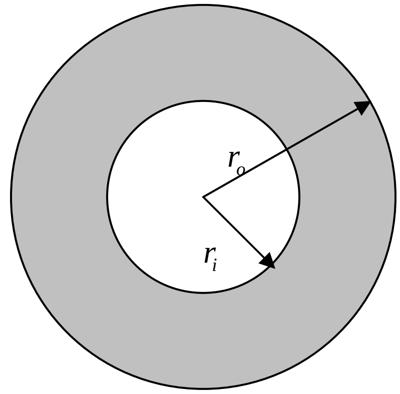
Chapter 7 section 2: Volume by Surface of Revolution (Disc method)



1) Establish the rotation axis and the interval we need to calculate (intersection points of the 2 functions).

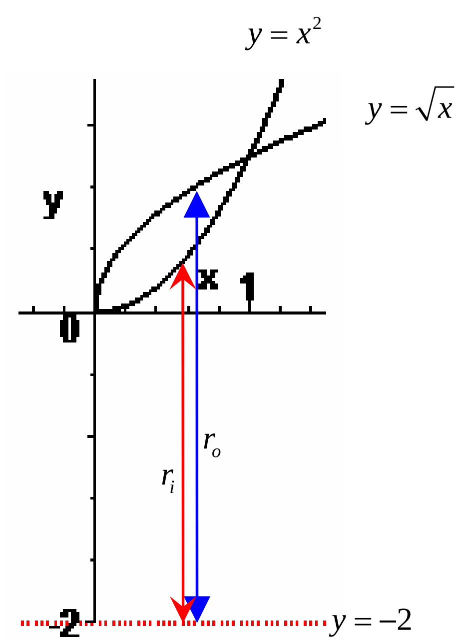
2) We need to get the area between 2 circles with same center (center is the rotation axis), see figure.
The area of circle is $A = \pi r^2$, therefore we need to figure the radii of the circles (The radius is from the rotation axis to the function). This will give us 2 radii, call them outer (r_o) and inner (r_i) radius. Thus we have $A_o = \pi r_o^2$ and $A_i = \pi r_i^2$.
How to find radius:
The height of the function is measured from *x*- or *y*-axis to the function, but the radius (either r_o or r_i) is from the rotation axis to the function. You may need to add or subtract to obtain your radius.

3) The area of the shaded region is $A = A_o - A_i$. This is the function that we will integrate (Draw the cross-section diagram). Remember that *A* is either a function of *x* or *y*.

4) A thin volume ΔV is needed and this can be established by multiplying *A*, obtained in step 3, by Δx or Δy, resulting $\Delta V = A \cdot \Delta x$ or $\Delta V = A \cdot \Delta y$.

5) The final volume is calculated by $V = \lim_{n\to\infty} \Delta V_j = \lim_{n\to\infty} A_j \cdot \Delta x_j$. Since we are calculating continuous function, we will calculate: $V = \int_a^b A\,dx$ or $V = \int_a^b A\,dy$.

Example2.1: Find the area bounded by $y = \sqrt{x}$, $y = x^2$, located in 1st quadrant rotated about $y = -2$.



Step 1: see figure to the left

$$x^2 = \sqrt{x}$$
$$x^2 - \sqrt{x} = 0 \quad \Rightarrow \quad \sqrt{x} = 0 \qquad x^{3/2} - 1 = 0$$
$$\sqrt{x}\left(x^{3/2} - 1\right) = 0 \qquad x = 0 \qquad x = 1$$

$$r_o = 2 + \sqrt{x} \qquad r_i = 2 + x^2$$

Step 2: $A_o = \pi\left(2 + \sqrt{x}\right)^2 = \pi(4 + 4\sqrt{x} + x)$ $\qquad$ $A_i = \pi\left(2 + x^2\right)^2 = \pi(4 + 4x^2 + x^4)$

Step 3:
$$A = A_o - A_i = \pi(4 + 4\sqrt{x} + x) - \pi(4 + 4x^2 + x^4)$$
$$= \pi[4 + 4\sqrt{x} + x - 4 - 4x^2 - x^4]$$
$$= \pi(4\sqrt{x} + x - 4x^2 - x^4)$$

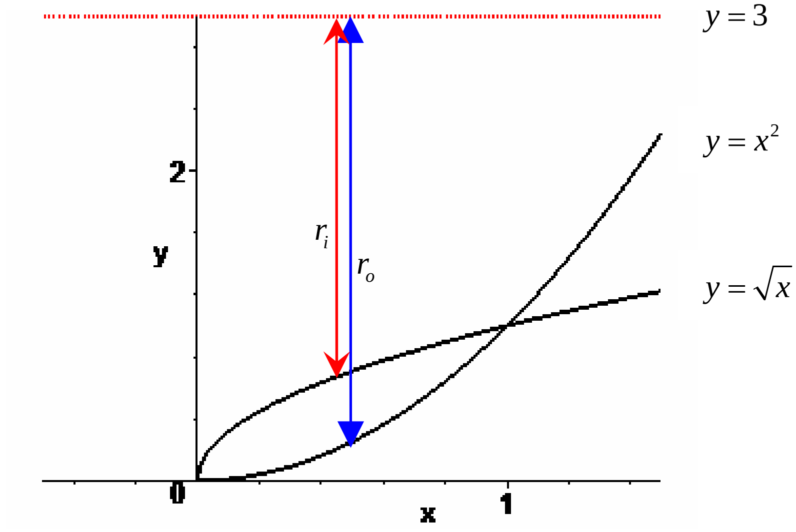
Step 4: $\Delta V = A \cdot \Delta x = \pi(4\sqrt{x} + x - 4x^2 - x^4)\Delta x$

