

Perfect Square Trinomials:

$$A^2 + 2AB + B^2 = (A + B)(A + B) = (A + B)^2$$

$$A^2 - 2AB + B^2 = (A - B)(A - B) = (A - B)^2$$

The Difference of Two Squares:

$$A^2 - B^2 = (A + B)(A - B)$$

Note: The expressions $A^2 + B^2$ and $-A^2 - B^2$ are not factorable.

Reason for the second one: $-A^2 - B^2 = -(A^2 + B^2)$

The Sum and Difference of Two Cubes:

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

The trinomial part of our factor is not factorable in real number. (So for this course, DO NOT factor!)

Read **To Factor a Polynomial** on Page 75 for guidance.

Additional examples:

2) $x^2 + 10x + 25$

This is a perfect square

$$\begin{aligned} x^2 + 10x + 25 &= (x + 5)(x + 5) = (x + 5)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad +(5) \quad +(5) \\ &= +10 \end{aligned}$$

4) $36 - 12a + a^2$

This is a perfect square

$$\begin{aligned} 36 - 12a + a^2 &= a^2 - 12a + 36 = (a - 6)(a - 6) = (a - 6)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad -(6) \quad -(6) \\ &= -12 \end{aligned}$$

6) $64 - 16t + t^2$

This is a perfect square

$$\begin{aligned} 64 - 16t + t^2 &= t^2 - 16t + 64 = (t - 8)(t - 8) = (t - 8)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad -(8) \quad -(8) \\ &= -16 \end{aligned}$$

8) $\frac{1}{4}x^2 - \frac{1}{3}x + \frac{1}{9}$

This is a perfect square

$$LCD = (4)(9) = (12)(3)$$

$$\begin{aligned} \frac{1}{4}x^2 - \frac{1}{3}x + \frac{1}{9} &= \frac{9}{9} \cdot \frac{1}{4}x^2 - \frac{12}{12} \cdot \frac{1}{3}x + \frac{4}{4} \cdot \frac{1}{9} = \frac{9x^2 - 12x + 4}{(4)(9)} = \frac{1}{(4)(9)} \{9x^2 - 12x + 4\} = \frac{1}{(4)(9)} \{(3x-2)(3x-2)\} \\ &= \frac{(3x-2)(3x-2)}{(4)(9)} = \frac{(3x-2)^2}{36} = \frac{(3x-2)^2}{(6)^2} = \left(\frac{3x-2}{6}\right)^2 \\ &= \left(\frac{3x}{6} - \frac{2}{6}\right)^2 = \left(\frac{x}{2} - \frac{1}{3}\right)^2 = \left(\frac{1}{2}x - \frac{1}{3}\right)^2 \end{aligned}$$

$\cancel{\uparrow} \quad \cancel{\uparrow}$
 $\cancel{-\{3\}(2)} - \cancel{\{3\}(2)}$
 $= -6 - 6$
 $= -12$

10) $25a^2 - 40ab + 16b^2$

This is a perfect square

$$\begin{aligned} 25a^2 - 40ab + 16b^2 &= (5a-4b)(5a-4b) = (5a-4b)^2 \\ &\quad \cancel{\uparrow} \quad \cancel{\uparrow} \\ &\quad -\{5\}(4) - \{5\}(4) \\ &= -20 - 20 \\ &= -40 \end{aligned}$$

12) $\frac{1}{9} - \frac{1}{3}t^3 + \frac{1}{6}t^6$

This is a perfect square

$$LCD = (9)(4) = (12)(3)$$

$$\begin{aligned} \frac{1}{9} - \frac{1}{3}t^3 + \frac{1}{6}t^6 &= \frac{9}{9} \cdot \frac{1}{4}t^6 - \frac{12}{12} \cdot \frac{1}{3}t^3 + \frac{4}{4} \cdot \frac{1}{9} = \frac{9t^6 - 12t^3 + 4}{(4)(9)} = \frac{1}{(4)(9)} \{9t^6 - 12t^3 + 4\} = \frac{1}{(4)(9)} \{(3t^3-2)(3t^3-2)\} \\ &= \frac{(3t^3-2)(3t^3-2)}{(4)(9)} = \frac{(3t^3-2)^2}{36} = \frac{(3t^3-2)^2}{(6)^2} = \left(\frac{3t^3-2}{6}\right)^2 \\ &= \left(\frac{3t^3}{6} - \frac{2}{6}\right)^2 = \left(\frac{t^3}{2} - \frac{1}{3}\right)^2 = \left(\frac{1}{2}t^3 - \frac{1}{3}\right)^2 \end{aligned}$$

