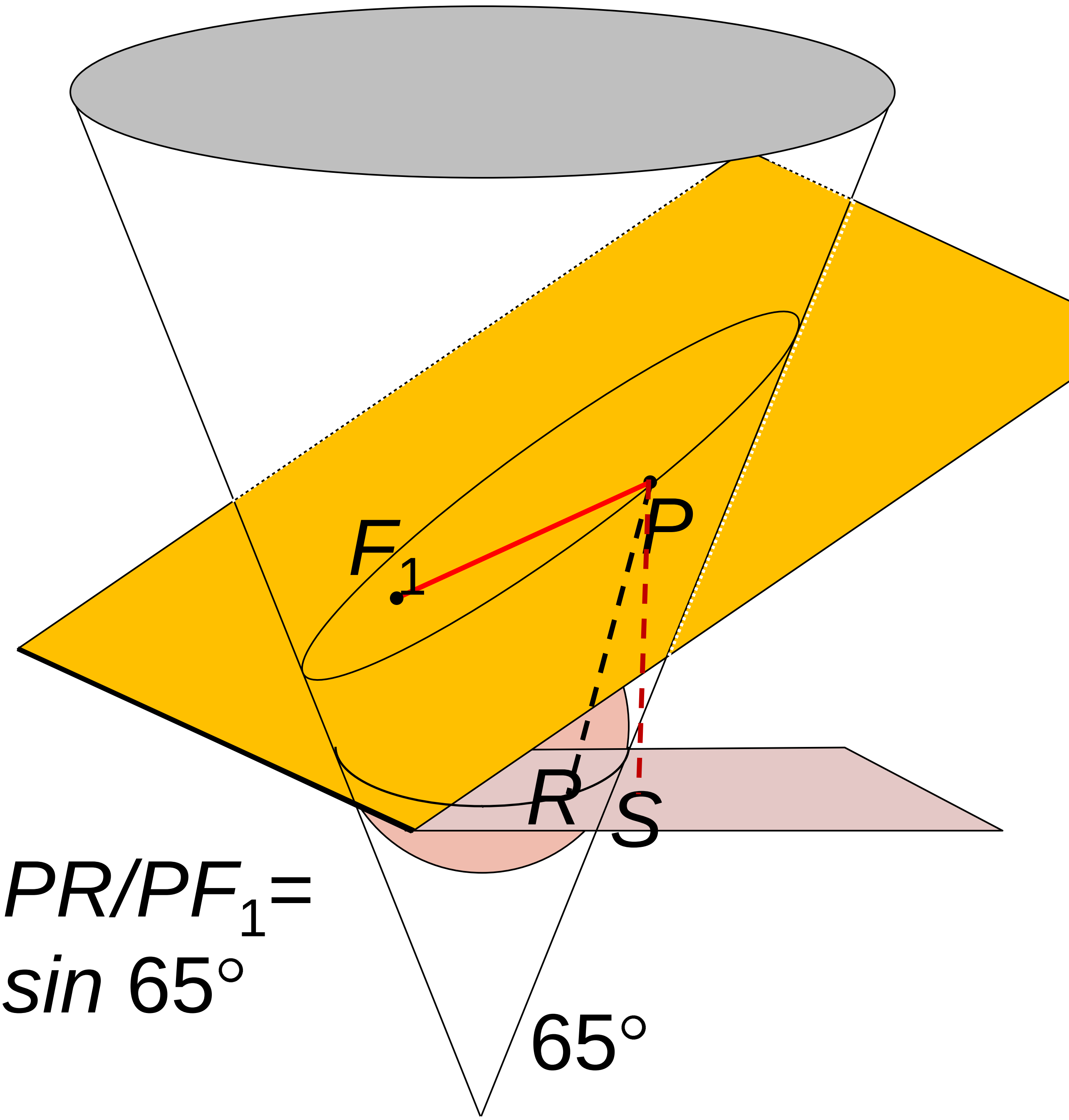
PS/PQ= *sin* 35°

More Geometry of the Sections



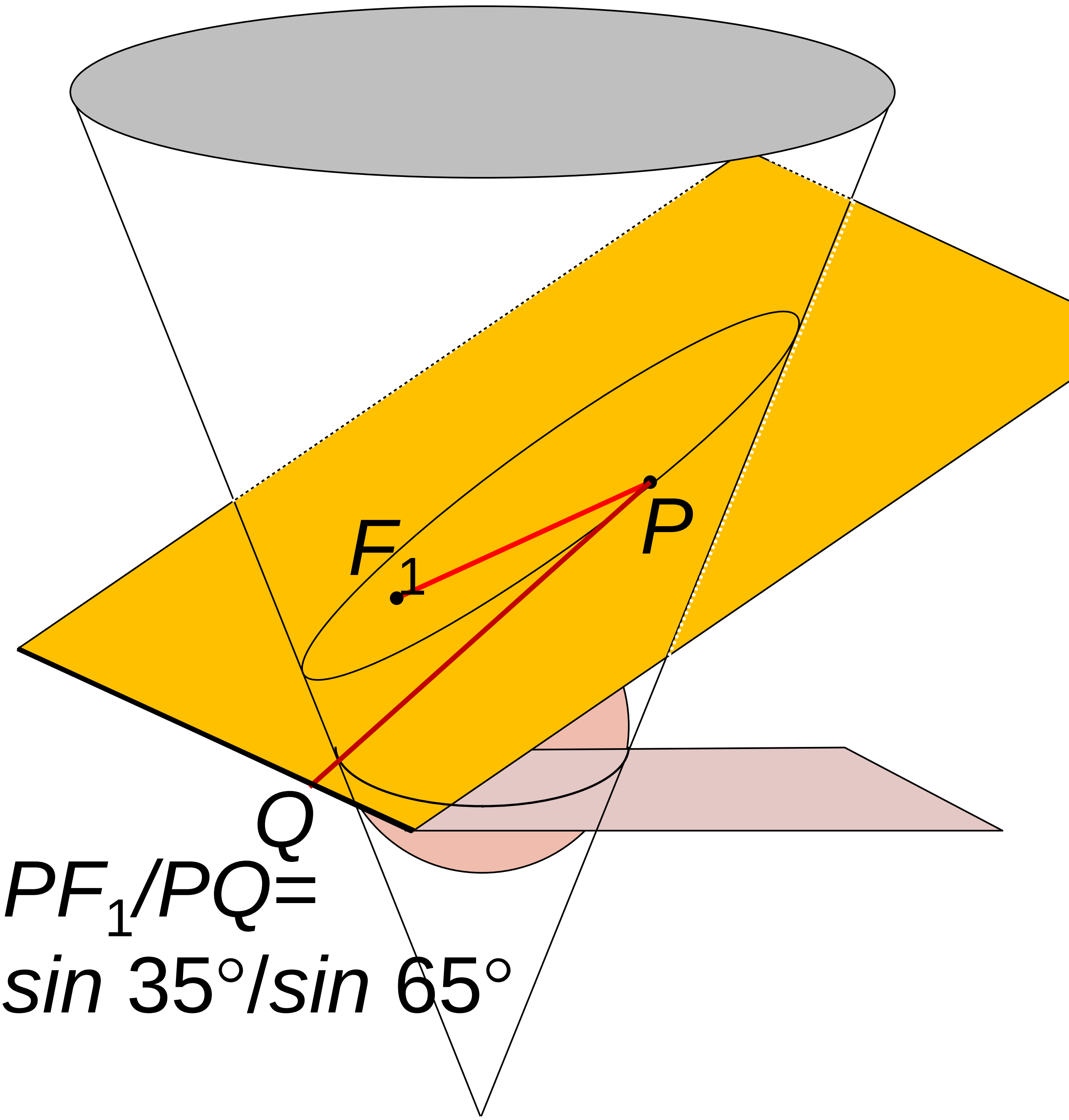
PS/*PR*= *sin* 65°

More Geometry of the Sections



$PR/PF_1 = \sin 65°$

More Geometry of the Sections

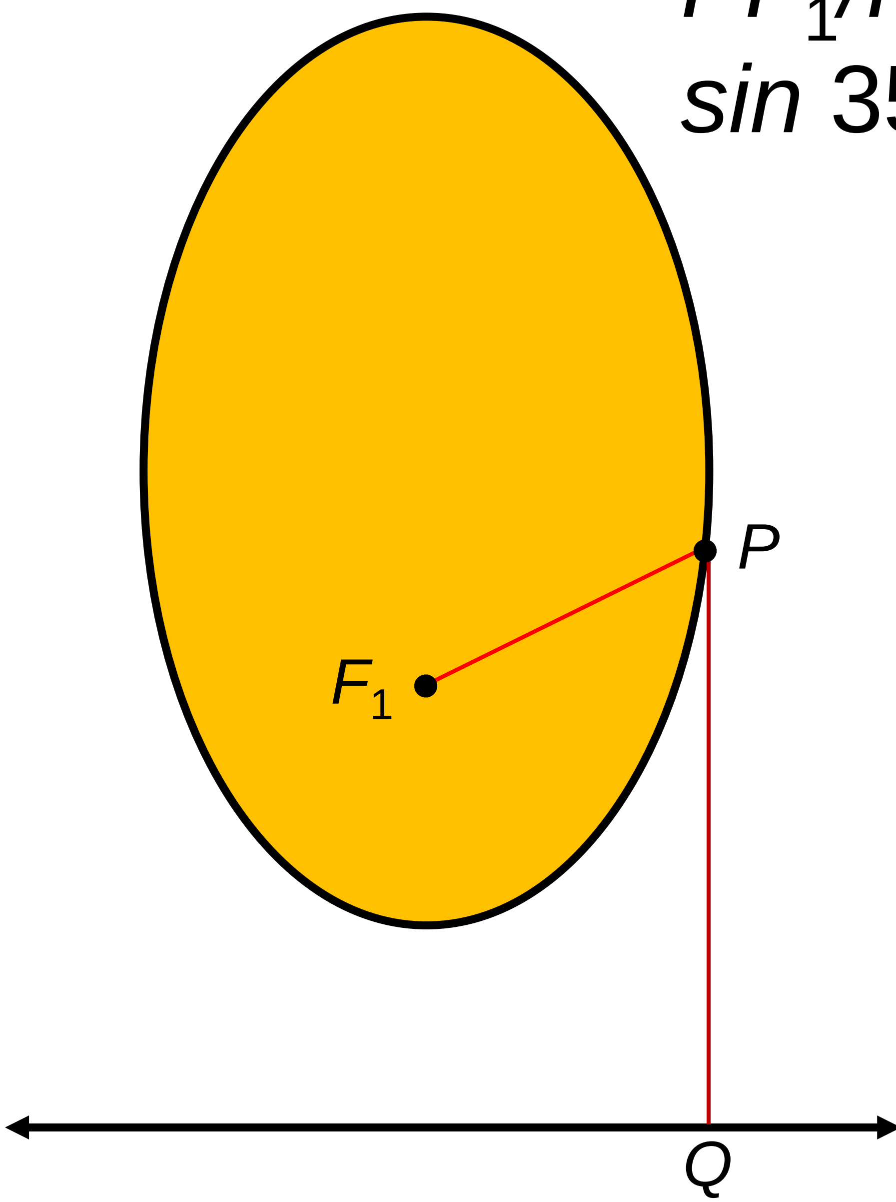


$PF_1/PQ=$

$\sin 35°/\sin 65°$

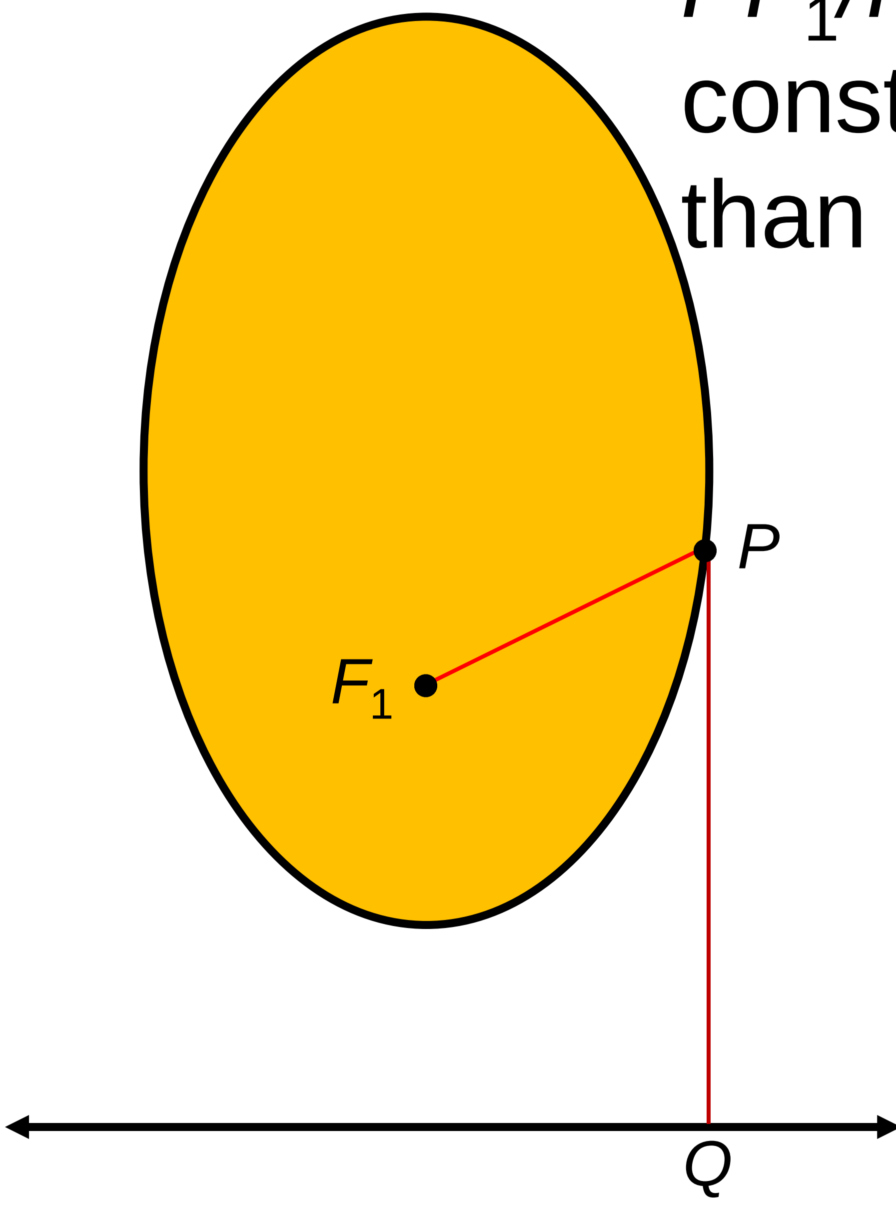
More Geometry of the Sections

$PF_1/PQ =$
$\sin 35°/\sin 65°$



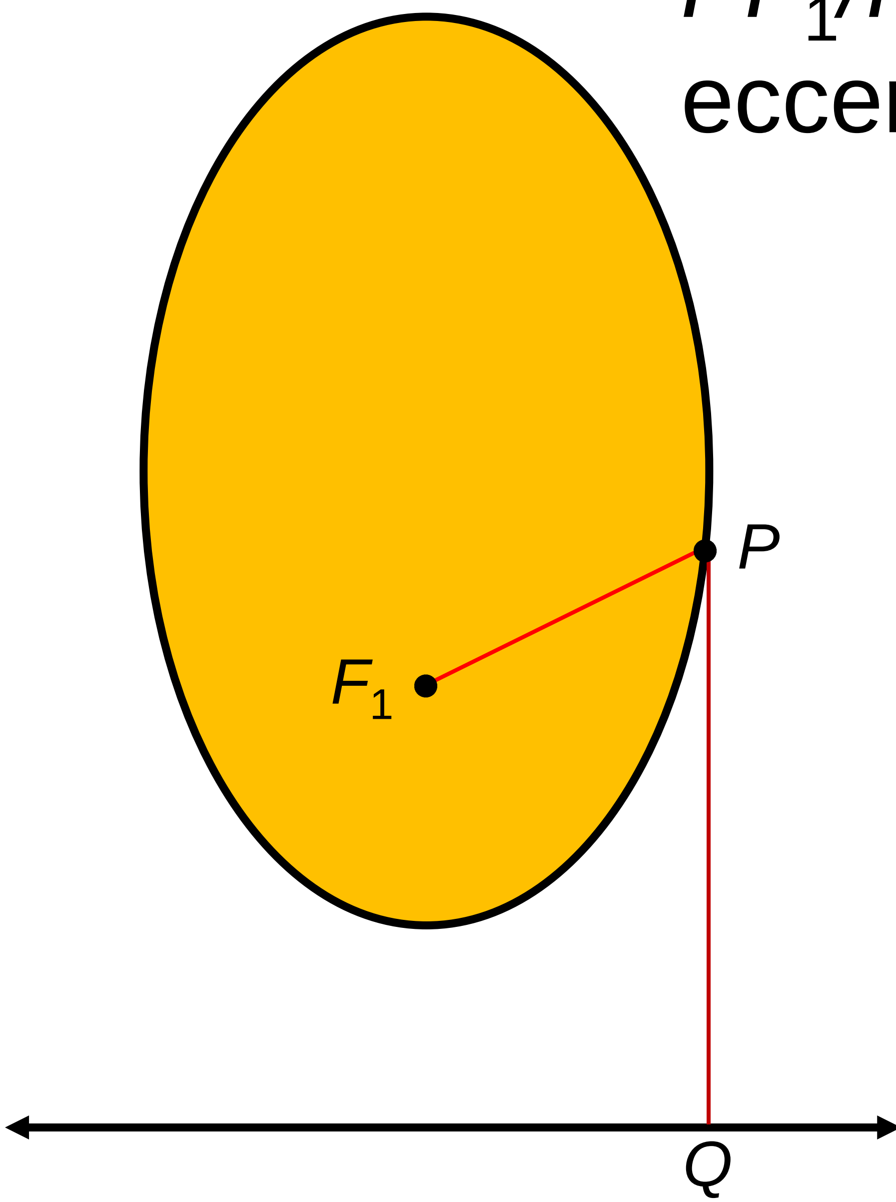
More Geometry of the Sections

PF_1/PQ = constant less than 1



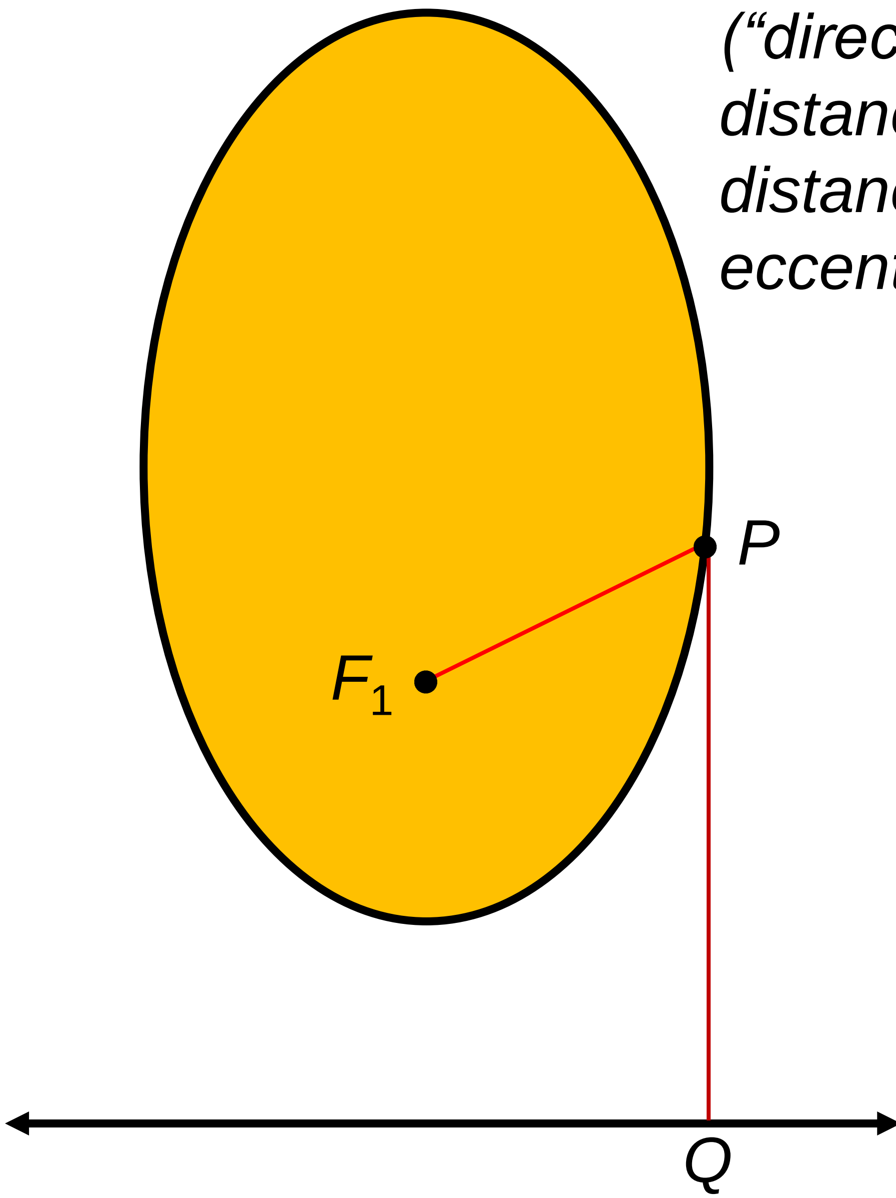
More Geometry of the Sections

PF_1/PQ = eccentricity