$$V = \int_0^1 \pi(4\sqrt{x} + x - 4x^2 - x^4)\,dx = \pi\left[\frac{8}{3}x^{3/2} + \frac{1}{2}x^2 - \frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^1$$

Step 5: $= \pi\left\{\left[\frac{8}{3}(1)^{3/2} + \frac{1}{2}(1)^2 - \frac{4}{3}(1)^3 - \frac{1}{5}(1)^5\right] - [0]\right\} = \pi\left\{\frac{8}{3} + \frac{1}{2} - \frac{4}{3} - \frac{1}{5}\right\} = \pi\left\{\frac{4}{3} + \frac{1}{2} - \frac{1}{5}\right\}$

$$= \pi\left\{\frac{40}{30} + \frac{15}{30} - \frac{6}{30}\right\} = \pi\left\{\frac{49}{30}\right\} = \frac{49\pi}{30}$$

Example 2.2: Same restriction as Example 2.1, but rotated about $y = 3$.



Step 1: see figure to the left and take same endpoints as Example 2.1.

Step 2: Notice that in this case we need to subtract the height of the function from a constant value of 3 to get our needed radii (unlike the Example 2.1 where we needed to add 2 to the height of the function). Thus resulting with our radii to be:

$r_o = 3 - x^2$ $\qquad r_i = 3 - \sqrt{x}$

$A_o = \pi\left(3 - x^2\right)^2 = \pi(9 - 6x^2 + x^4)$

$A_i = \pi\left(3 - \sqrt{x}\right)^2 = \pi(9 - 6\sqrt{x} + x)$

$A = A_o - A_i = \pi(9 - 6x^2 + x^4) - \pi(9 - 6\sqrt{x} + x)$

Step 3: $= \pi[9 - 6x^2 + x^4 - 9 + 6\sqrt{x} - x]$

$= \pi(x^4 - 6x^2 - x + 6\sqrt{x})$

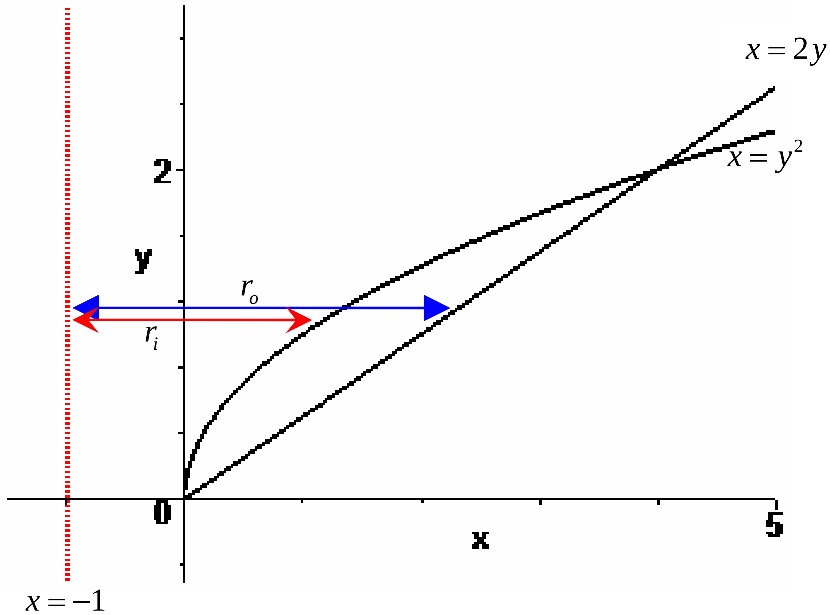
Step 4: $\Delta V = A \cdot \Delta x = \pi(x^4 - 6x^2 - x + 6\sqrt{x})\Delta x$

$$V = \int_0^1 \pi(x^4 - 6x^2 - x + 6\sqrt{x})\,dx = \pi\left[\frac{1}{5}x^5 - 2x^3 - \frac{1}{2}x^2 + 4x^{3/2}\right]_0^1$$

Step 5: $= \pi\left\{\left[\frac{1}{5}(1)^5 - 2(1)^3 - \frac{1}{2}(1)^2 + 4(1)^{3/2}\right] - [0]\right\} = \pi\left\{\frac{1}{5} - 2 - \frac{1}{2} + 4\right\} = \pi\left\{\frac{1}{5} - \frac{1}{2} + 2\right\}$

$$= \pi\left\{\frac{2}{10} - \frac{5}{10} + \frac{20}{10}\right\} = \pi\left\{\frac{17}{10}\right\} = \frac{17\pi}{10}$$

Example 2.3: Calculate the volume generated by rotating 1st quadrant of $x = y^2$ and $x = 2y$, rotated about $x = -1$.



