SUPPLEMENTARY NOTE 5b FOR MATH 212

Conic Sections and Quadric Surfaces

CONIC SECTIONS

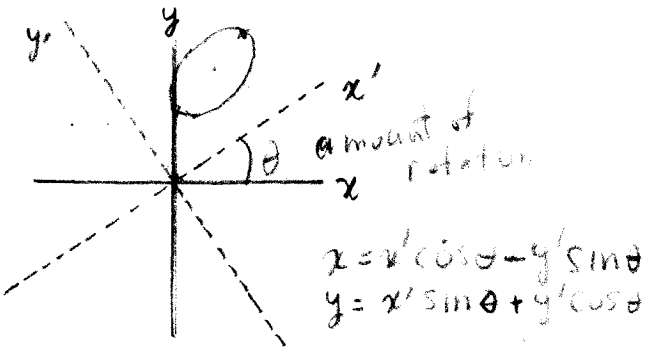
A *conic section* is the intersection of a right circular cone and a plane.

THE GENERAL SECOND DEGREE EQUATION

The degree of a term of a polynomial in more than one variable is the sum of the exponents of the variables in the term. For example, the term $2x^3yz^2$ has degree $3+1+2=6$. By the general second degree polynomial in two variables, we mean all polynomials in two variables whose terms have degree at most two and at least one term is of degree two. The general second degree equation is the equation that a general second degree polynomial equals zero, i.e., it is of the form

$$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0.$$

We ask the question, seemingly unrelated to conic sections, of what are all possible graphs of second degree polynomial equations. To answer this question we rewrite the equation several different ways.



It is possible to express in a rotated coordinate system, any graph of a second degree polynomial so that the coefficient C of the xy term is zero. Thus, we will only consider second degree equations with no xy term.