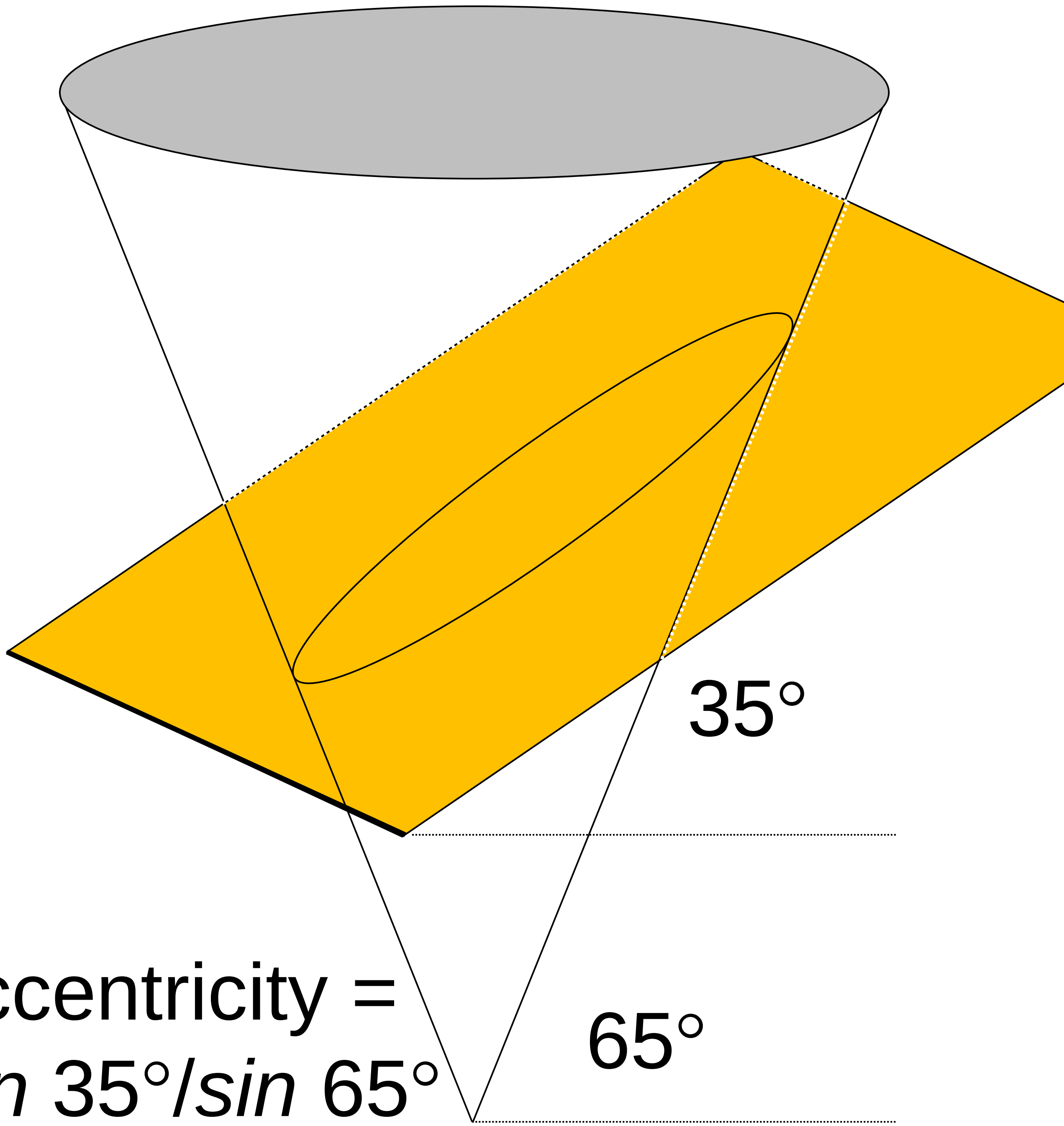


Alternate Description of the Ellipse

There is a line ("directrix") such that distance to focus ÷ distance to line = eccentricity

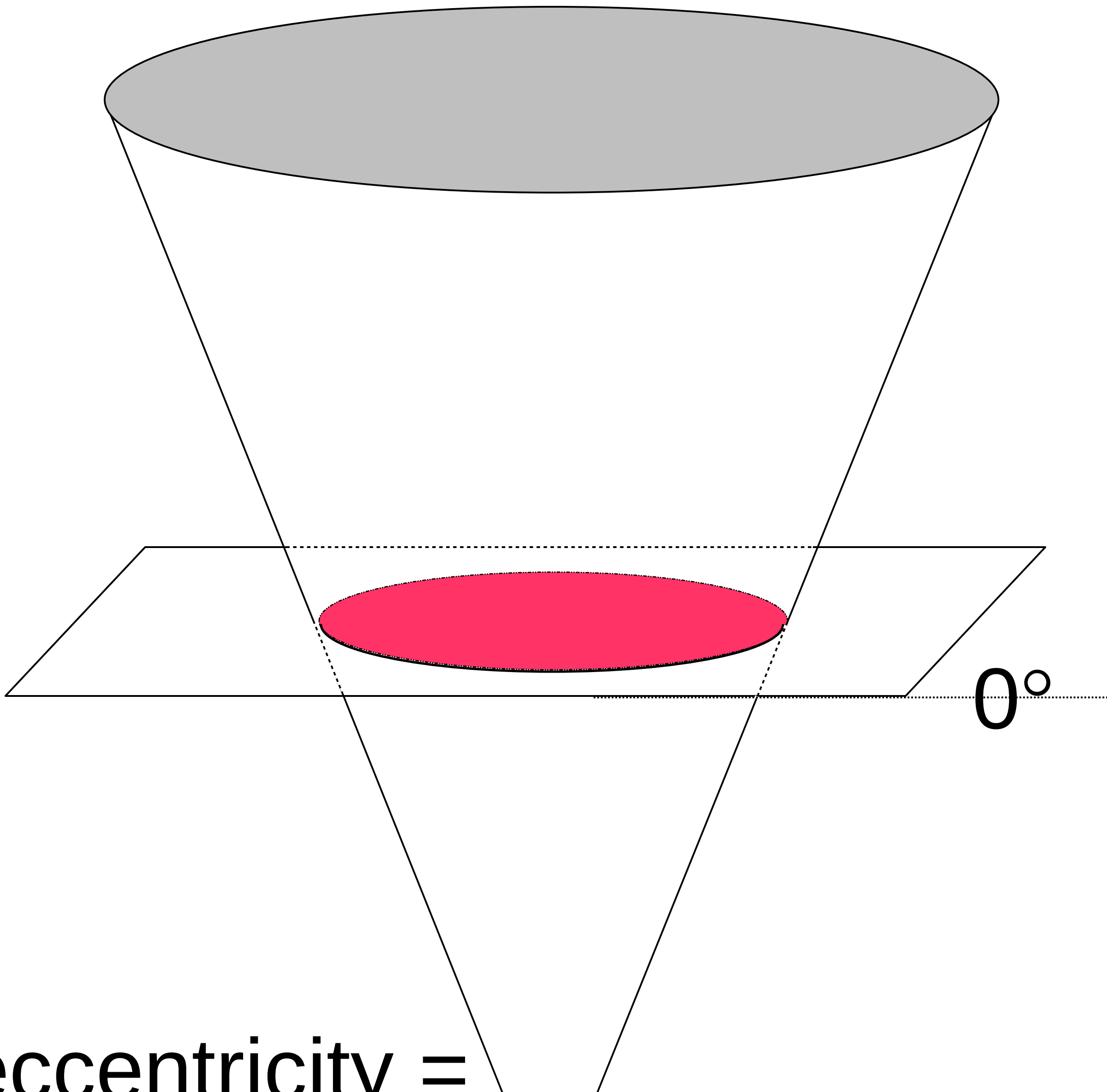


Eccentricity in the Sections



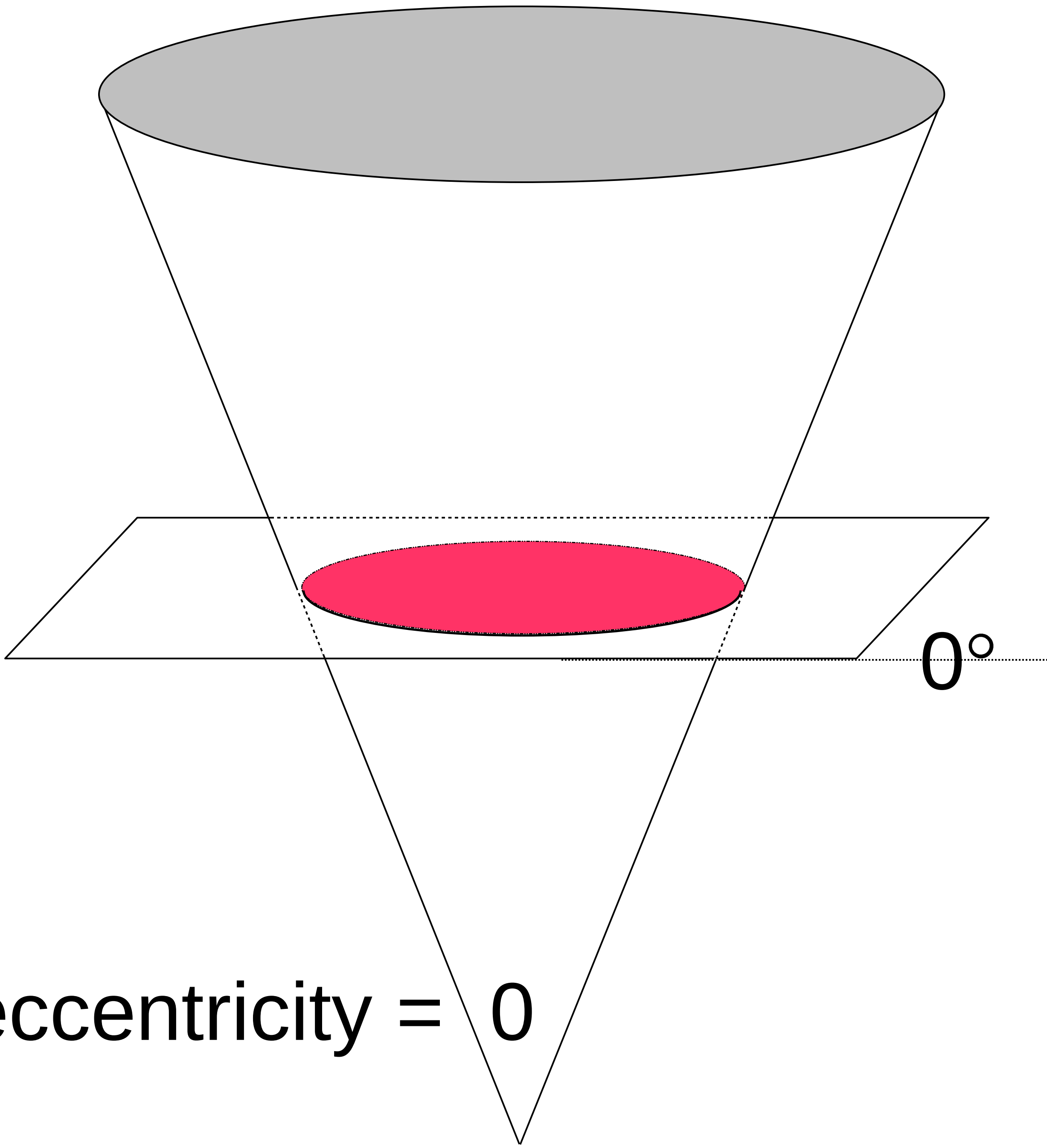
eccentricity =
$\sin 35° / \sin 65°$

Eccentricity in the Sections



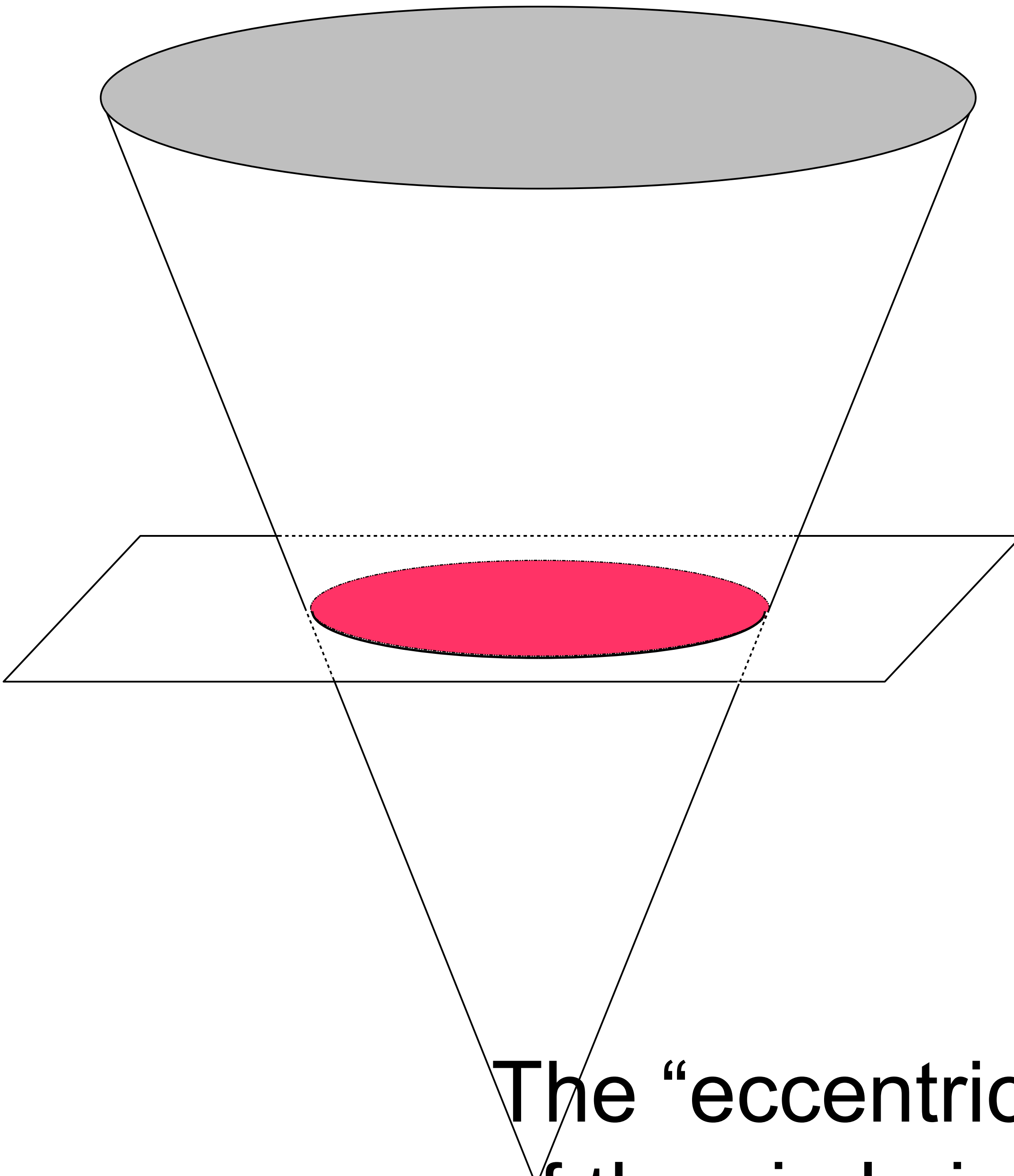
eccentricity =
sin 0°/*sin* 65°

Eccentricity in the Sections



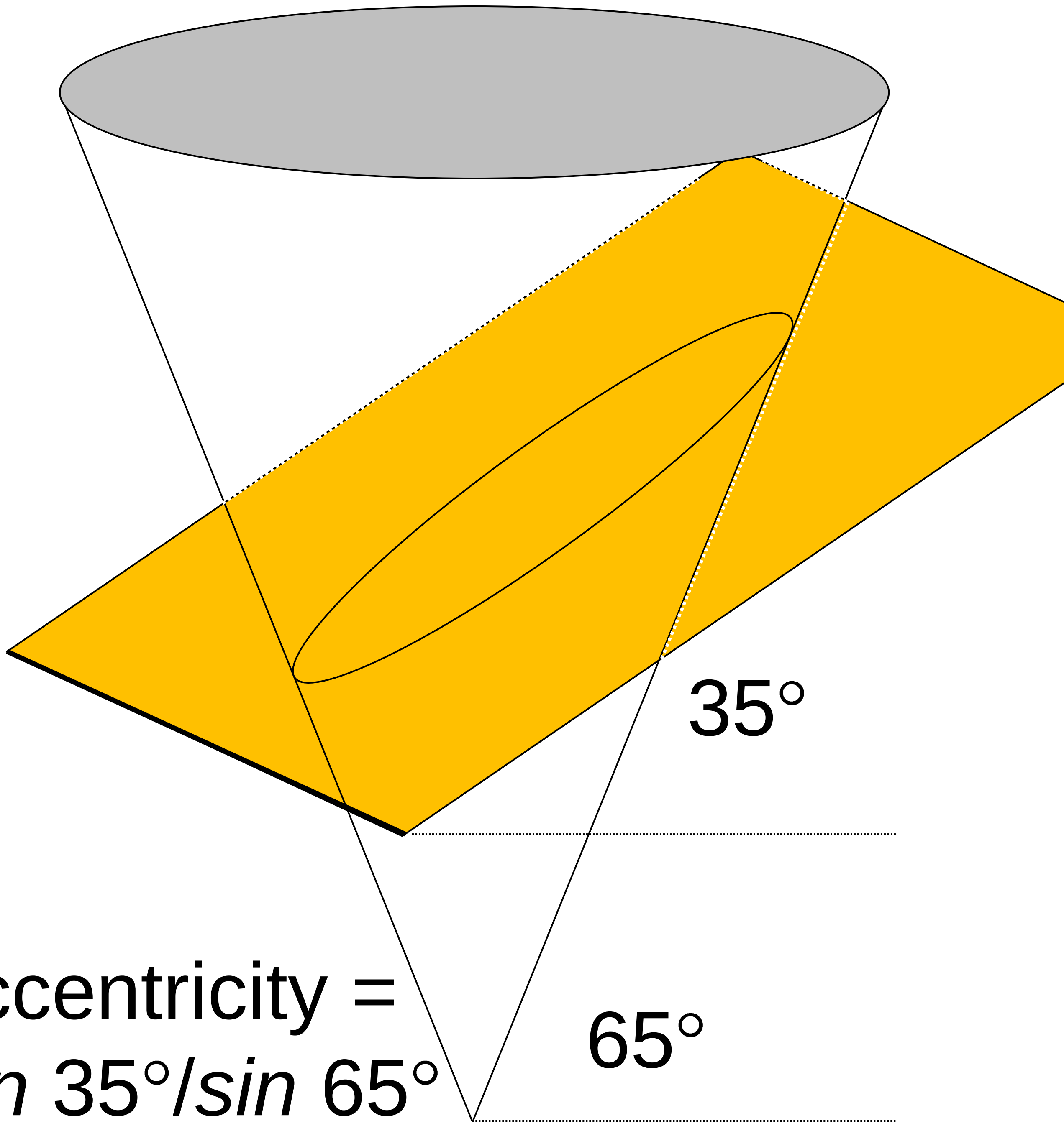
eccentricity = 0

Eccentricity in the Sections



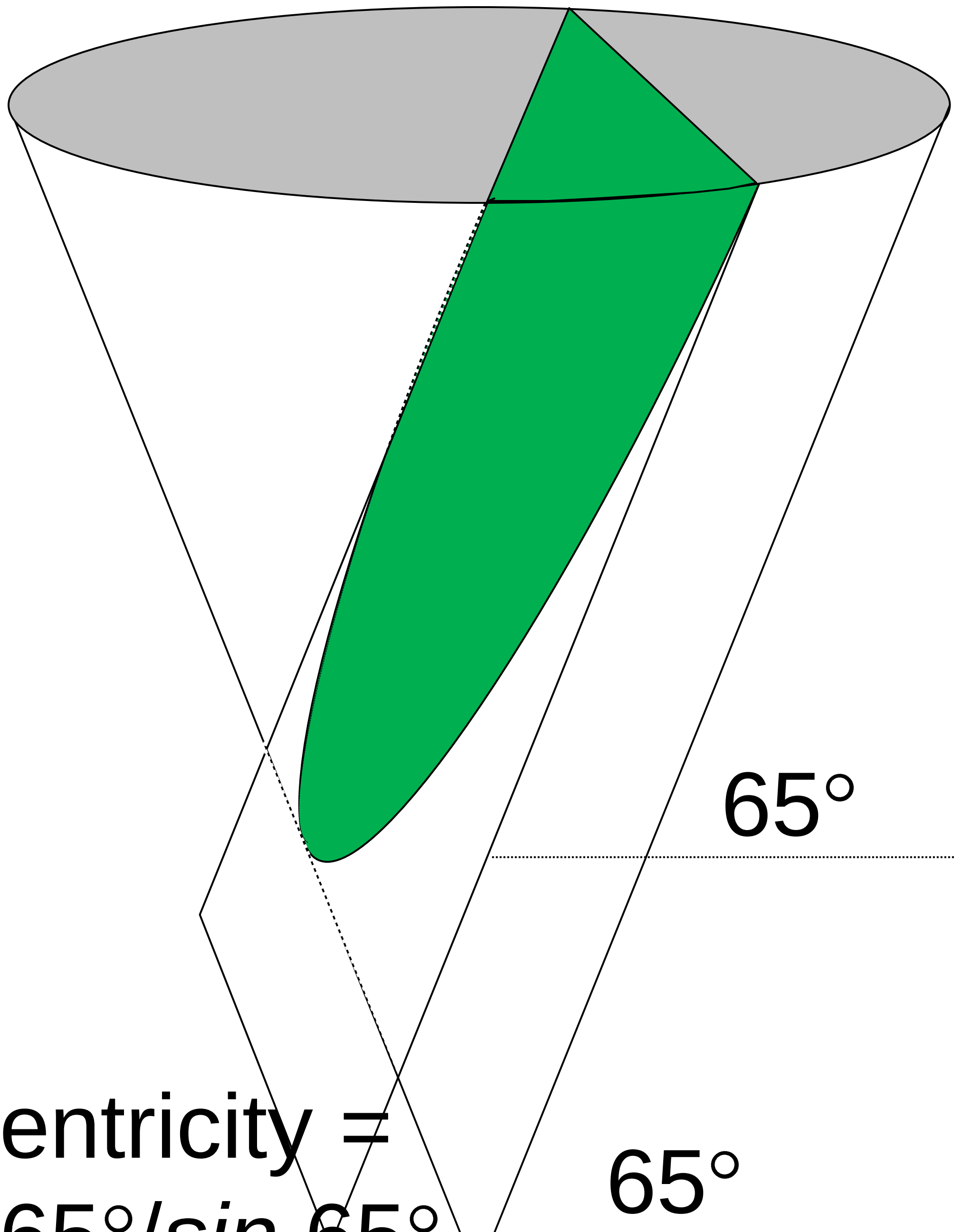
The "eccentricity" of the circle is 0.