Step 1: see figure to the left

$$y^2 = 2y$$
$$y^2 - 2y = 0$$
$$y(y-2) = 0$$
$$y = 0 \quad y = 2$$

Step 2: Notice that unlike the previous 2 examples, the radii is measured horizontally not vertically because we are generating volume by rotating about vertical axis.

$r_o = 1 + 2y$ $\qquad$ $r_i = 1 + y^2$

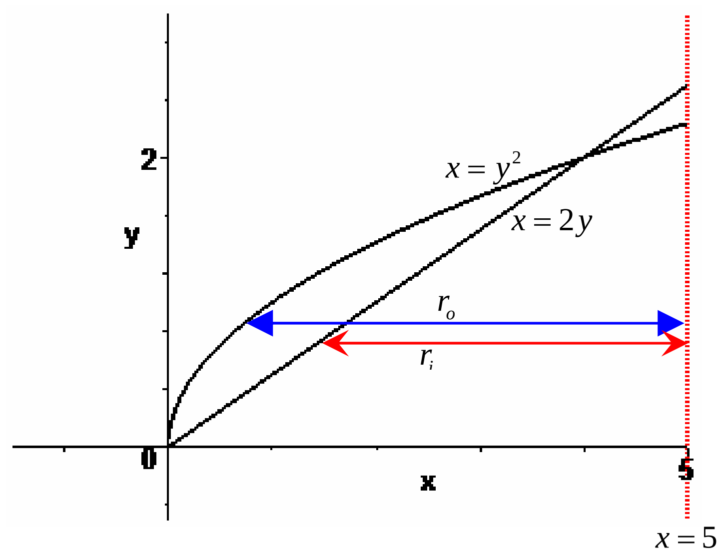
$A_o = \pi(1+2y)^2 = \pi(1+4y+4y^2)$ $\qquad$ $A_i = \pi(1+y^2)^2 = \pi(1+2y^2+y^4)$

Step 3: $A = A_o - A_i = \pi(1+4y+4y^2) - \pi(1+2y^2+y^4) = \pi[1+4y+4y^2-1-2y^2-y^4] = \pi(4y+2y^2-y^4)$

Step 4: $\Delta V = A \cdot \Delta y = \pi(4y+2y^2-y^4)\Delta y$

Step 5:
$$V = \int_0^2 \pi(4y+2y^2-y^4)\,dy = \pi\left[2y^2 + \frac{2}{3}y^3 - \frac{1}{5}y^5\right]_0^2 = \pi\left\{\left[2(2)^2 + \frac{2}{3}(2)^3 - \frac{1}{5}(2)^5\right] - [0]\right\}$$
$$= \pi\left\{8 + \frac{16}{3} - \frac{32}{5}\right\} = \pi\left\{\frac{120}{15} + \frac{80}{15} - \frac{96}{15}\right\} = \pi\left\{\frac{104}{15}\right\} = \frac{104\pi}{15}$$

Example 2.4: Same restriction as Example 2.3, but rotated about $x = 5$.



Step 1: see figure to the left and take same endpoints as Example 2.3.

Step 2: Notice that in this case we need to subtract the height of the function from a constant value of 3 to get our needed radii (unlike the Example 2.3 where we needed to add 1 to the height of the function). Thus resulting with our radii to be:

$r_o = 5 - y^2$ $\qquad$ $r_i = 5 - 2y$

$A_o = \pi\left(5 - y^2\right)^2 = \pi(25 - 10y^2 + y^4)$ $\qquad$ $A_i = \pi\left(5 - 2y\right)^2 = \pi(25 - 20y + 4y^2)$

Step 3: $A = A_o - A_i = \pi(25-10y^2+y^4) - \pi(25-20y+4y^2) = \pi[25-10y^2+y^4-25+20y-4y^2]$
$= \pi(y^4 - 14y^2 + 20y)$

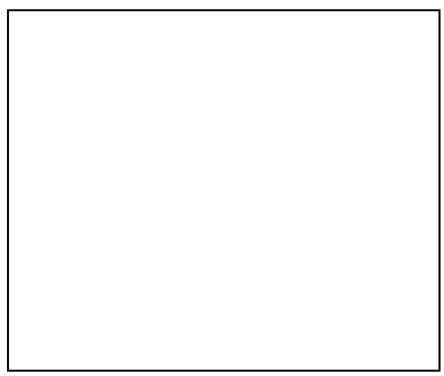
Step 4: $\Delta V = A \cdot \Delta y = \pi(y^4 - 14y^2 + 20y)\Delta y$

Step 5: $V = \int_0^2 \pi(y^4 - 14y^2 + 20y)\, dy = \pi\left[\frac{1}{5}y^5 - \frac{14}{3}y^3 + 10y^2\right]_0^2 = \pi\left\{\left[\frac{1}{5}(2)^5 - \frac{14}{3}(2)^3 + 10(2)^2\right] - [0]\right\}$
$= \pi\left\{\frac{32}{5} - \frac{112}{3} + 40\right\} = \pi\left\{\frac{96}{15} - \frac{560}{15} + \frac{600}{15}\right\} = \pi\left\{\frac{136}{15}\right\} = \frac{136\pi}{15}$

<u>Chapter 7 section 3: Volume by Surface of Revolution (Shell method)</u>

1) This method is generating volume by wrapping rectangular sheets on top, layer after layer. Imagine wrapping aluminum foil on a ball point pen; as we apply more foil, the overall diameter gets thicker. Draw a cross section diagram so we can establish the rotation axis and the interval we need to calculate (intersection points of the 2 functions).