Eccentricity in the Sections



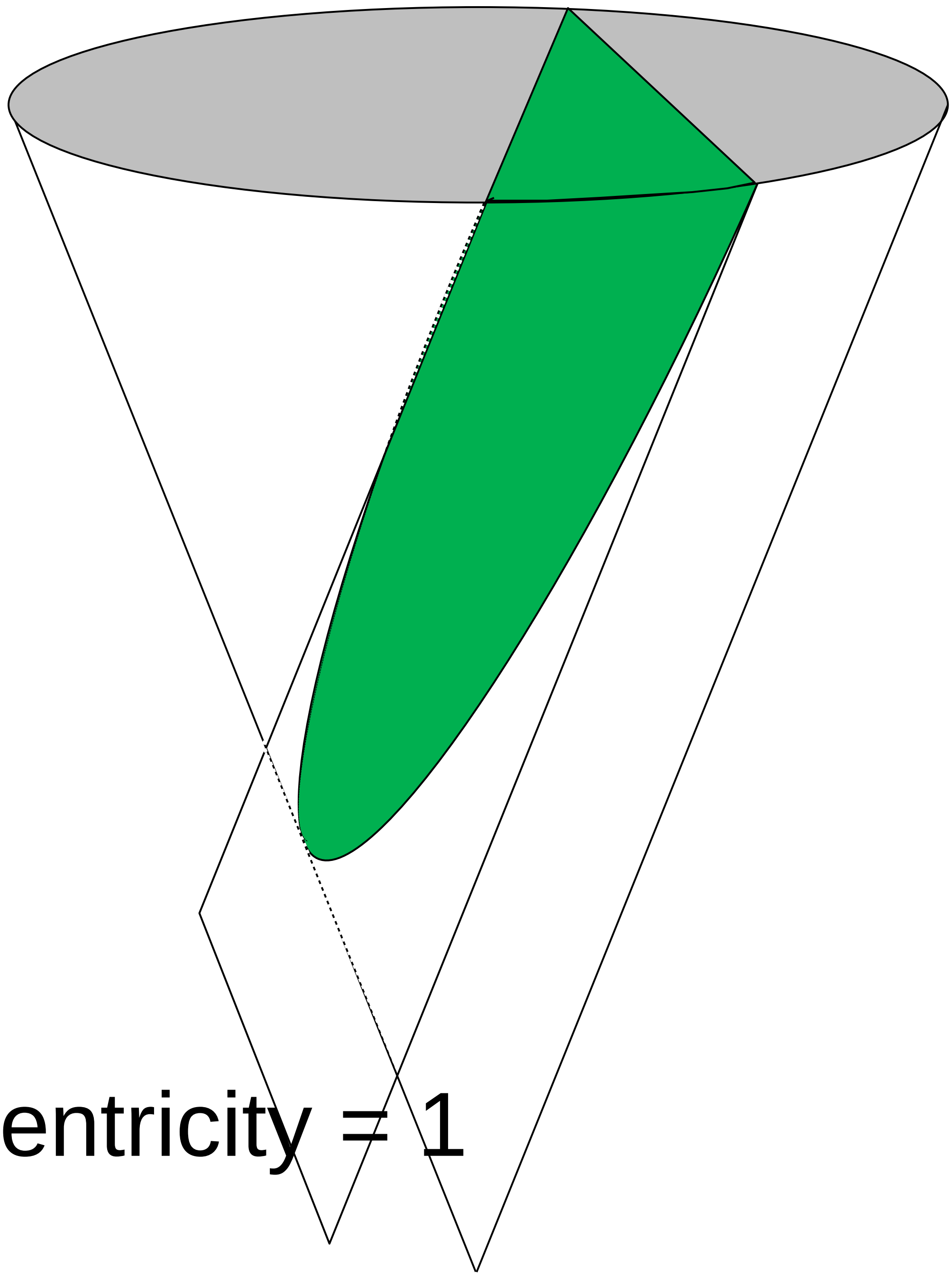
eccentricity =
sin 35°/*sin* 65°

Eccentricity in the Sections



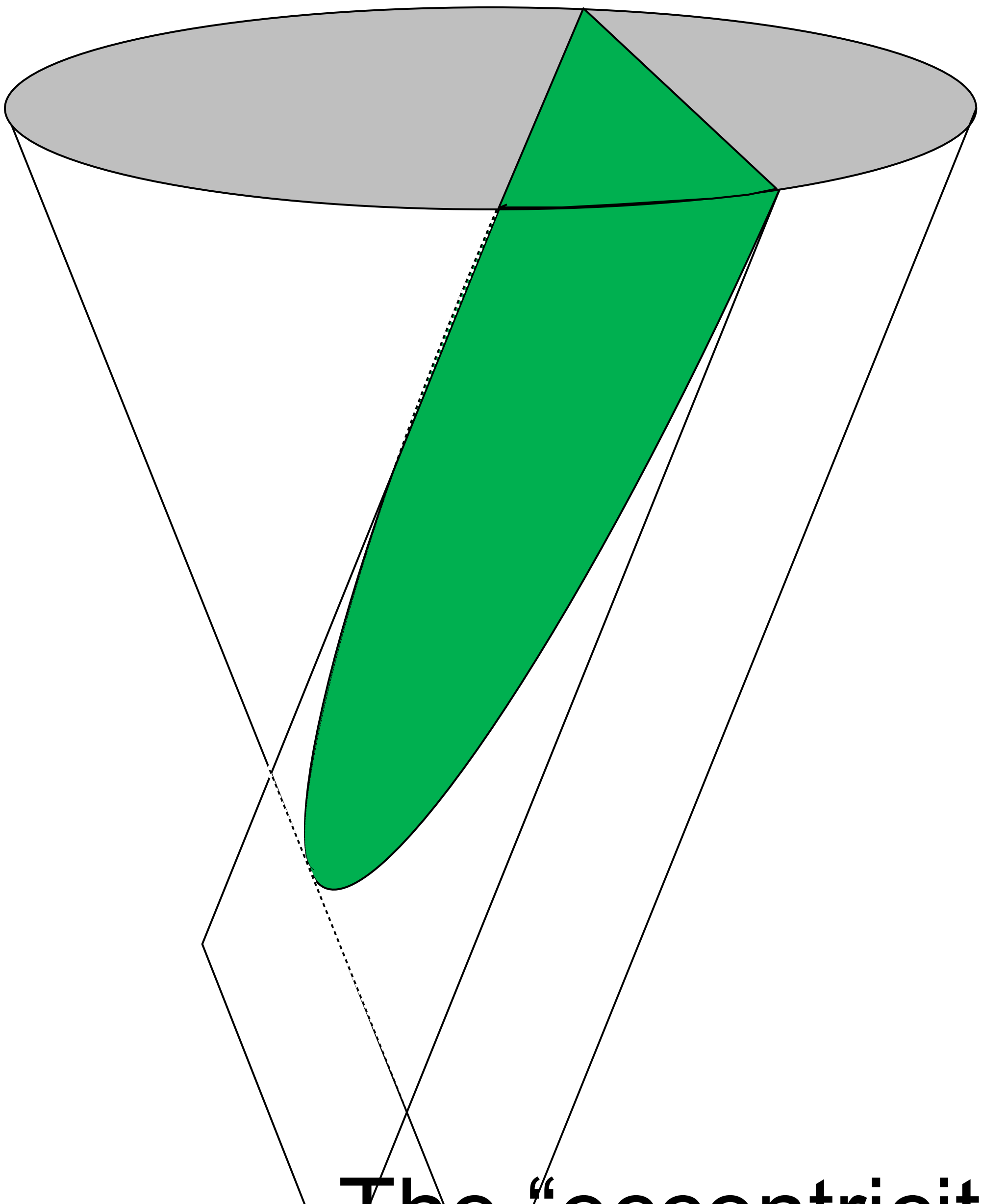
eccentricity =
sin 65°/*sin* 65°

Eccentricity in the Sections



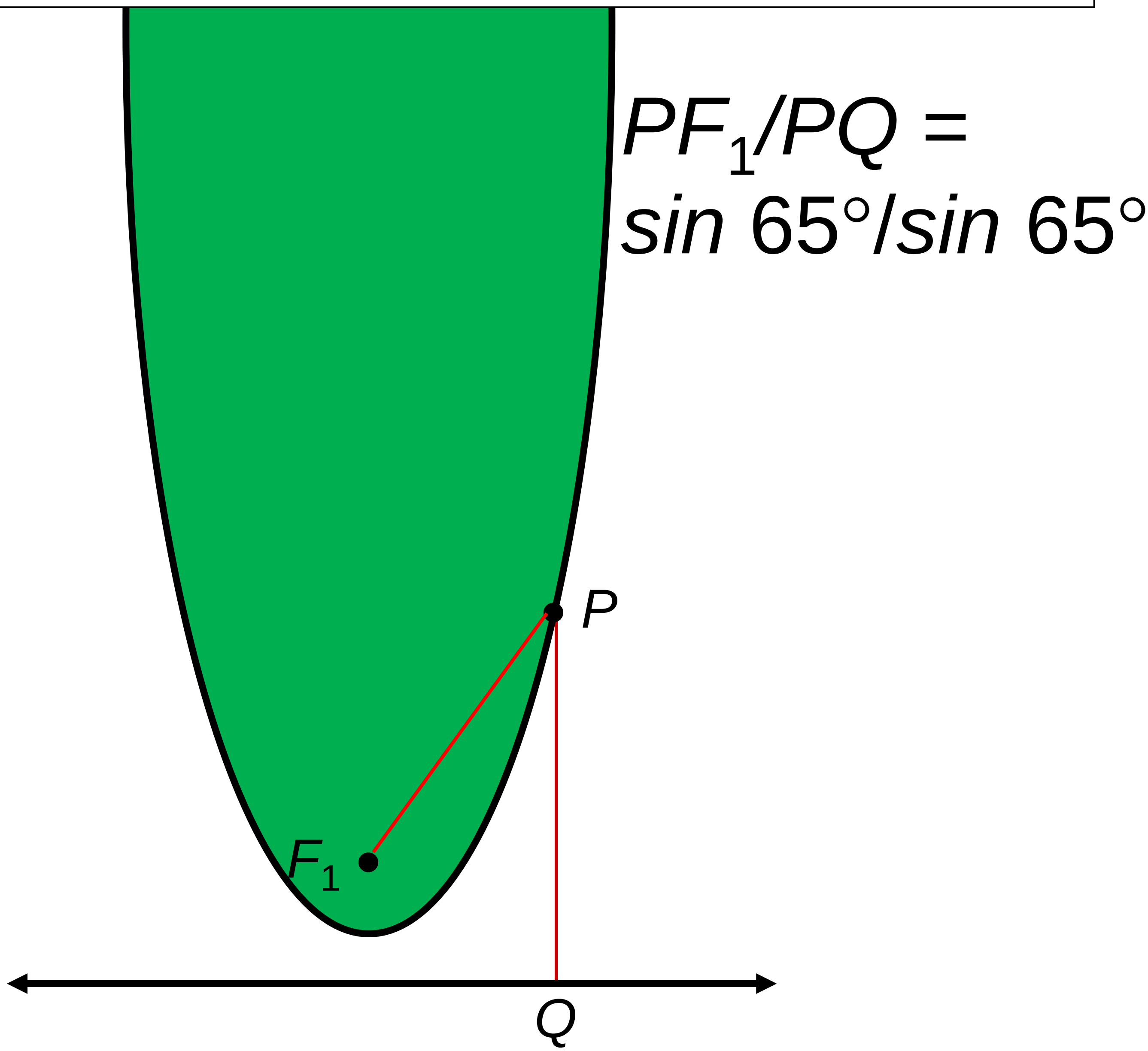
eccentricity = 1

Eccentricity in the Sections



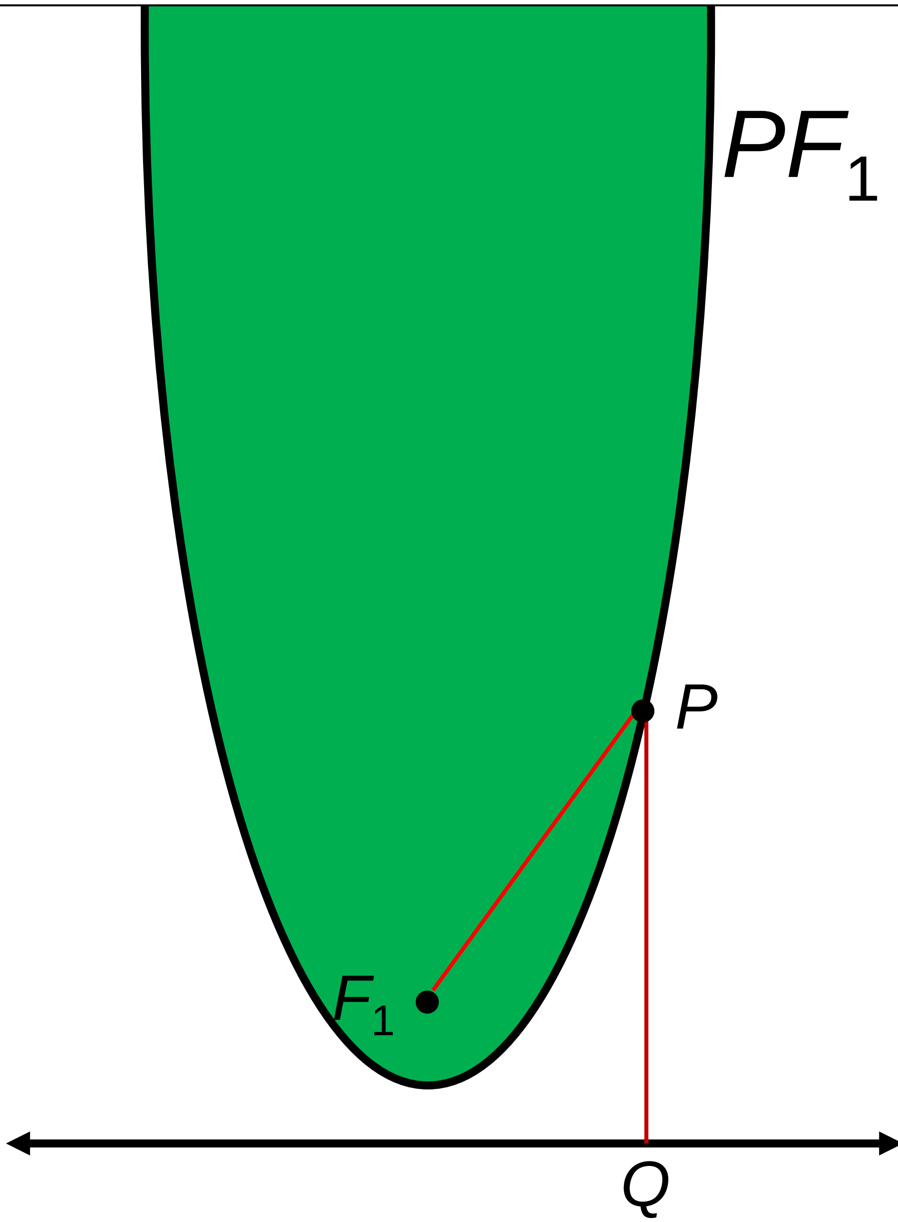
The “eccentricity” of the parabola is 1.

Definition of the Parabola

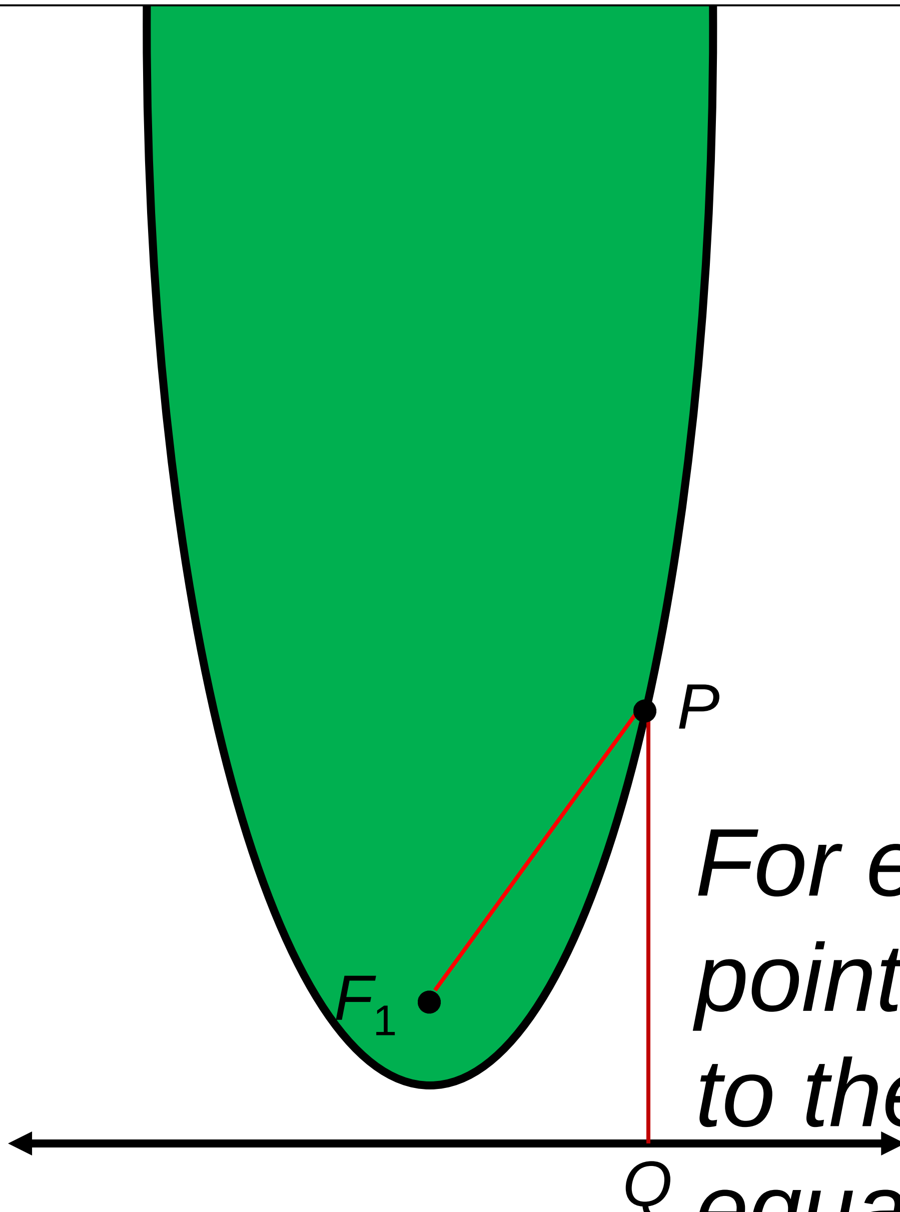


Definition of the Parabola

$$PF_1 = PQ$$

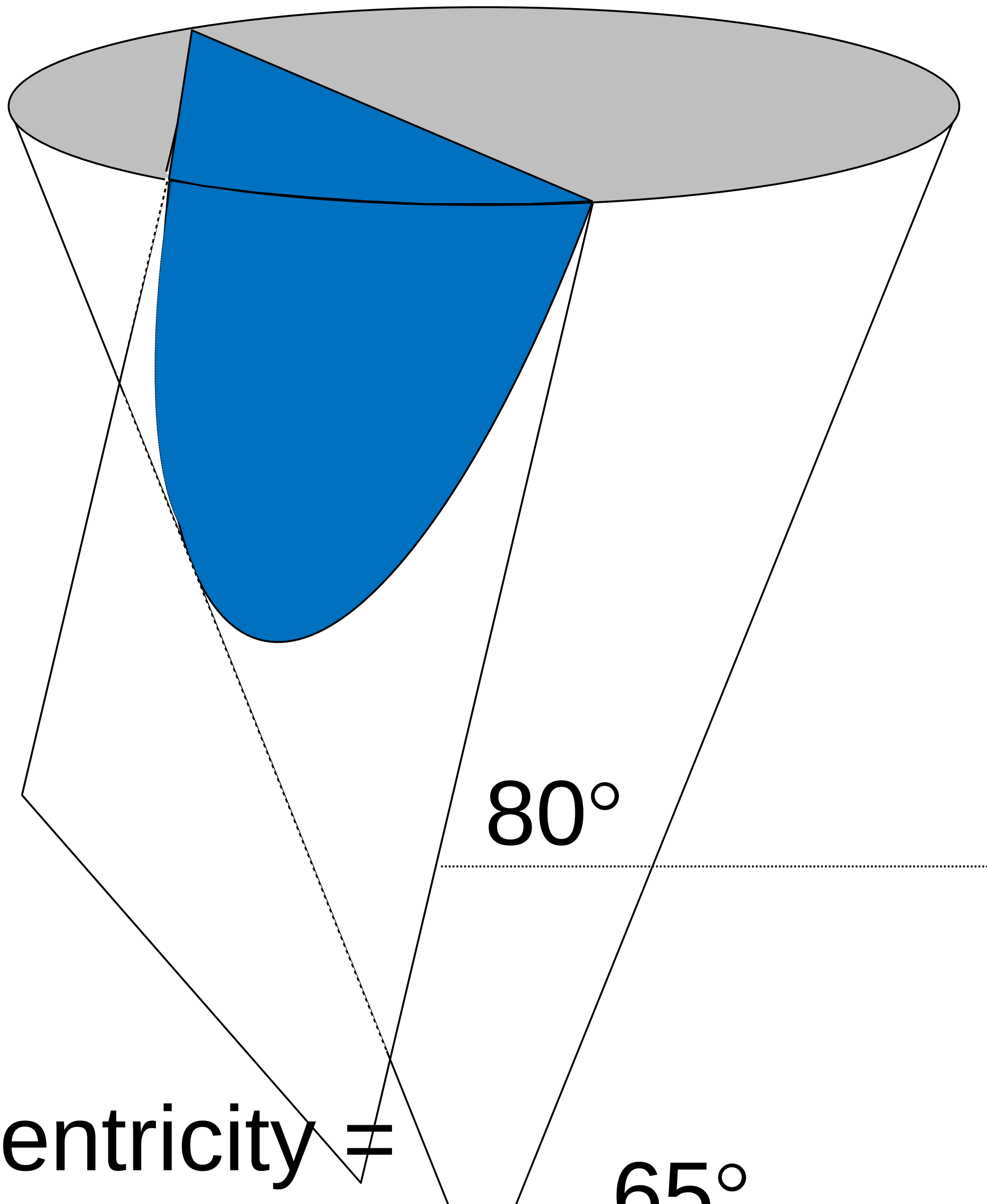


Definition of the Parabola



For every point, distance to the focus equals distance to the directrix.

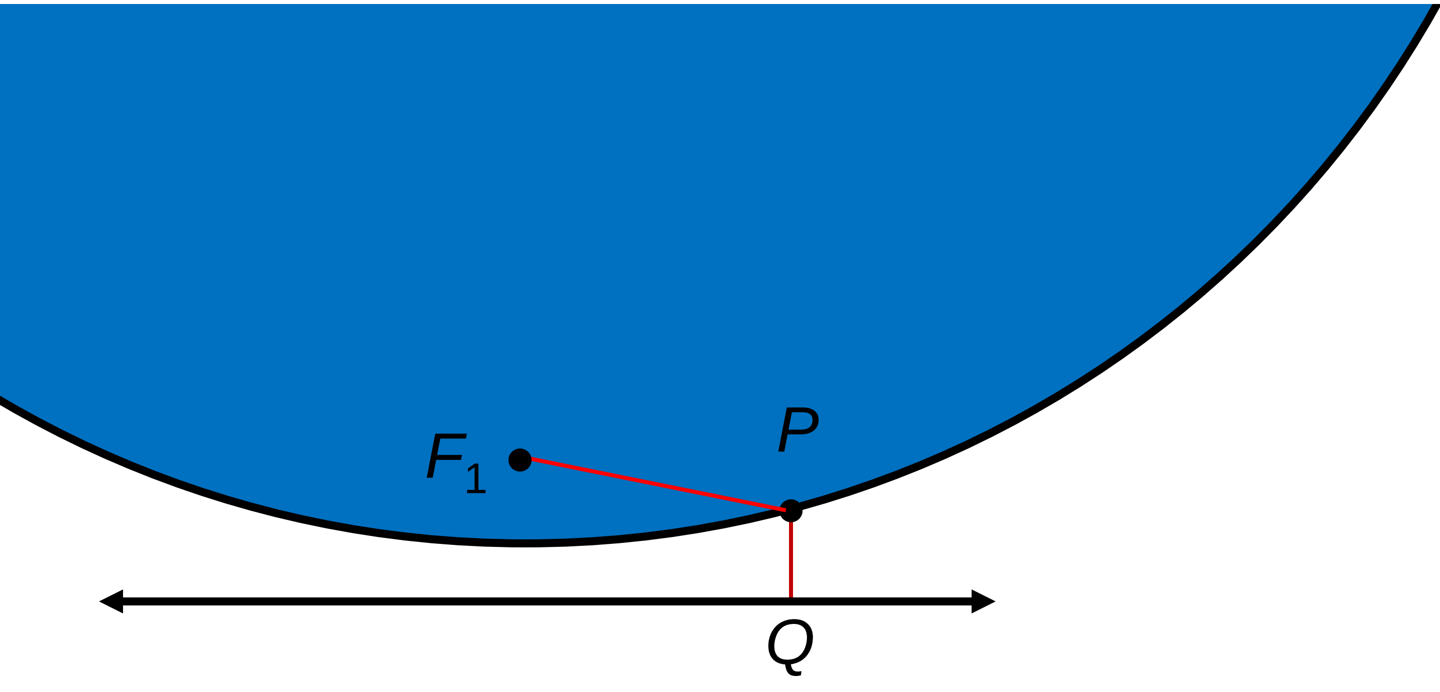
Eccentricity in the Sections



eccentricity = $\sin 80° / \sin 65°$

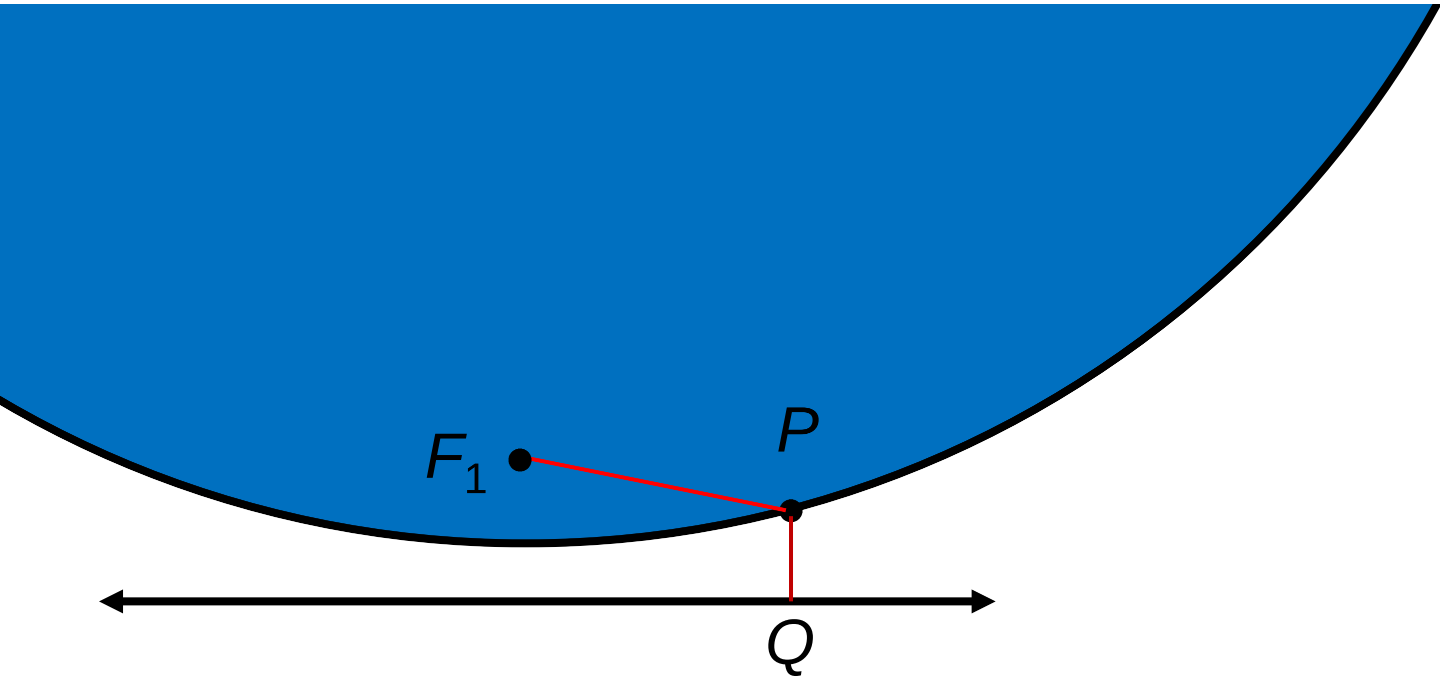
Eccentricity in the Sections

$PF_1/PQ =$
$\sin 80°/\sin 65°$



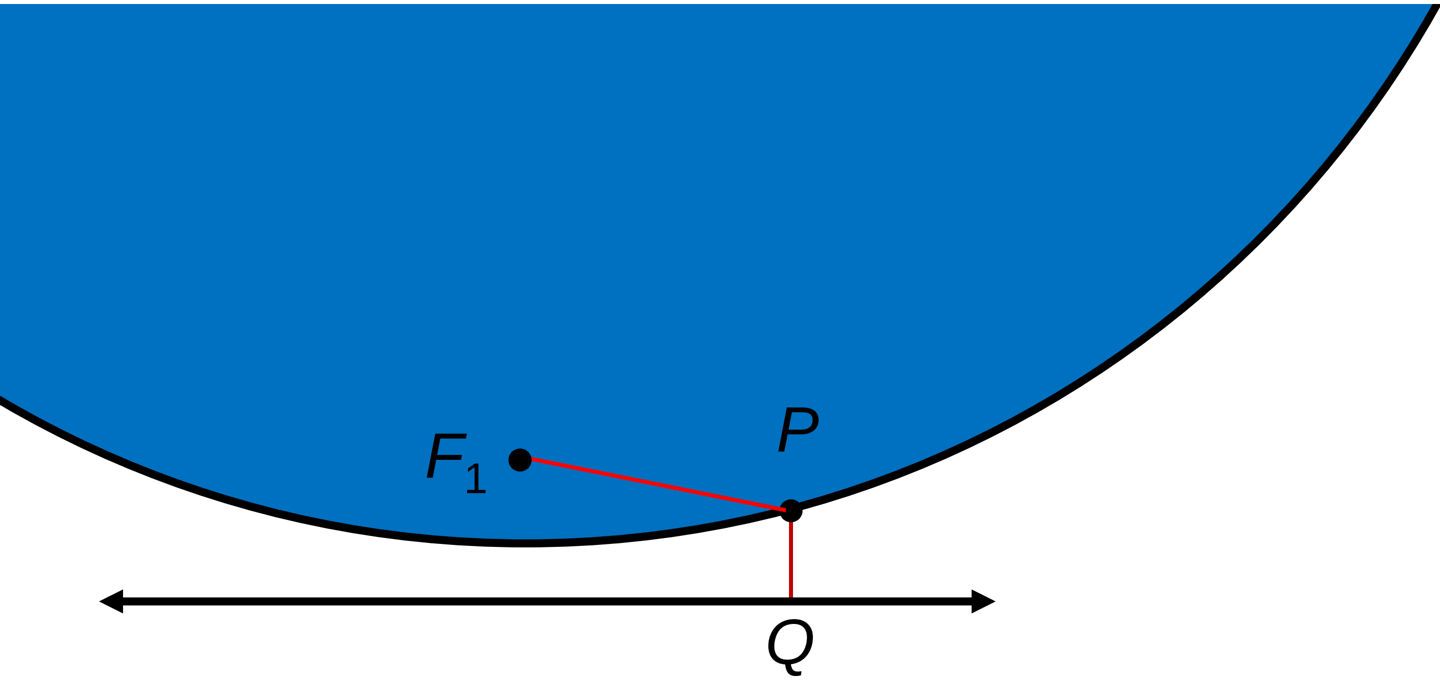
Eccentricity in the Sections

PF_1/PQ = constant greater than 1

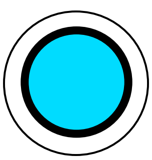


Geometry of the Steep Section

"Eccentricity" of the hyperbola exceeds 1.