2) We are wrapping rectangular sheets see figure. What we need to get is the area of rectangle (arbitrary). Width is easier; just take the difference of the upper and lower functions (like Section 6.1). The length is the trickier one. This is the circumference of the circular part that is wrapping around. The radius (there is only one) is measured from the rotating axis to the point where the functions are expressed (Reminder: radius is not always $r = x$ or $r = y$). Make sure to draw a cross-section diagram so we can determine if we need to take the sum or the difference in expressing our radius r.



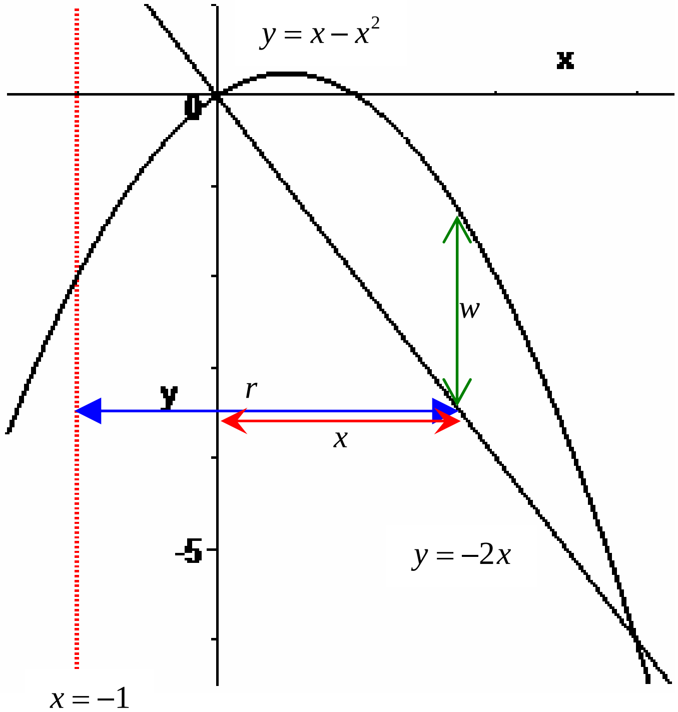
$w = [f(x) - g(x)]$
$f(x)$ upper function
$g(x)$ lower function

$l = C = 2\pi r$

3) Determine $A_R = l \cdot w$ by using l and w from part 2. Then our $\Delta V = A_R \cdot \Delta x$ or $\Delta V = A_R \cdot \Delta y$ depending on the rotating axis.

4) The final volume is calculating $V = \lim_{n\to\infty} \sum_{j=1}^{n} V_j$. Since we are calculating continuous functions $V = \int_a^b A_R\, dx$ or $V = \int_a^b A_R\, dy$.

Example 3.1: Calculate the volume generated by rotating the area of intersection points of $y = x - x^2$ and $y = -2x$, and $x = 0$, rotated about $x = -1$.



Step 1: see figure to the left

$$x - x^2 = -2x$$
$$0 = x^2 - 3x$$
$$0 = x(x-3)$$
$$x = 0 \quad x = 3$$

Step 2: $w = (x - x^2) - (-2x) = 3x - x^2$

$$r = 1 + x$$
$$l = C = 2\pi r = 2\pi(1+x)$$

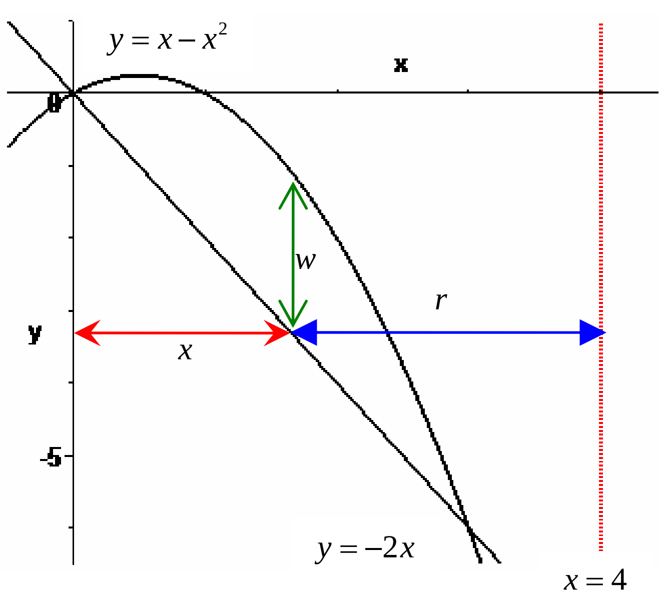
Step 3:

$$A_R = l \cdot w = 2\pi(1+x)(3x - x^2)$$
$$= 2\pi(3x - x^2 + 3x^2 - x^3)$$
$$= 2\pi(3x + 2x^2 - x^3)$$
$$\Delta V = A_R \cdot \Delta x$$
$$= 2\pi(3x + 2x^2 - x^3)\Delta x$$

Step 4:

$$V = \int_0^3 2\pi(3x + 2x^2 - x^3)\, dx = 2\pi\left[\frac{3}{2}x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^3 = 2\pi\left\{\left[\frac{3}{2}(3)^2 + \frac{2}{3}(3)^3 - \frac{1}{4}(3)^4\right] - [0]\right\}$$
$$= 2\pi\left\{\frac{27}{2} + 18 - \frac{81}{4}\right\} = 2\pi\left\{\frac{54}{4} + \frac{72}{4} - \frac{81}{4}\right\} = 2\pi\left\{\frac{45}{4}\right\} = \frac{45\pi}{2}$$

Example 3.2: Same restriction as Example 3.1, but rotated about $x = 4$.



Step 1: see figure to the left and take same endpoints as Example 3.1.

Step 2: The width is same as Example 3.1. But notice that in this case we need to subtract x from a constant value of 4 to get our needed radius (unlike the Example 3.1 where we needed to add 1 to x). Thus resulting with our radius to be:

$$r = 4 - x$$
$$l = C = 2\pi r = 2\pi(4 - x)$$

Step 3:

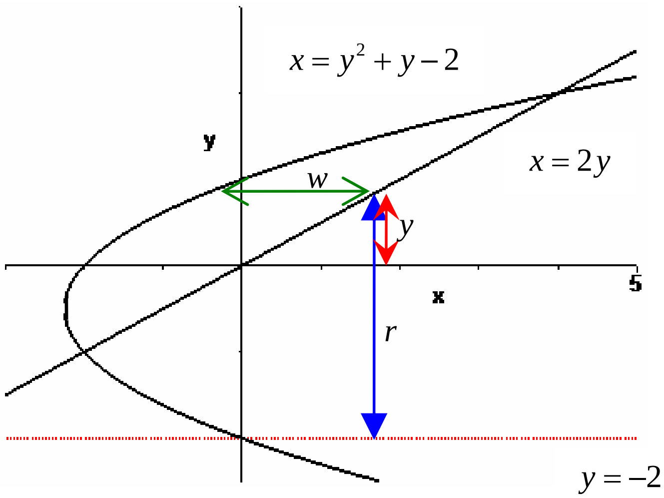
$$A_R = l \cdot w = 2\pi(4 - x)(3x - x^2)$$
$$= 2\pi(12x - 4x^2 - 3x^2 + x^3)$$
$$= 2\pi(12x - 7x^2 + x^3)$$
$$\Delta V = A_R \cdot \Delta x = 2\pi(12x - 7x^2 + x^3)\Delta x$$

Step 4:

$$V=\int_0^3 2\pi(12x-7x^2+x^3)\,dx=2\pi\left[6x^2-\frac{7}{3}x^3+\frac{1}{4}x^4\right]_0^3=2\pi\left\{\left[6(3)^2-\frac{7}{3}(3)^3+\frac{1}{4}(3)^4\right]-[0]\right\}$$

$$=2\pi\left\{54-63+\frac{81}{4}\right\}=2\pi\left\{-9+\frac{81}{4}\right\}=2\pi\left\{\frac{-36}{4}+\frac{81}{4}\right\}=2\pi\left\{\frac{45}{4}\right\}=\frac{45\pi}{2}$$

Example 3.3: Calculate the volume generated by rotating the area of intersection points of $x=y^2+y-2$ and $x=2y$, rotated about $y=-2$.



Step 1: see figure to the left

$$y^2+y-2=2y$$
$$y^2-y-2=0$$
$$(y+1)(y-2)=0$$
$$y=-1 \quad y=2$$

Step 2: $w=(2y)-(y^2+y-2)=2+y-y^2$

$$r=2+y$$
$$l=C=2\pi r=2\pi(2+y)$$

Step 3:

$$A_R=l\cdot w=2\pi(2+y)(2+y-y^2)$$
$$=2\pi(4+2y-2y^2+2y+y^2-y^3)$$
$$=2\pi(4+4y-y^2-y^3)$$
$$\Delta V=A_R\cdot\Delta y$$
$$=2\pi(4+4y-y^2-y^3)\Delta y$$

Step 4:

$$V=\int_{-1}^{2} 2\pi(4+4y-y^2-y^3)\,dy=2\pi\left[4y+2y^2-\frac{1}{3}y^3-\frac{1}{4}y^4\right]_{-1}^{2}$$

$$=2\pi\left\{\left[4(2)+2(2)^2-\frac{1}{3}(2)^3-\frac{1}{4}(2)^4\right]-\left[4(-1)+2(-1)^2-\frac{1}{3}(-1)^3-\frac{1}{4}(-1)^4\right]\right\}$$