Speed and Eccentricity



17,600 mph

Speed and Eccentricity

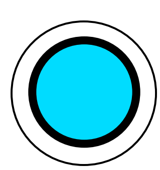
circle

eccentricity

$= (v/v_0)^2 - 1$

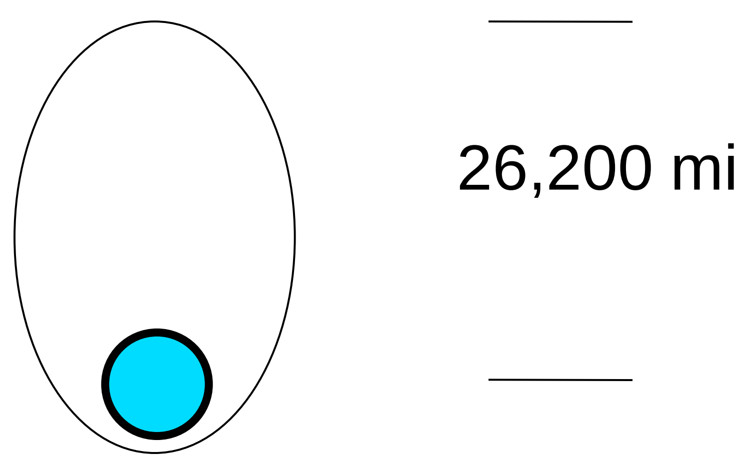
$= 1^2 - 1$

$= 0$



17,600 mph

Speed and Eccentricity



Add 32%
23,200 mph

Speed and Eccentricity

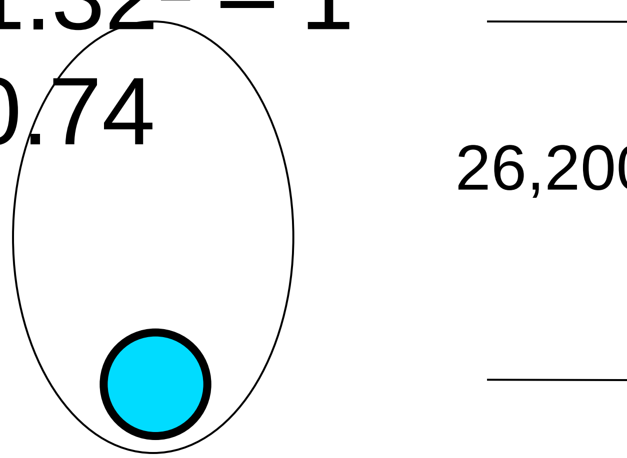
ellipse

eccentricity

$= (v/v_0)^2 - 1$

$= 1.32^2 - 1$

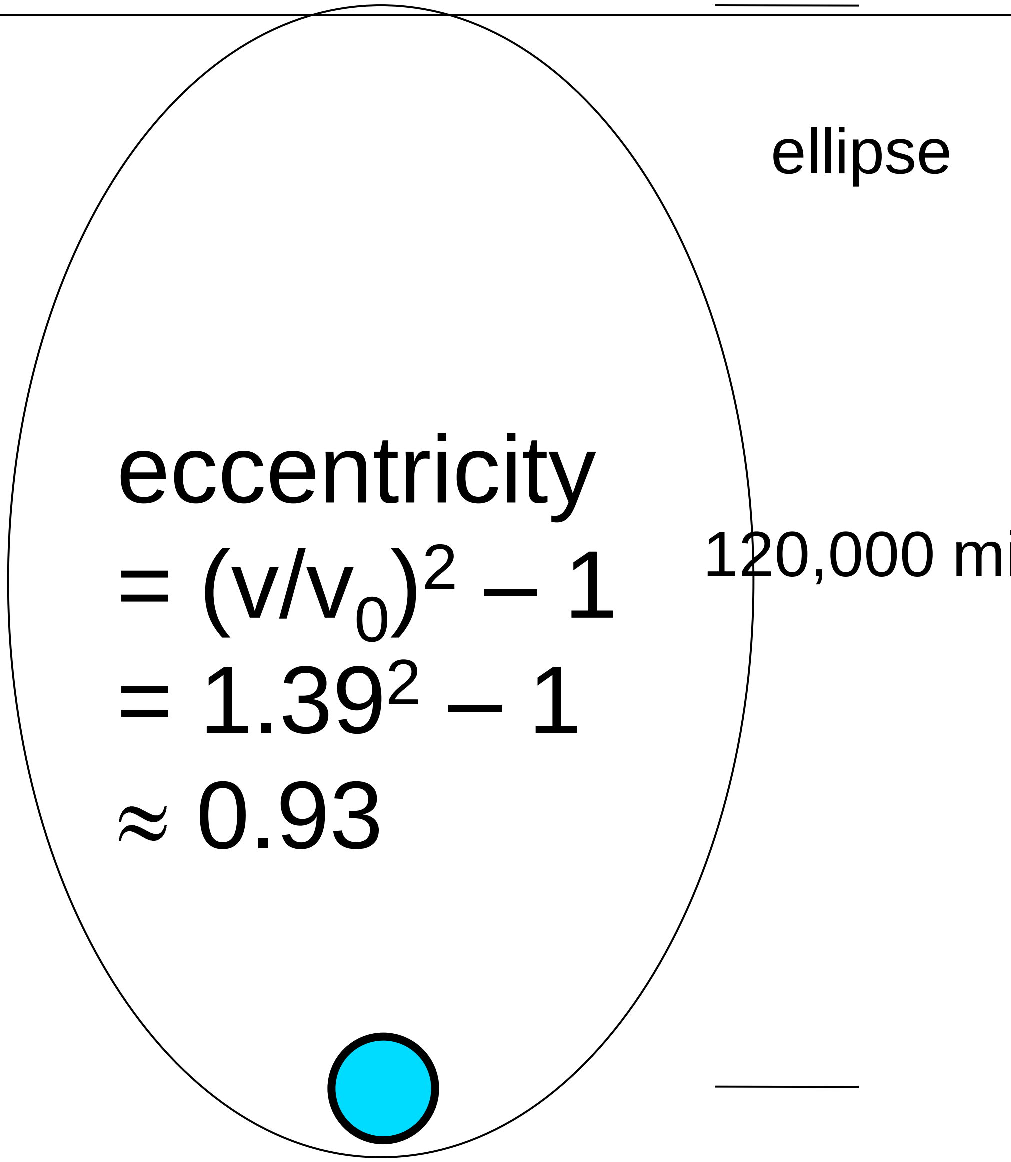
≈ 0.74



Add 32%

23,200 mph

Speed and Eccentricity

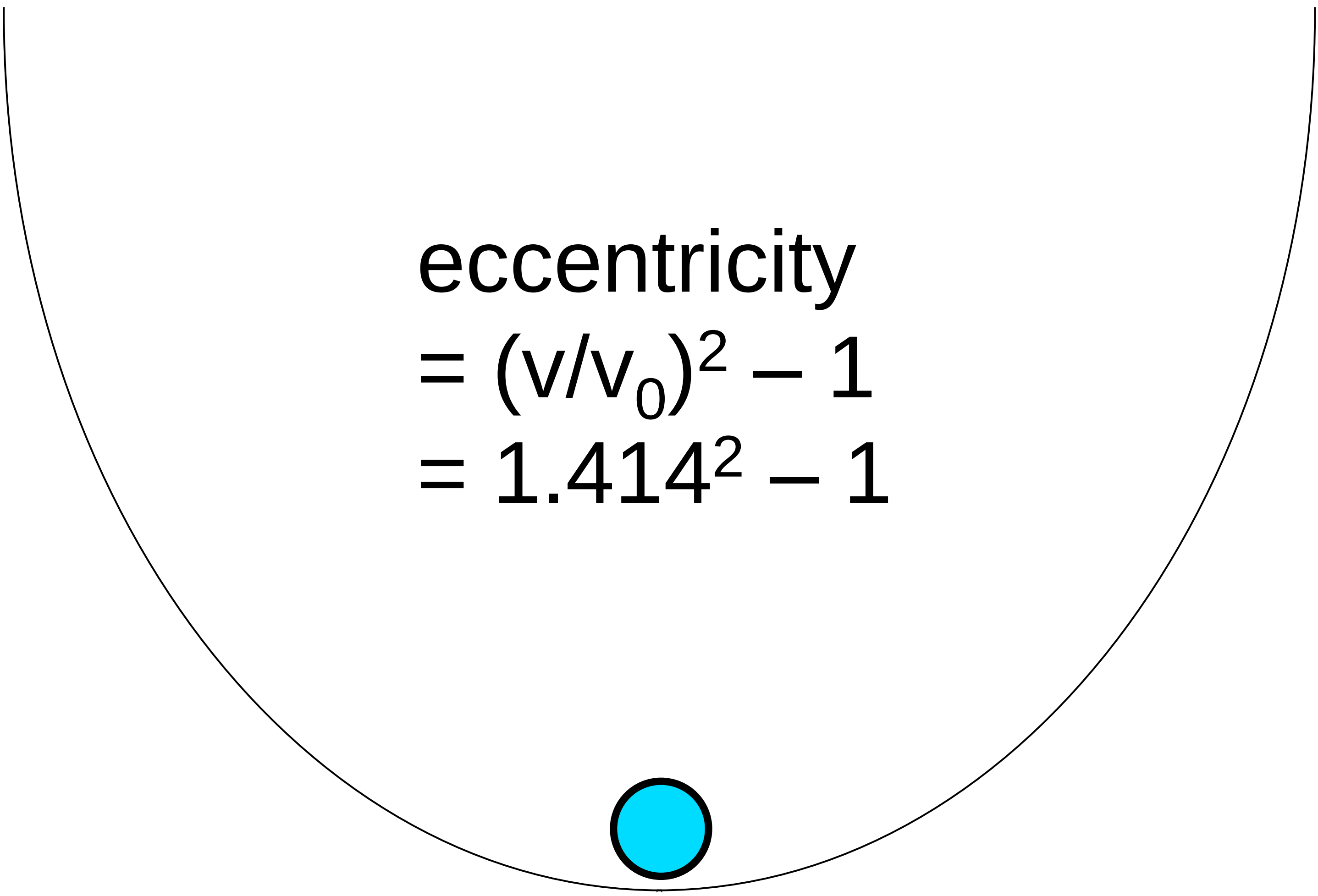


eccentricity

$$= (v/v_0)^2 - 1$$
$$= 1.39^2 - 1$$
$$\approx 0.93$$

Add 39%
24,500 mph

Speed and Eccentricity



eccentricity