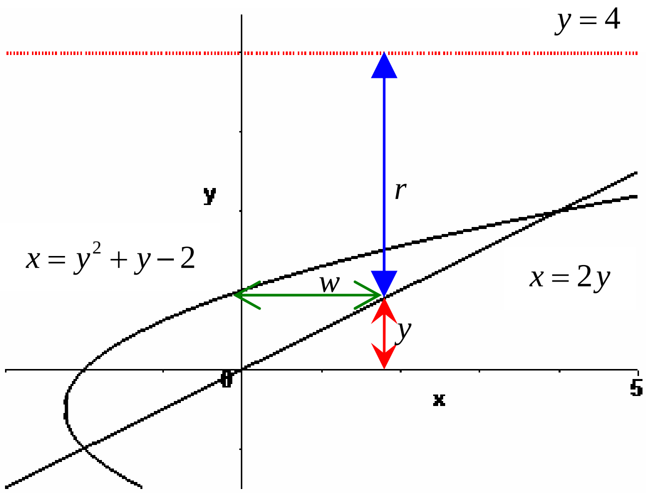
$$=2\pi\left\{\left[8+8-\frac{8}{3}-4\right]-\left[-4+2+\frac{1}{3}-\frac{1}{4}\right]\right\}$$

$$=2\pi\left\{8+8-\frac{8}{3}-4+4-2-\frac{1}{3}+\frac{1}{4}\right\}$$

$$=2\pi\left\{14-\frac{9}{3}+\frac{1}{4}\right\}=2\pi\left\{14-3+\frac{1}{4}\right\}=2\pi\left\{11+\frac{1}{4}\right\}$$

$$=2\pi\left\{\frac{44}{4}+\frac{1}{4}\right\}=2\pi\left\{\frac{45}{4}\right\}=\frac{45\pi}{2}$$

Example 3.4: Same restriction as Example 3.3, but rotated about $y = 4$.



Step 1: see figure to the left and take same endpoints as Example 3.3.

Step 2: The width is same as Example 3.3. But notice that in this case we need to subtract *x* from a constant value of 4 to get our needed radius (unlike the Example 3.3 where we needed to add 2 to *y*). Thus resulting with our radius to be:

$$r = 4 - y$$
$$l = C = 2\pi r = 2\pi(4-y)$$

Step 3:
$$\begin{aligned} A_R &= l \cdot w = 2\pi(4-y)(2+y-y^2) \\ &= 2\pi(8+4y-4y^2-2y-y^2+y^3) \\ &= 2\pi(8+2y-5y^2+y^3) \end{aligned}$$
$$\Delta V = A_R \cdot \Delta y = 2\pi(8+2y-5y^2+y^3)\Delta y$$

Step 4:
$$\begin{aligned} V &= \int_{-1}^{2} 2\pi(8+2y-5y^2+y^3)\,dy = 2\pi\left[8y+y^2-\frac{5}{3}y^3+\frac{1}{4}y^4\right]_{-1}^{2} \\ &= 2\pi\left\{\left[8(2)+(2)^2-\frac{5}{3}(2)^3+\frac{1}{4}(2)^4\right]-\left[8(-1)+(-1)^2-\frac{5}{3}(-1)^3+\frac{1}{4}(-1)^4\right]\right\} \\ &= 2\pi\left\{\left[16+4-\frac{40}{3}+4\right]-\left[-8+1+\frac{5}{3}+\frac{1}{4}\right]\right\} = 2\pi\left\{16+4-\frac{40}{3}+4+8-1-\frac{5}{3}-\frac{1}{4}\right\} \\ &= 2\pi\left\{31-\frac{45}{3}-\frac{1}{4}\right\} = 2\pi\left\{31-15-\frac{1}{4}\right\} = 2\pi\left\{16-\frac{1}{4}\right\} = 2\pi\left\{\frac{64}{4}-\frac{1}{4}\right\} = 2\pi\left\{\frac{63}{4}\right\} = \frac{63\pi}{2} \end{aligned}$$

Chapter 7 section 6: Work (Liquid Pumping method)

The set up for work of pumping liquid is similar to the set up of related rates problems from 1st Calculus class. Therefore, we need to be able to formulate correctly the volume equation in single variable. But instead of a general volume *V*, we need to find a thin volume ΔV.

1) Draw a cross-section diagram (side view and top view). Top view is needed for us to find out the geometric shape of the volume we are lifting to calculate work. Side view is needed to determine the pumping distance and formulation of the area, *A*, which is the figure obtained from the top view and eventually generate ΔV. Make sure to figure out the interval where the liquid is being pumped ($a \le x \le b$).