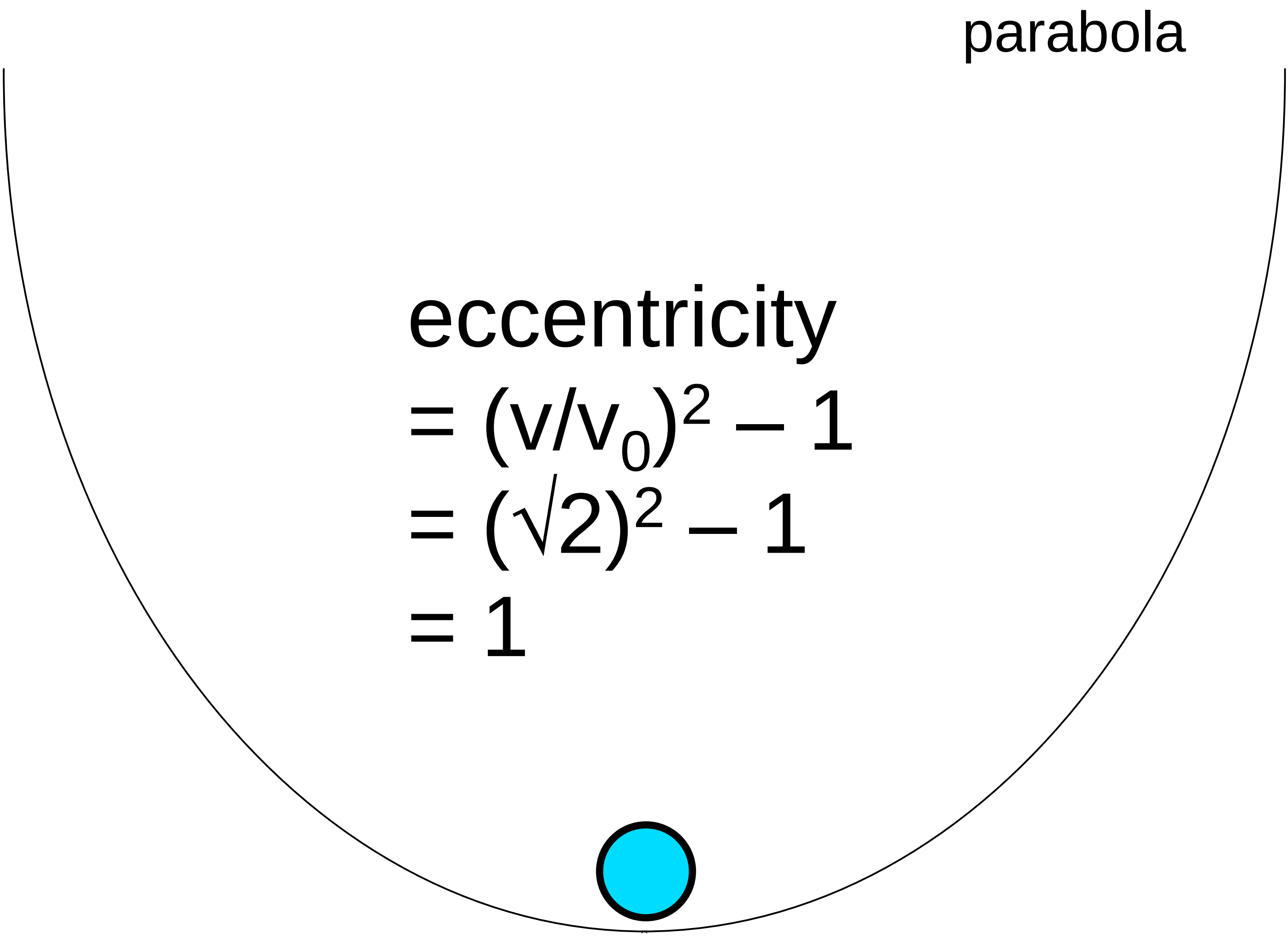
$$= (v/v_0)^2 - 1$$

$$= 1.414^2 - 1$$

Add 41.4%

24,900 mph

Speed and Eccentricity



eccentricity

$$= (v/v_0)^2 - 1$$

$$= (\sqrt{2})^2 - 1$$

$$= 1$$

Add 41.4%

24,900 mph

Speed and Eccentricity

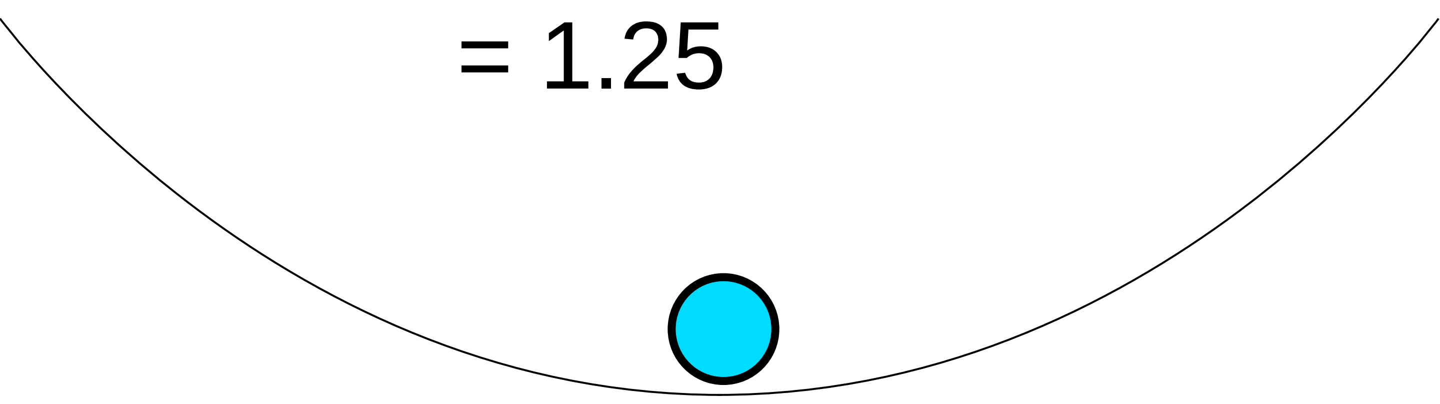
hyperbola

eccentricity

$= (v/v_0)^2 - 1$

$= 1.5^2 - 1$

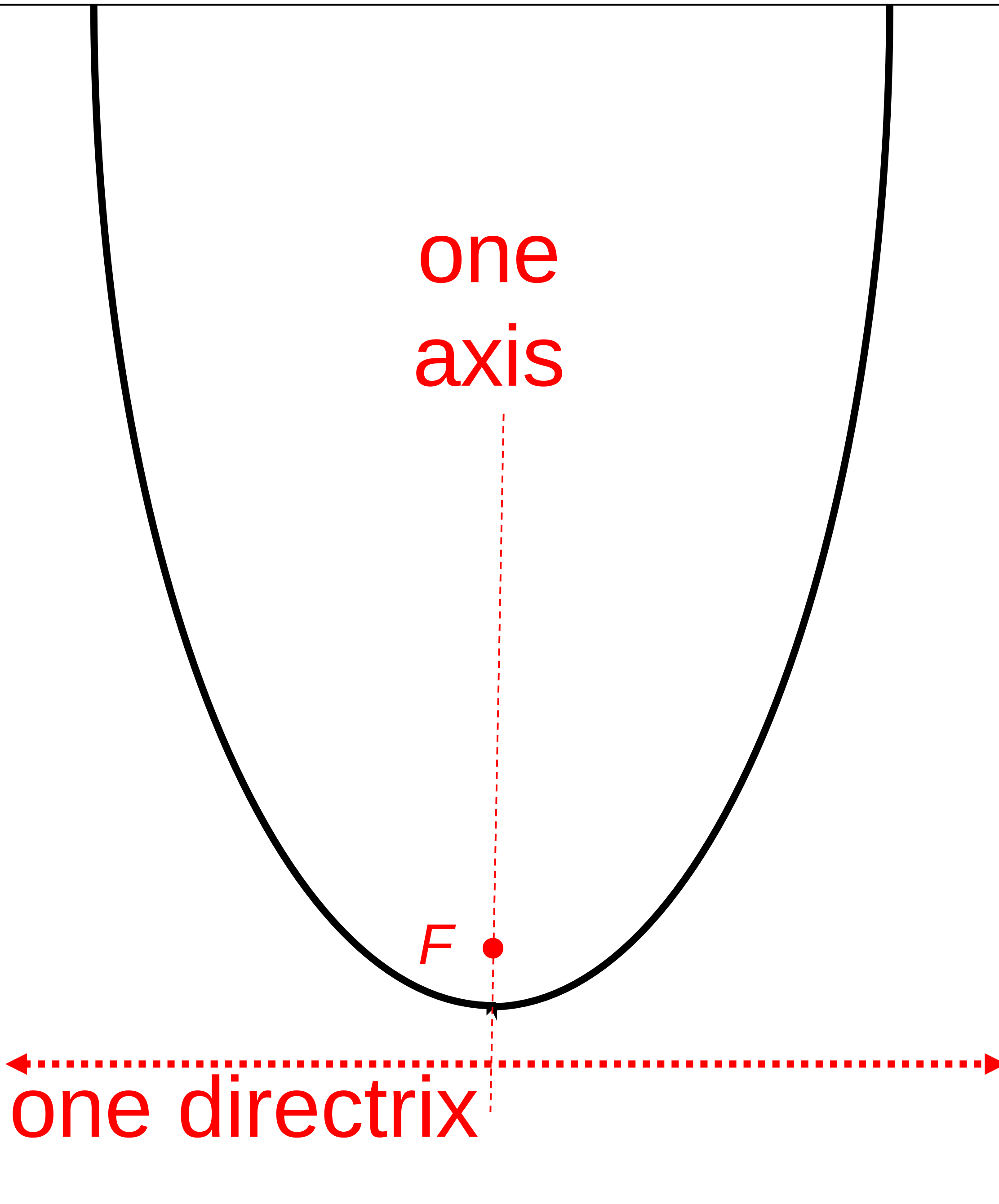
$= 1.25$



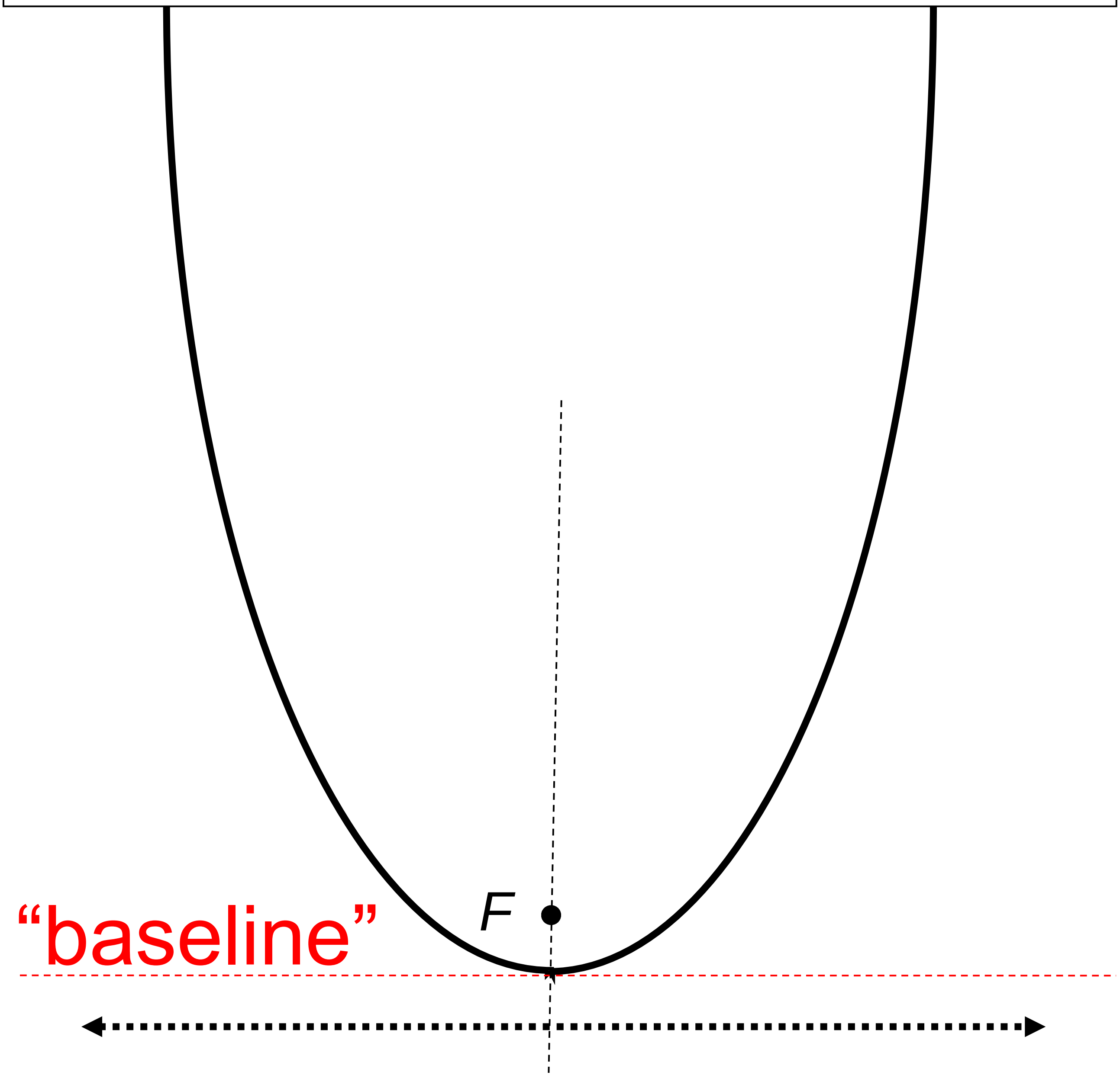
Add 50%

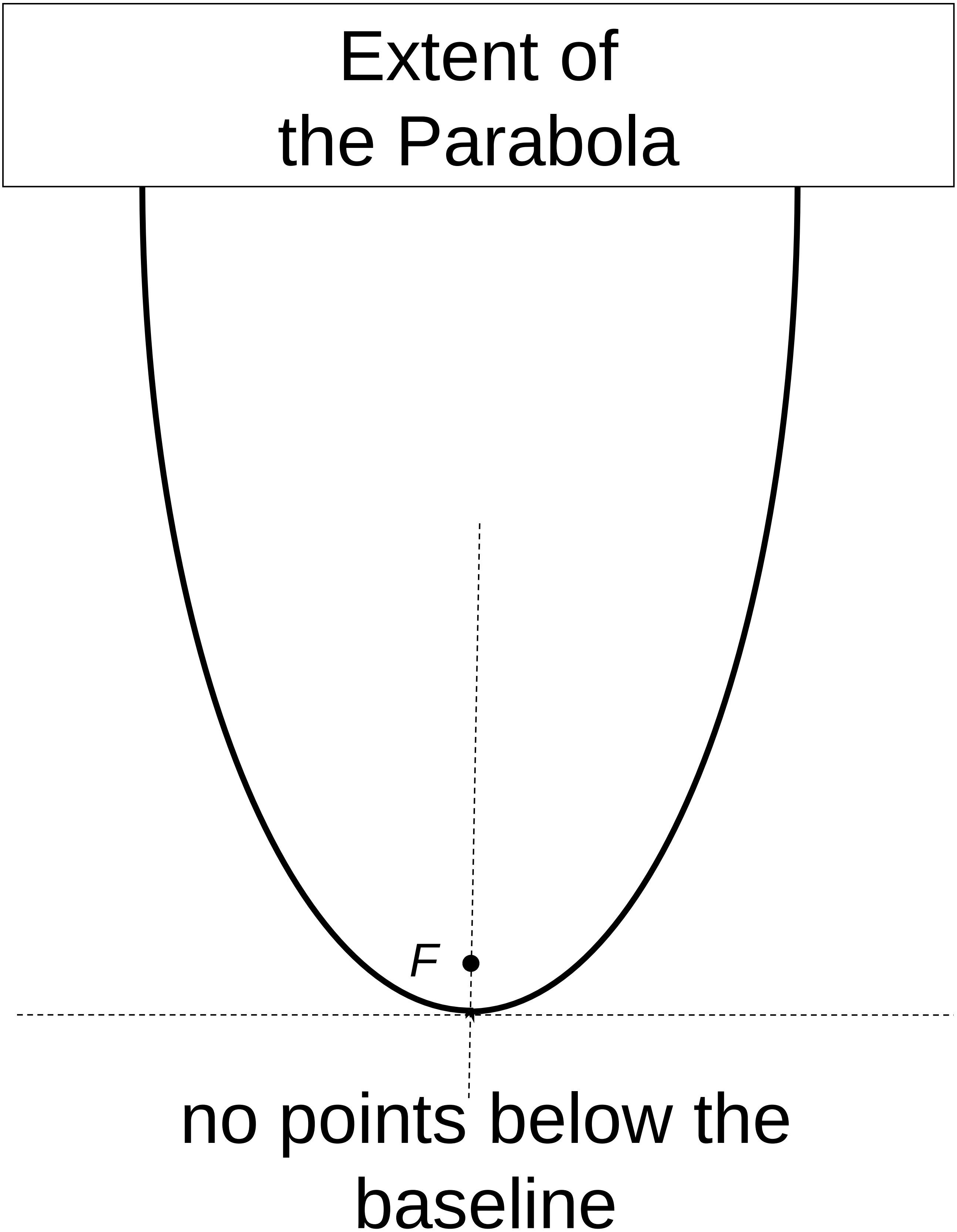
26,400 mph

Elements of the Parabola

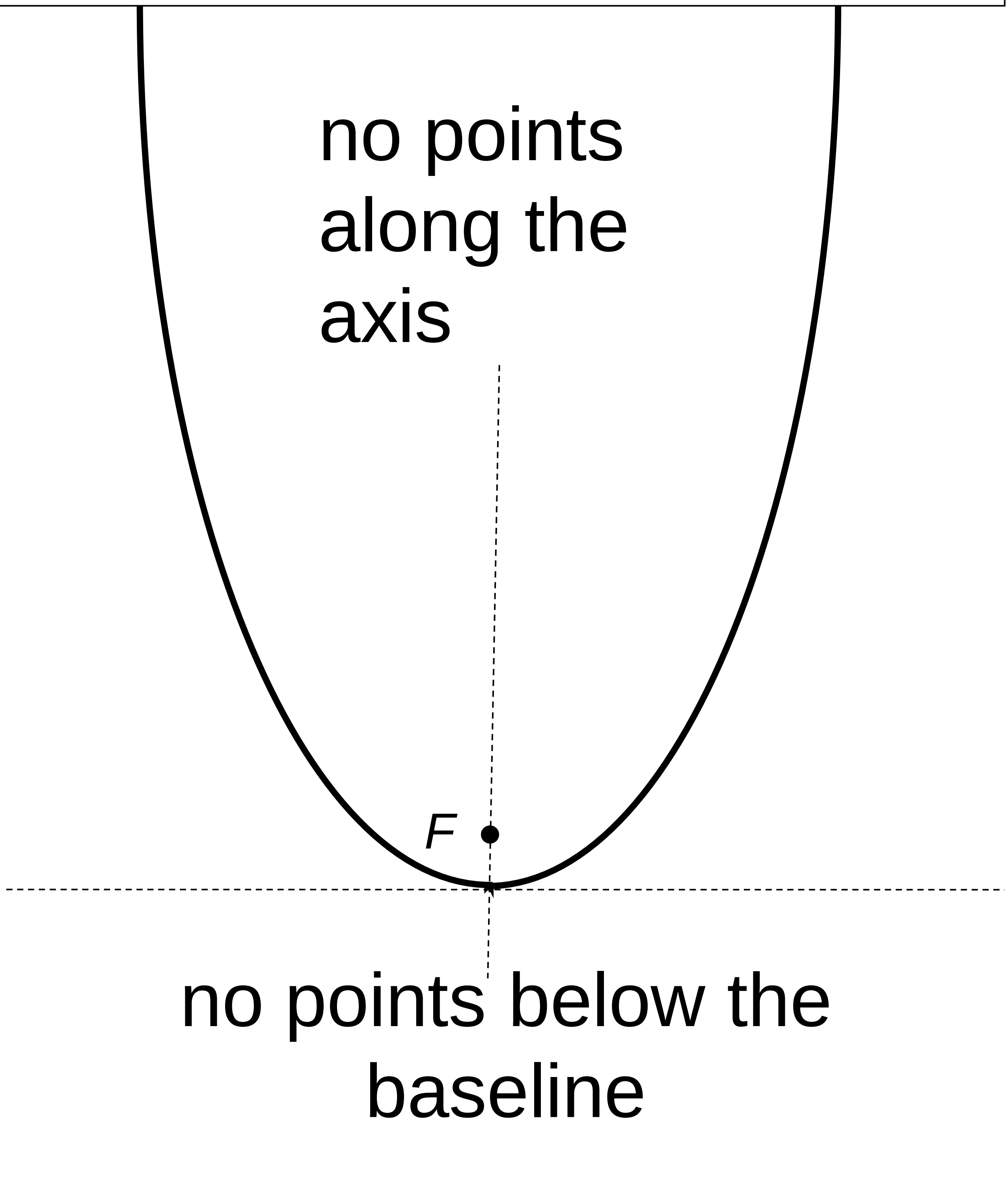


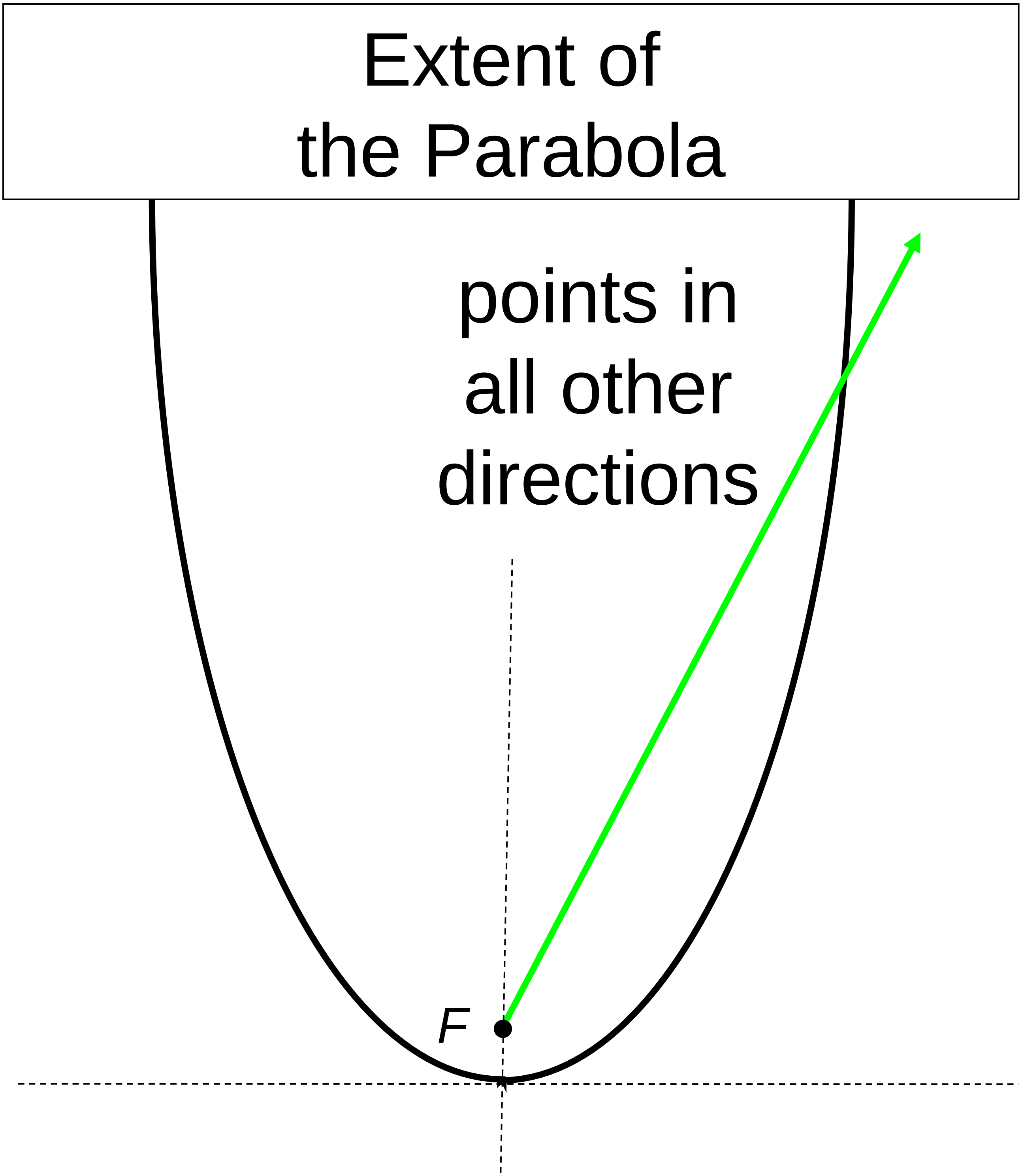
Elements of the Parabola



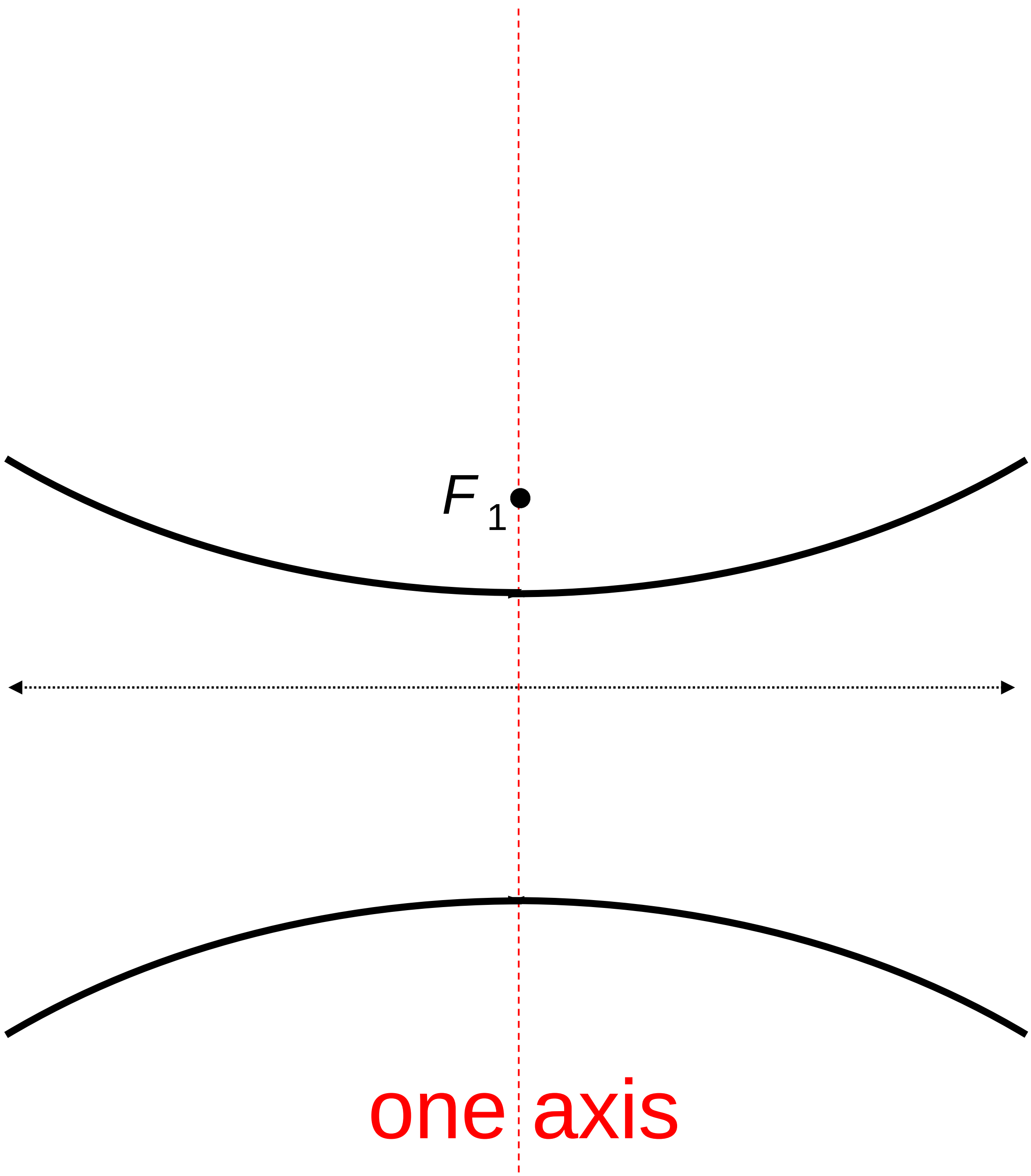

Extent of
the Parabola
F
no points below the
baseline

Elements of the Parabola



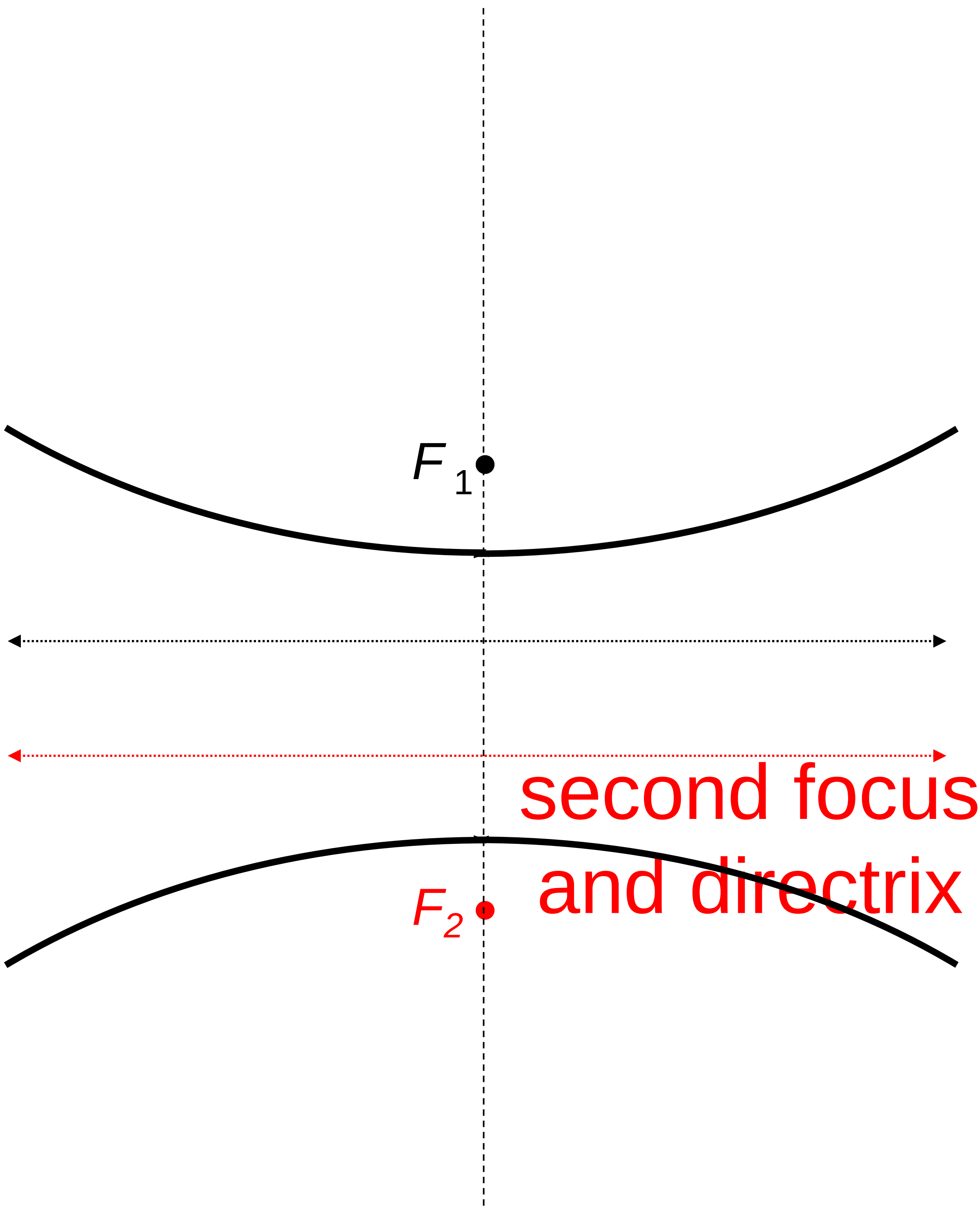

Extent of
the Parabola
points in
all other
directions
F

Elements of the Hyperbola

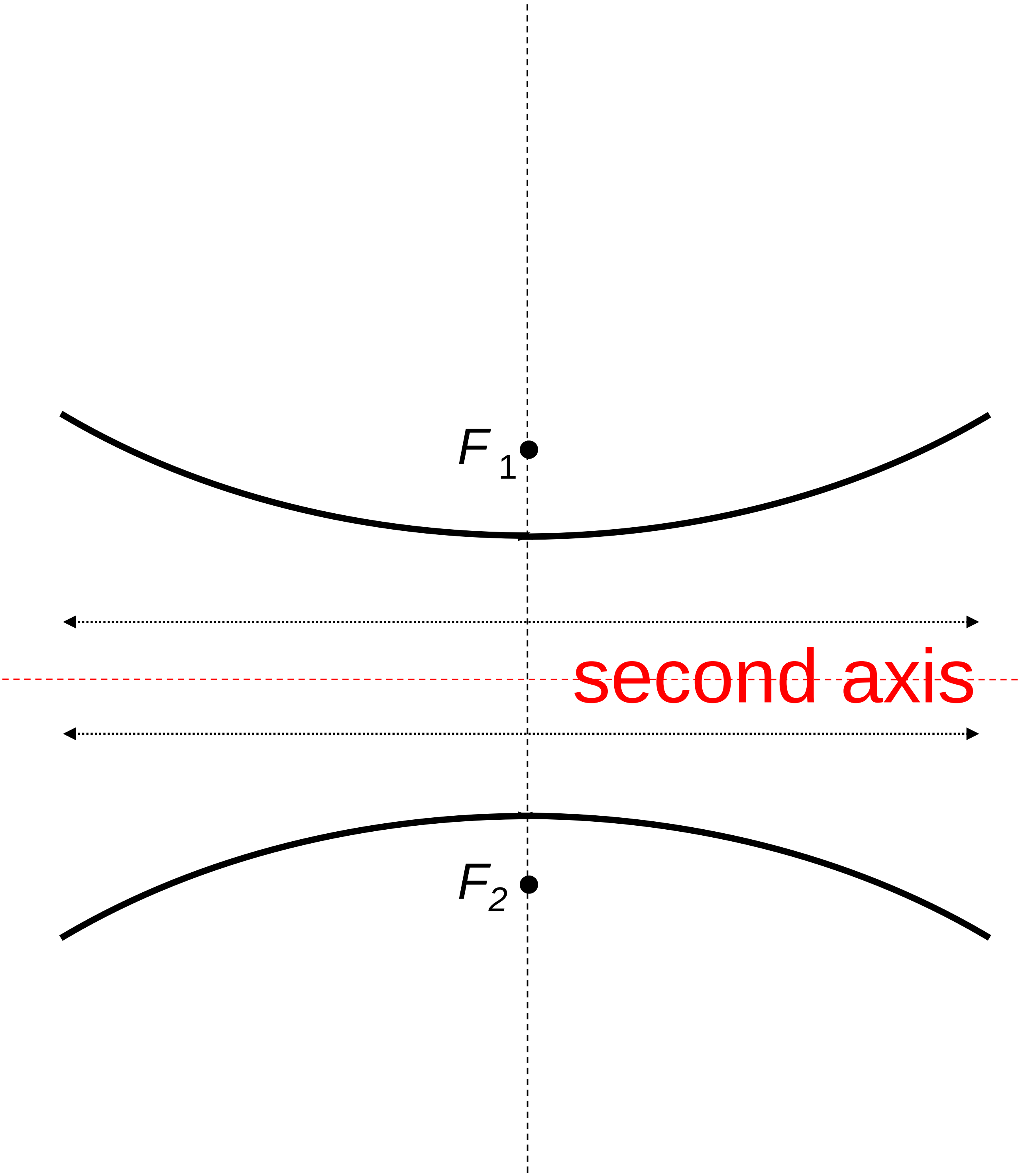


one axis

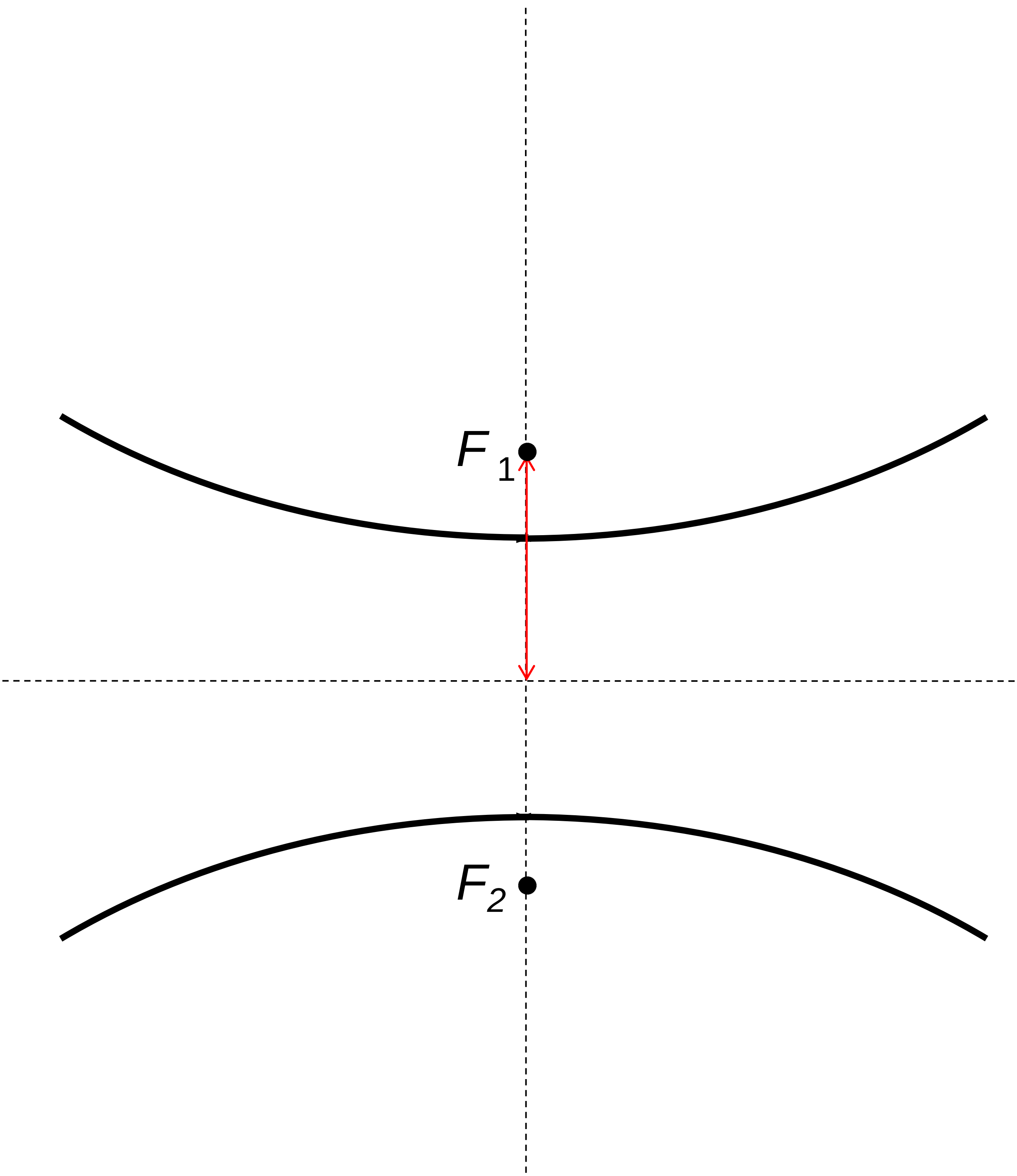
Elements of the Hyperbola



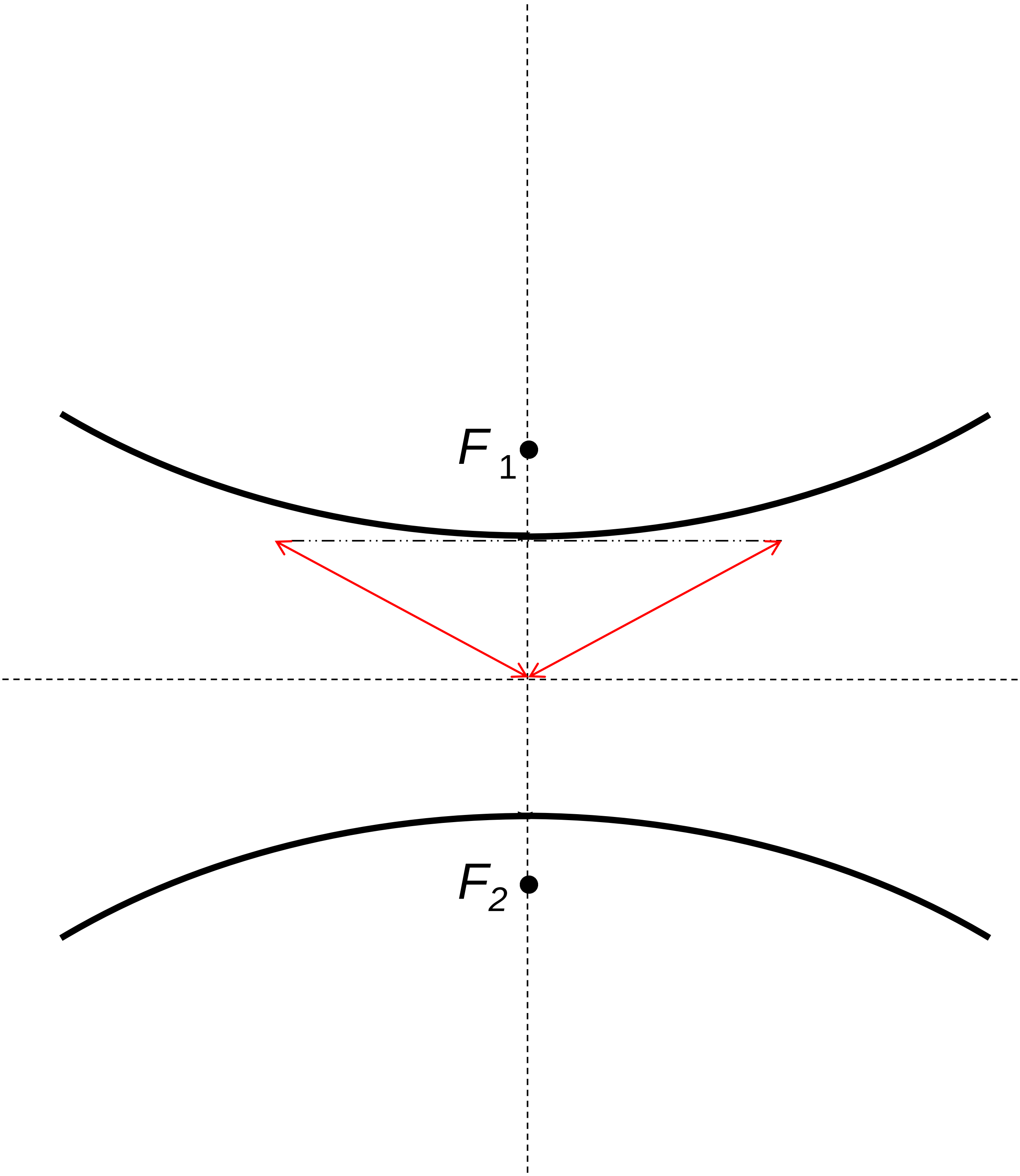
Elements of the Hyperbola



Elements of the Hyperbola

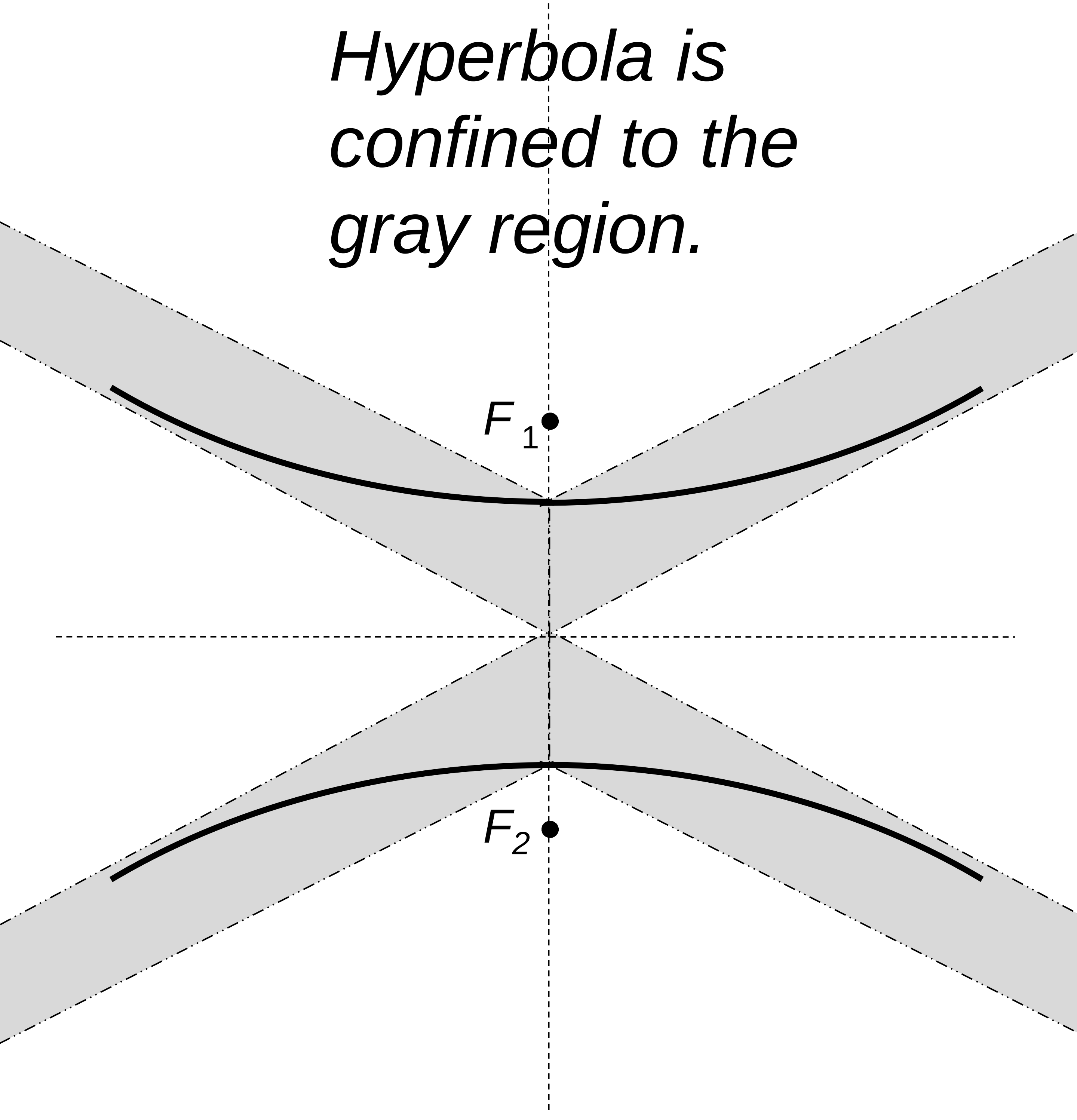


Elements of the Hyperbola

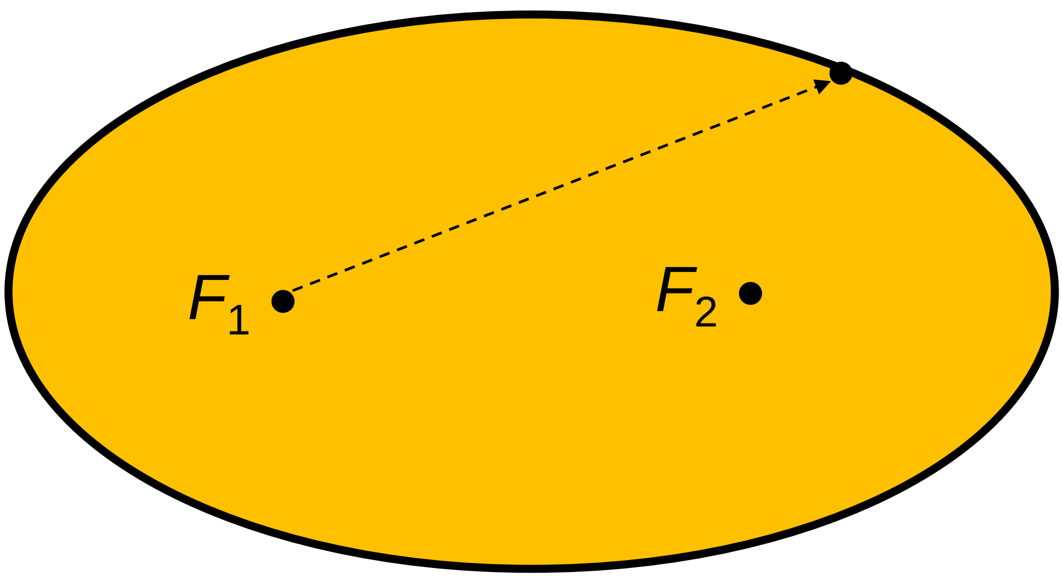


Extent of the Hyperbola

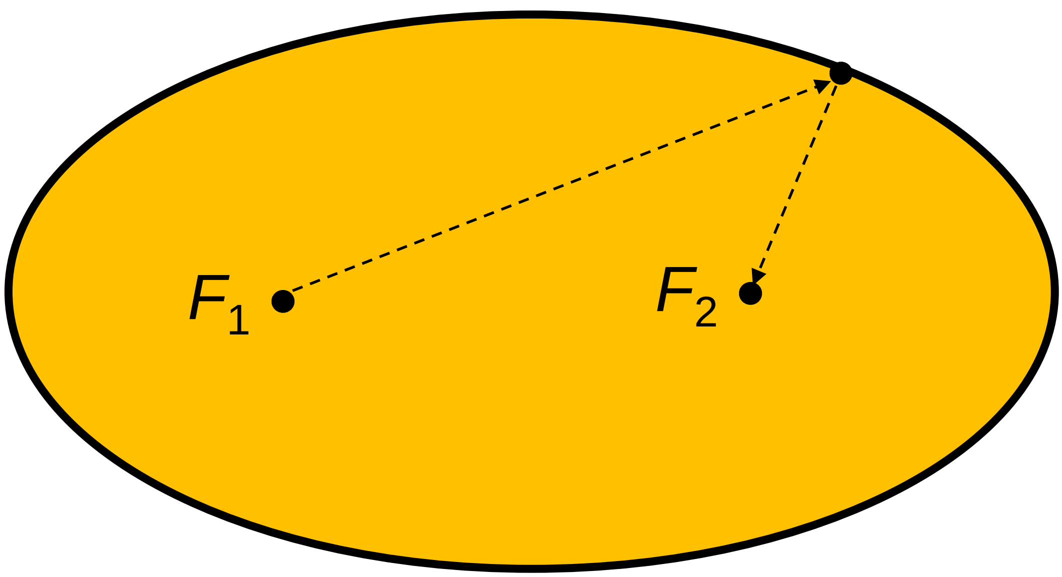
Hyperbola is confined to the gray region.