2) Generate the area *A* (which can be rectangular, circular, triangular, etc.), then multiply by Δx and we will get ΔV. This is the thin volume that we will pump out of the tank (think of lifting a ream of paper but instead of lifting entire ream once, think of lifting a sheet at a time until the entire ream is lifted to a higher location). Then we can formulate ΔF (there are usually 2 cases of ΔF that we need to worry about, metric form and English form) shown below:

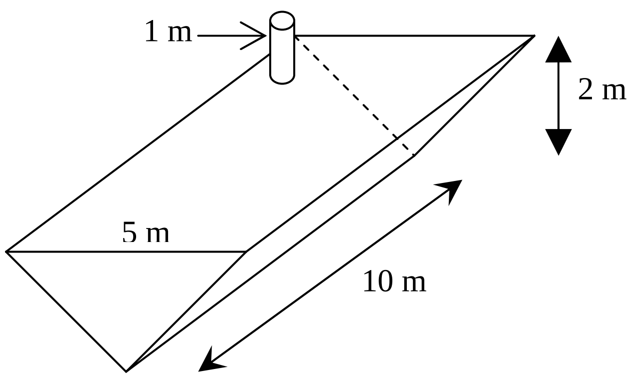
$\Delta V = A \cdot \Delta x$	
Metric [measurement done using meters (m)]	English [measurement done with feet (ft)]
Get mass: $m = (density)(volume)$ Usually density of water is used 1000 kg/m^3 Therefore: $\Delta m = (1000)\Delta V$	This case is simpler because a constant number or variable is given such that from volume we can obtain force without calculating the mass (call it δ). $F = \delta \cdot (volume)$ $\Delta F = \delta \cdot \Delta V$ $= \delta A \cdot \Delta x$
Get force: $F = m \cdot a$, Usually a is gravity which is $9.8\ m/\sec^2$ $\Delta F = (9.8)\Delta m$ $= (9.8)(1000)\Delta V = (9800)\Delta V$ $= (9800)A \cdot \Delta x$	

Keep in mind that usually the method using metric measurement is longer because from volume we need to get mass before obtaining the force; while the method using English measurement is a bit shorter from volume we get the force immediately. If we use some other liquid instead of water in metric measurement, then the density will also change thus changing the constant in force equation to a different value than the one shown above.

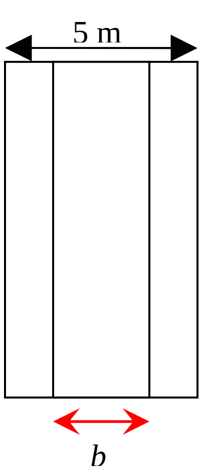
3) Work formula is $W = F \cdot p$, where $F = \text{force}$ and $p = \text{pumping distance}$. Therefore, we get $\Delta W = \Delta F \cdot p = p \cdot \Delta F = pA \cdot \Delta x$. Both expression p and A is a function of x; the expression pA should be distributed and simplified, in order for us to have a simple integration.

4) Since we are dealing with continuous function: $W = \int_a^b pA\, dx$.

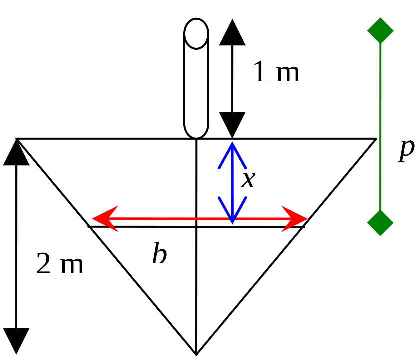
Example 6.1: Consider a V shaped tank 10 meters wide, 5 meters at the top, and 2 meters high filled full of water (see figure to the right). Attached to the top is a pipe of 1 meter where water will be pumped. Find the work done when 1 meter of water (measured from the top) is pumped out of the tank.



Top view



Side view



Step 1: See figures to the left. Also from side view we can calculate our pumping distance of $p = 1 + x$. Considering that the top of the tank is $x = 0$, we can conclude that our integrating interval is $0 \le x \le 1$.

Step 2: The top view shows us that the area we need to calculate is a rectangle. And the length of this rectangle is b. Using triangular proportion, we get:

$$\frac{b}{5} = \frac{2-x}{2} \quad \Rightarrow \quad b = \frac{5}{2}(2-x)$$

$$A = 10b = (10)\left(\frac{5}{2}(2-x)\right) = 25(2-x) \quad \Rightarrow \quad \Delta V = A \cdot \Delta x = 25(2-x)\cdot \Delta x$$

$$\Delta m = (density)\Delta V = (1000)(25(2-x))\Delta x$$

$$\Delta F = a \cdot \Delta m = (9.8)(1000)(25(2-x))\Delta x = (9800)(25(2-x))\Delta x = 245000(2-x)\Delta x$$

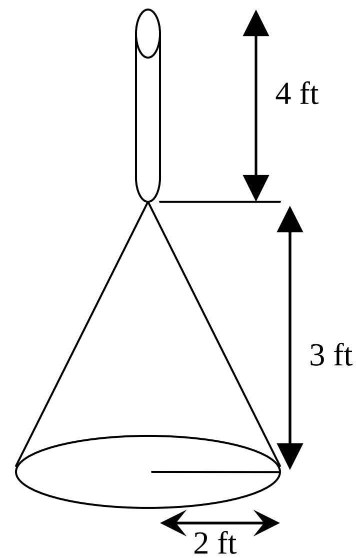
Step 3: $\Delta W = p \cdot \Delta F = (1+x)(245000(2-x))\Delta x = 245000(2+x-x^2)\Delta x$

Step 4:

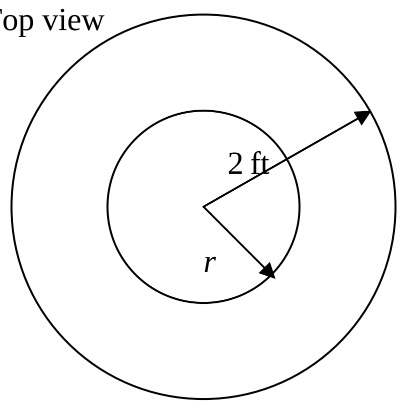
$$W = \int_0^1 245000(2+x-x^2)\,dx = 245000\left[2x+\frac{1}{2}x^2-\frac{1}{3}x^3\right]_0^1 = 245000\left\{\left[2(1)+\frac{1}{2}(1)^2-\frac{1}{3}(1)^3\right]-[0]\right\}$$

$$= 245000\left\{2+\frac{1}{2}-\frac{1}{3}\right\} = 245000\left\{\frac{12}{6}+\frac{3}{6}-\frac{2}{6}\right\} = 245000\left\{\frac{13}{6}\right\} = 122500\left\{\frac{13}{3}\right\} = \frac{1592500}{3} \text{ joules}$$

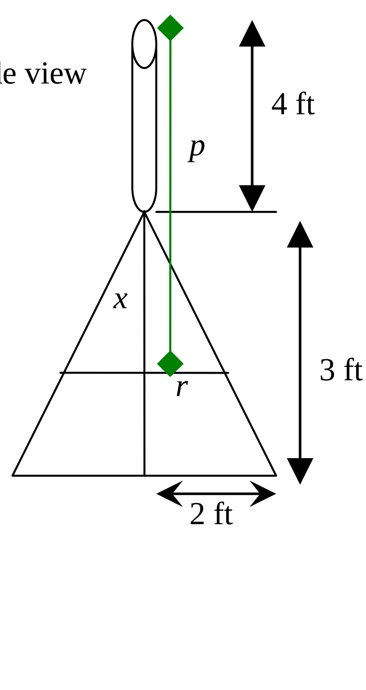
Example 6.2: Consider a conical tank (see figure to the right), full of mysterious liquid, with pointed side up; the base radius is 2 feet and height is 3 feet; attached to the top is a pipe of 4 feet where the liquid will be pumped. The density of liquid is $10\ \text{lbs/ft}^3$. Find the work done when the entire mysterious liquid is pumped from the tank.



Top view



Side view



Step 1: See figures to the left. Also from side view we can calculate our pumping distance of $p = 4 + x$. Considering that the top of the tank is $x = 0$, we can conclude that our integrating interval is $0 \le x \le 3$.

Step 2: The top view shows us that the area we need to calculate is a circle. And the radius of this circle is *r*. Using triangular proportion, we get:

$$\frac{r}{2} = \frac{x}{3} \quad \Rightarrow \quad r = \frac{2}{3}x$$

$$A = \pi r^2 = \pi\left(\frac{2}{3}x\right)^2 = \frac{4\pi}{9}x^2$$

$$\Delta V = A \cdot \Delta x = \left(\frac{4\pi}{9}x^2\right)\Delta x$$

Density: $\delta = 10\ \text{lbs/ft}^3$

$$\Delta F = \delta \cdot \Delta V = 10\left(\frac{4\pi}{9}x^2\right)\Delta x = \left(\frac{40\pi}{9}x^2\right)\Delta x$$

Step 3: $\Delta W = p \cdot \Delta F = (4+x)\left(\frac{40\pi}{9}x^2\right)\Delta x = \frac{40\pi}{9}(4x^2+x^3)\Delta x$

Step 4:

$$W = \int_0^3 \frac{40\pi}{9}(4x^2 + x^3)\,dx = \frac{40\pi}{9}\left[\frac{4}{3}x^3 + \frac{1}{4}x^4\right]_0^3 = \frac{40\pi}{9}\left\{\left[\frac{4}{3}(3)^3 + \frac{1}{4}(3)^4\right] - [0]\right\} = \frac{40\pi}{9}\left\{4(9) + \frac{81}{4}\right\}$$

$$= \frac{40\pi}{9}\left\{4(9) + \frac{9}{4}(9)\right\} = \frac{40\pi}{9}(9)\left\{4 + \frac{9}{4}\right\} = 40\pi\left\{\frac{16}{4} + \frac{9}{4}\right\} = 40\pi\left\{\frac{25}{4}\right\} = 10\pi(25) = 250\pi \text{ foot-pounds}$$

Now if you're up to the challenge, try this following exercise:

Exercise 6.3: Find the work done in pumping the water over the rim of a tank, which is 50 feet long and has a semicircular end of radius 10 feet, if the tank is filled to a depth of 7 feet. [note: Density of water is $\delta = 62.4$].