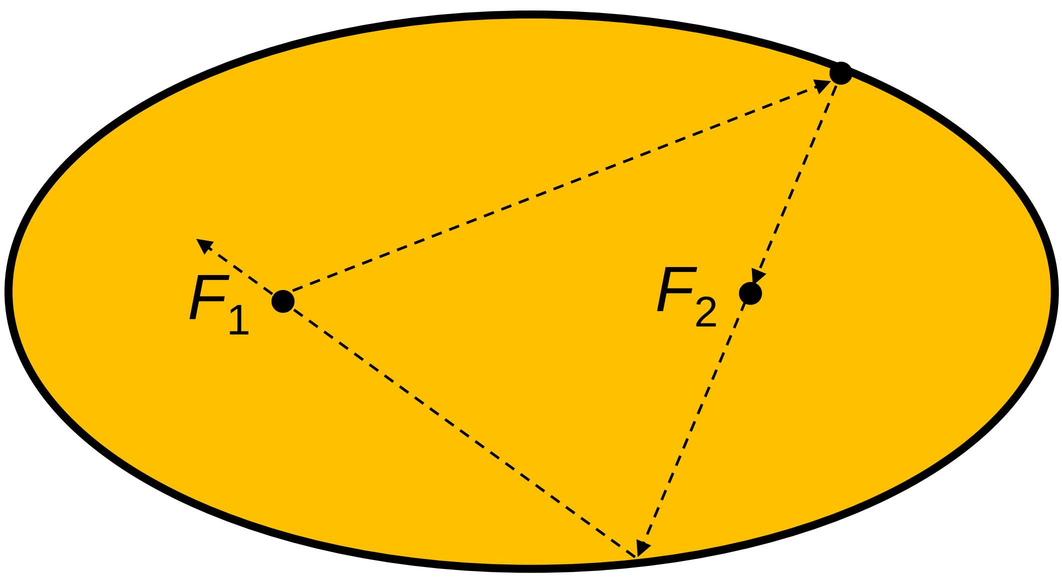
Reflection Properties: the Ellipse



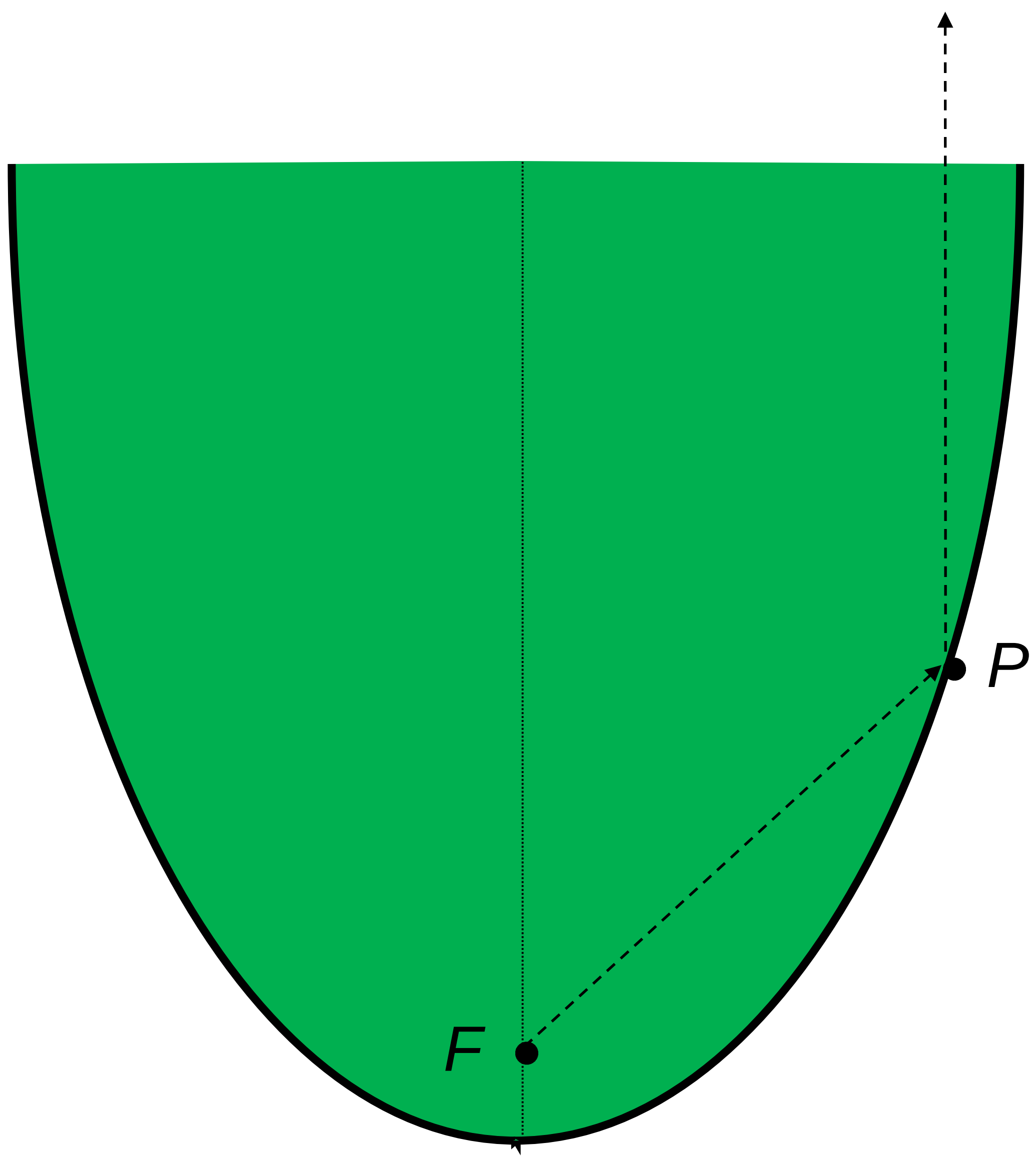
Reflection Properties: the Ellipse



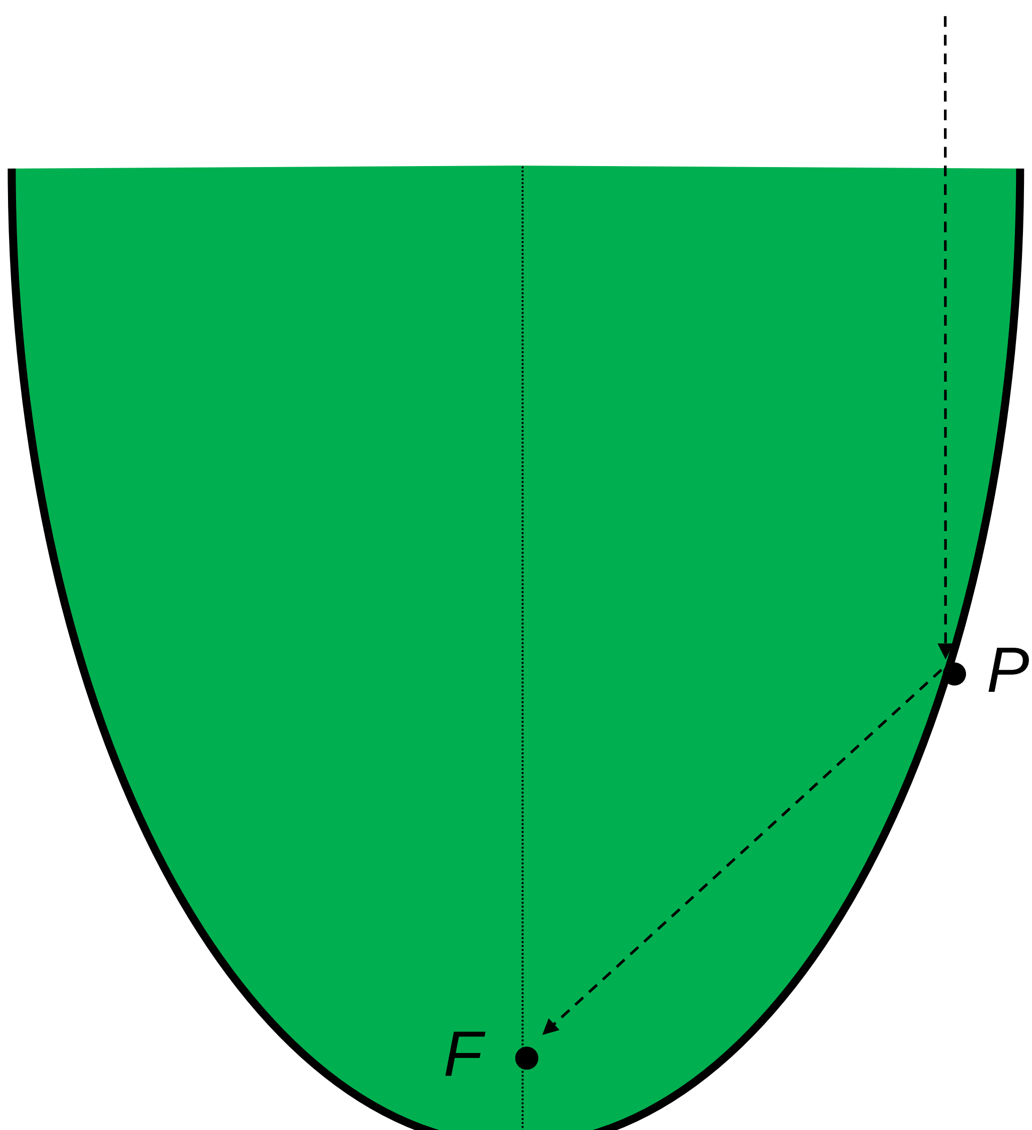
Reflection Properties: the Ellipse



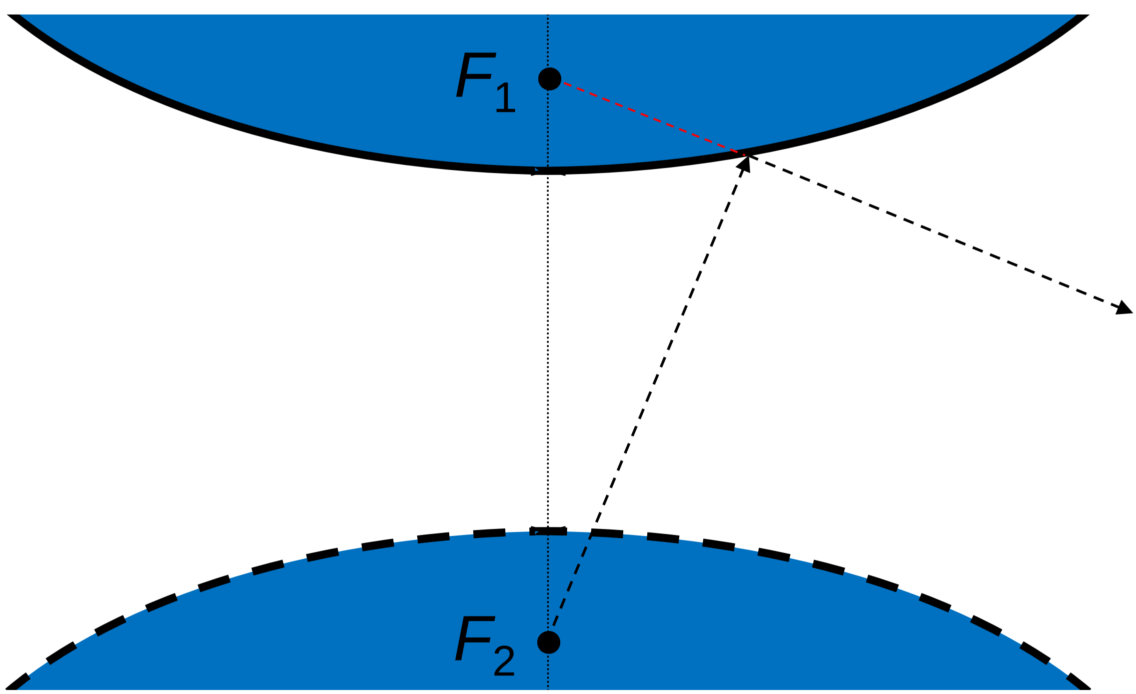
Reflection Properties: the Parabola



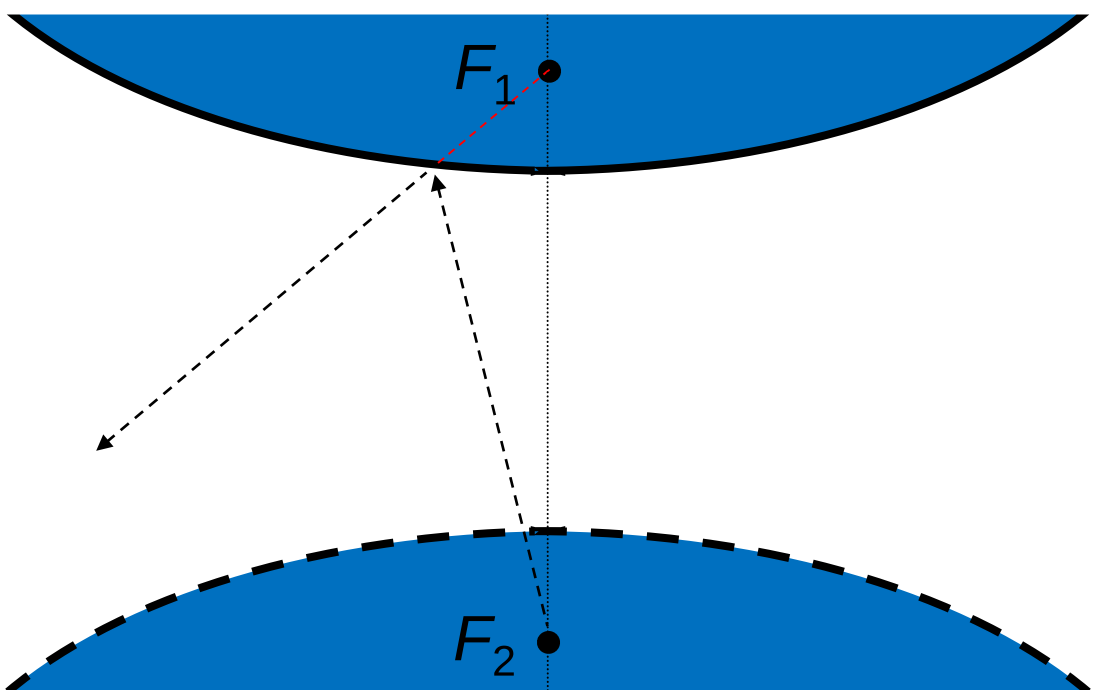
Reflection Properties: the Parabola



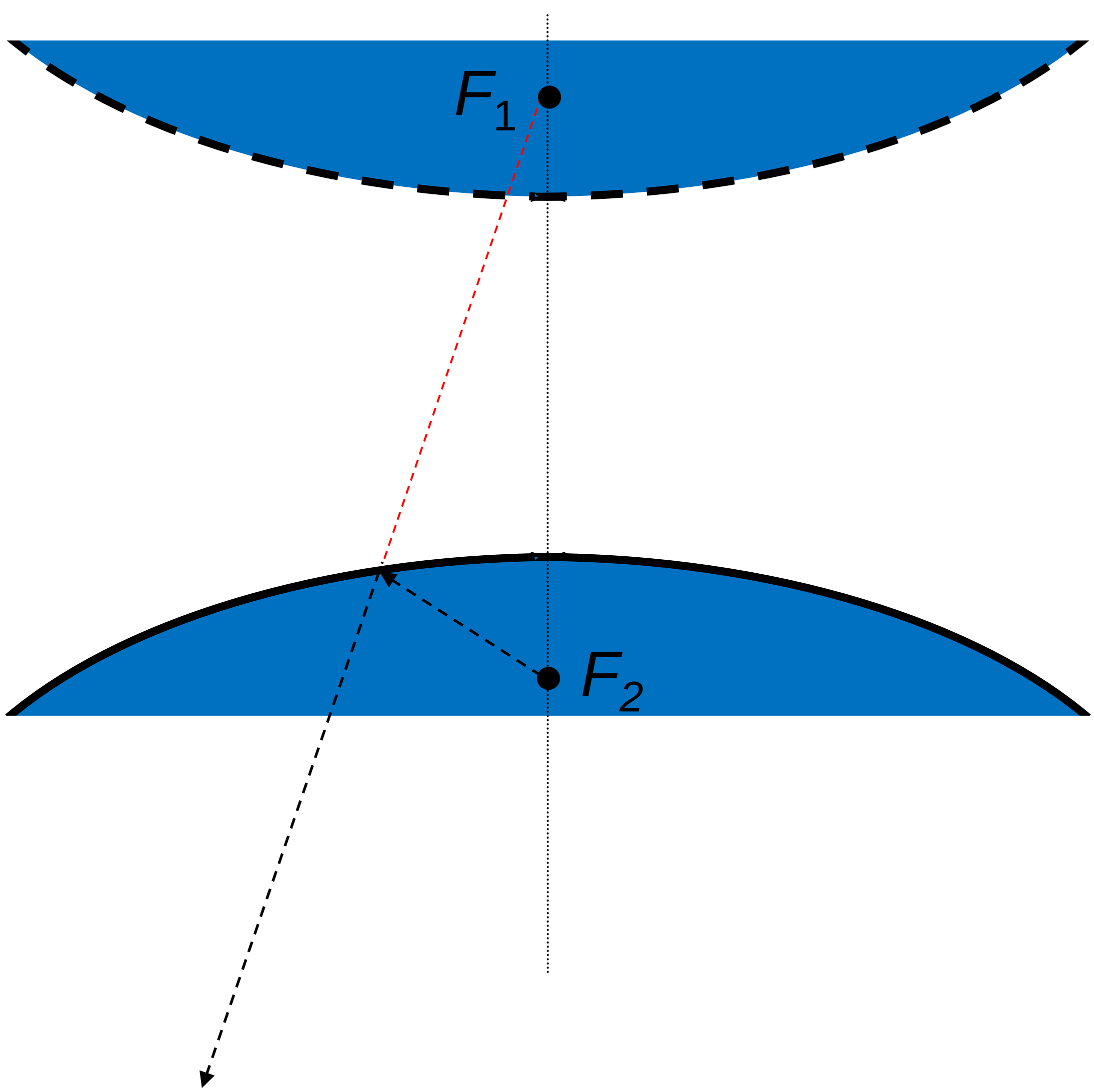
Reflection Properties: the Hyperbola



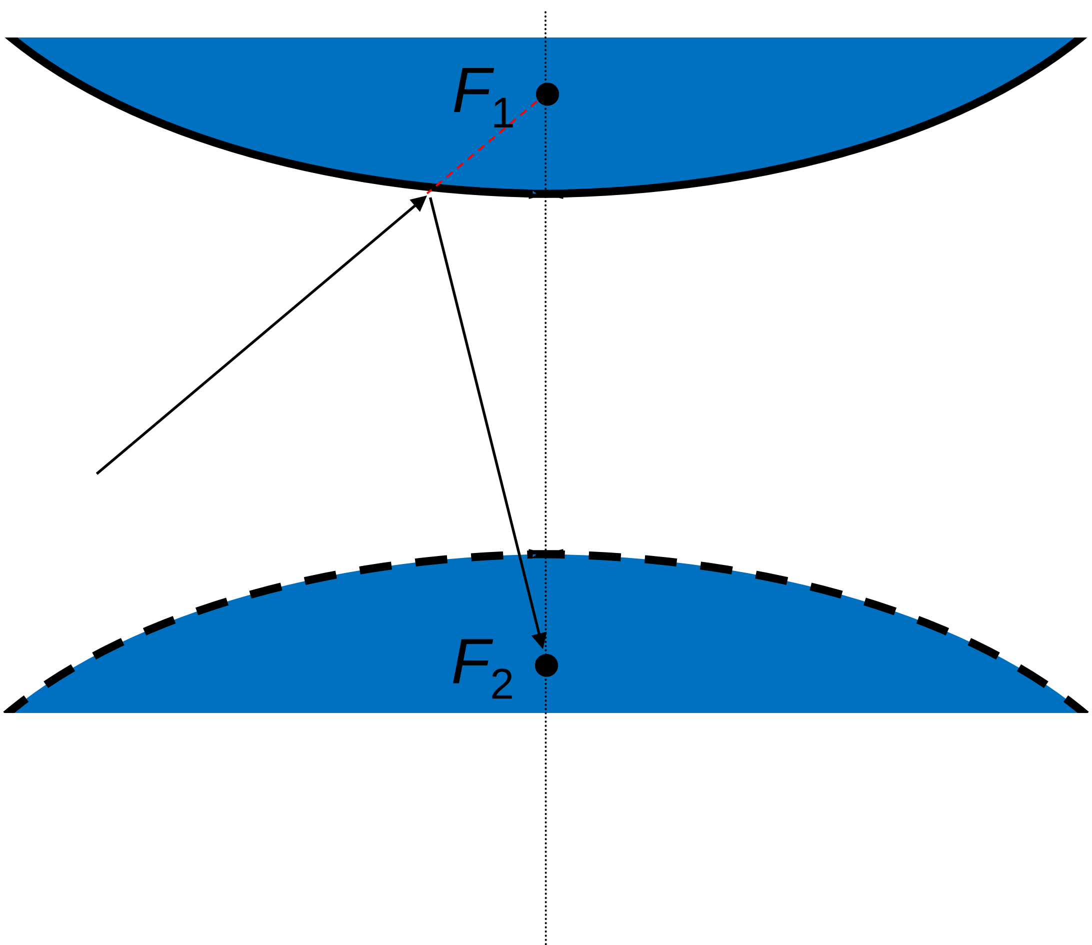
Reflection Properties: the Hyperbola



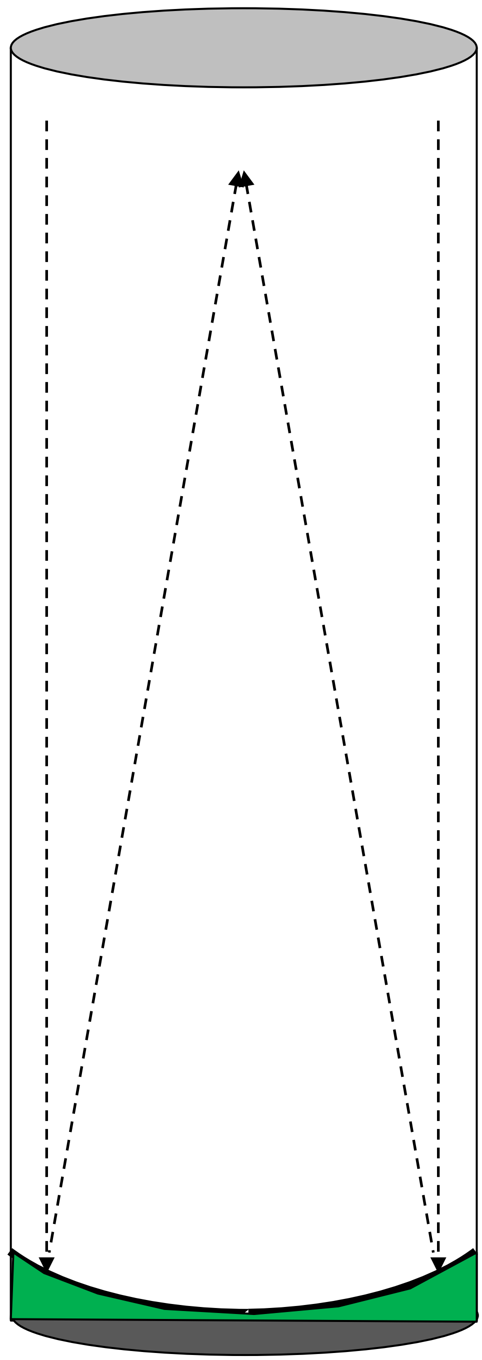
Reflection Properties: the Hyperbola



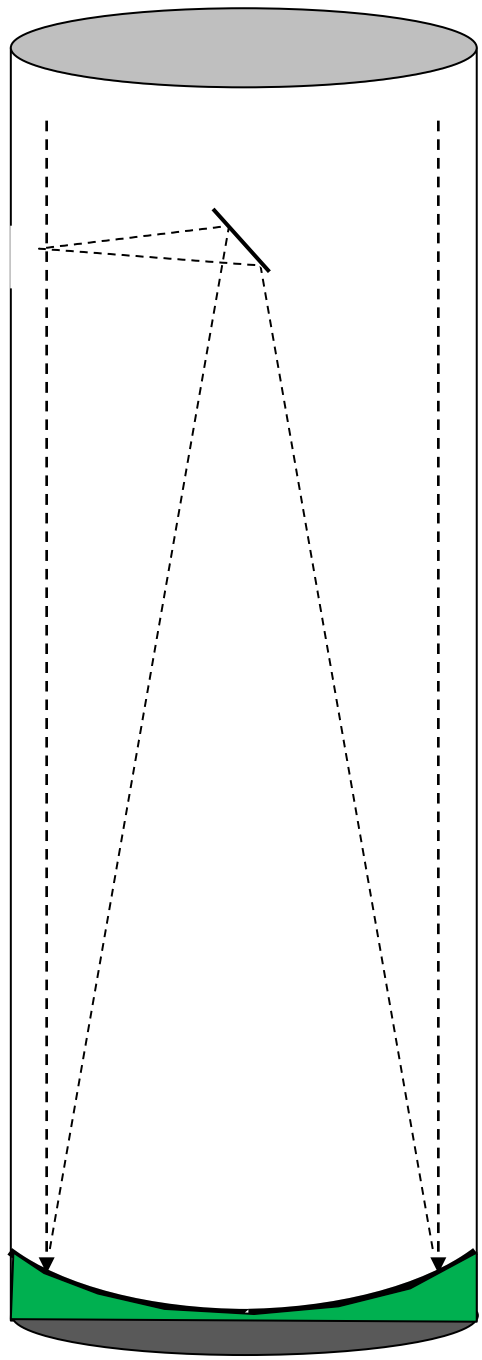
Reflection Properties: the Hyperbola



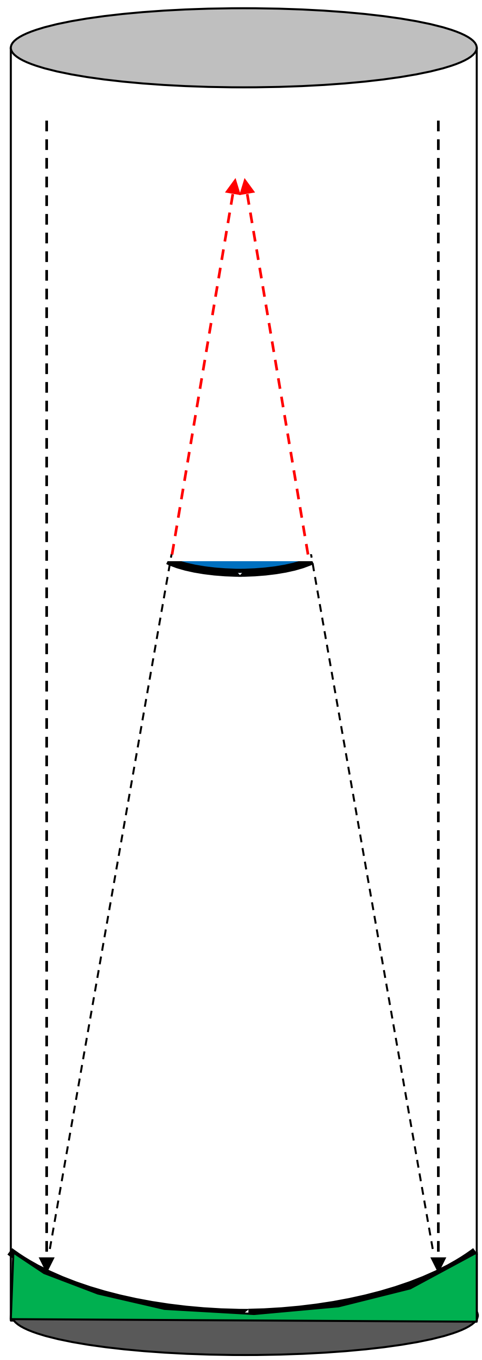
Telescopes and the Conics



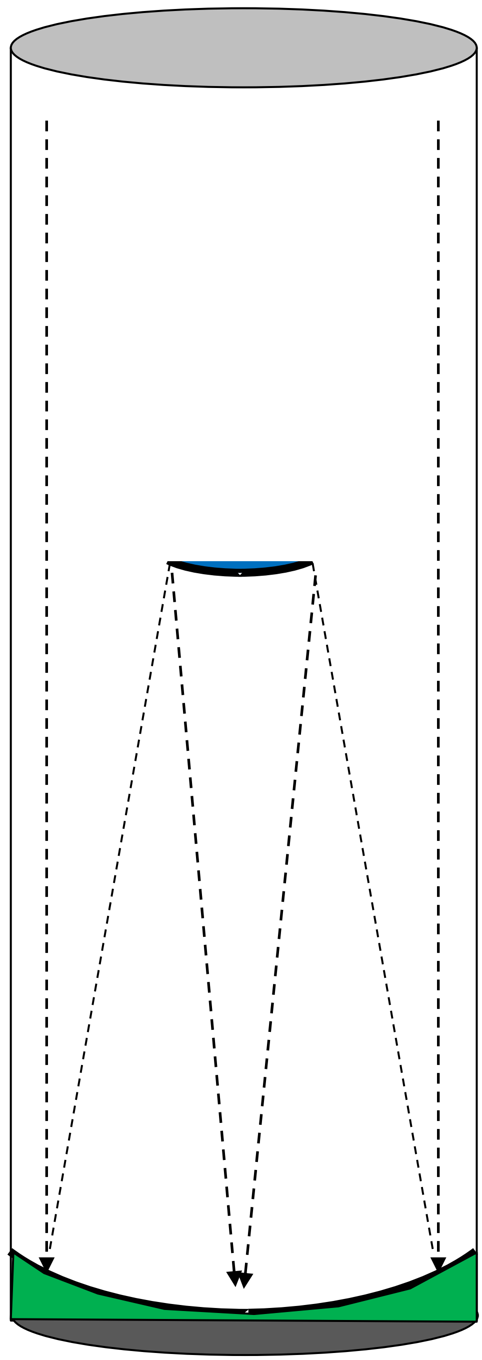
Telescopes and the Conics



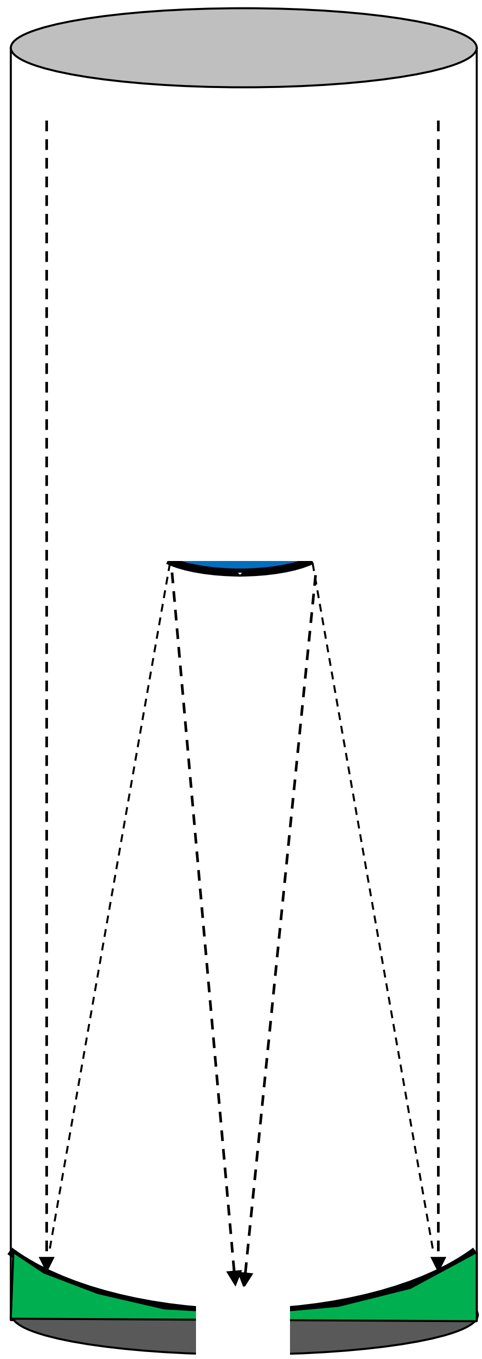
Telescopes and the Conics



Telescopes and the Conics



Telescopes and the Conics



Telescopes and the Conics