$\cancel{\uparrow} \quad \cancel{\uparrow}$
 $\cancel{-\{3\}(2)} - \cancel{\{3\}(2)}$
 $= -6 - 6$
 $= -12$

14) $(x+5)^2 + 4(x+5) + 4$

Let $y = (x+5)$

$$\begin{aligned} (x+5)^2 + 4(x+5) + 4 &= y^2 + 4y + 4 = (y+2)(y+2) = (y+2)^2 = ((x+5)+2)^2 = (x+5+2)^2 = (x+7)^2 \\ &\quad \cancel{\uparrow} \quad \cancel{\uparrow} \\ &\quad +(2) \quad +(2) \\ &= +4 \end{aligned}$$

Alternate technique; view $(x+5)$ like as if it is a single variable.

$$(x+5)^2 + 4(x+5) + 4 = ((x+5)+2)((x+5)+2) = ((x+5)+2)^2 = (x+5+2)^2(x+7)^2$$

16) $81x^2 - 49y^2$

$$81x^2 - 49y^2 = (9x)^2 - (7y)^2 = (9x + 7y)(9x - 7y)$$

18) $25a^2 - \frac{1}{25}$

$$25a^2 - \frac{1}{25} = (5a)^2 - \left(\frac{1}{5}\right)^2 = \left(5a + \frac{1}{5}\right)\left(5a - \frac{1}{5}\right)$$

Alternate technique:

$$LCD = 25$$

$$\begin{aligned} 25a^2 - \frac{1}{25} &= \frac{25}{25} \cdot 25a^2 - \frac{1}{25} = \frac{(25a)^2 - (1)^2}{25} = \frac{(25a+1)(25a-1)}{25} = \frac{(25a+1)(25a-1)}{(5)^5} = \frac{(25a+1)(25a-1)}{(5)(5)} \\ &= \left(\frac{25a+1}{5}\right)\left(\frac{25a-1}{5}\right) = \left(\frac{25a}{5} + \frac{1}{5}\right)\left(\frac{25a}{5} - \frac{1}{5}\right) = \left(5a + \frac{1}{5}\right)\left(5a - \frac{1}{5}\right) \end{aligned}$$

20) $x^2 - \frac{25}{36}$

$$x^2 - \frac{25}{36} = x^2 - \left(\frac{5}{6}\right)^2 = \left(x + \frac{5}{6}\right)\left(x - \frac{5}{6}\right)$$

Alternate technique:

$$LCD = 36$$

$$\begin{aligned} x^2 - \frac{25}{36} &= \frac{36}{36} \cdot x^2 - \frac{25}{36} = \frac{36x^2 - 25}{36} = \frac{(6x)^2 - (5)^2}{36} = \frac{(6x+5)(6x-5)}{36} = \frac{(6x+5)(6x-5)}{(6)(6)} \\ &= \left(\frac{6x+5}{6}\right)\left(\frac{6x-5}{6}\right) = \left(\frac{6x}{6} + \frac{5}{6}\right)\left(\frac{6x}{6} - \frac{5}{6}\right) = \left(x + \frac{5}{6}\right)\left(x - \frac{5}{6}\right) \end{aligned}$$

22) $64 - \frac{1}{16}t^2$

$$64 - \frac{1}{16}t^2 = 64 - \frac{t^2}{16} = (8)^2 - \left(\frac{t}{4}\right)^2 = \left(8 + \frac{t}{4}\right)\left(8 - \frac{t}{4}\right) = \left(8 + \frac{1}{4}t\right)\left(8 - \frac{1}{4}t\right)$$

24) $81a^4 - 16b^4$

$$\begin{aligned} 81a^4 - 16b^4 &= (9a^2)^2 - (4b^2)^2 = (9a^2 + 4b^2)(9a^2 - 4b^2) \\ &= (9a^2 + 4b^2)((3a)^2 - (2b)^2) = (9a^2 + 4b^2)(3a + 2b)(3a - 2b) \end{aligned}$$

26) $x^2 - 6x + 9 - y^2$

This is perfect square and then difference of two squares

$$\begin{aligned} x^2 - 6x + 9 &= (x-3)(x-3) = (x-3)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad -(3) \quad -(3) \\ &= -6 \end{aligned}$$

$$x^2 - 6x + 9 - y^2 = (x^2 - 6x + 9) - y^2 = (x-3)^3 - y^2 = ((x-3)+y)((x-3)-y) = (x-3+y)(x-3-y)$$

28) $a^2 + 12a + 36 - 100b^2$

This is perfect square and then difference of two squares

$$\begin{aligned} a^2 + 12a + 36 &= (a+6)(a+6) = (a+6)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad +(6) \quad +(6) \\ &= +12 \end{aligned}$$

$$\begin{aligned} a^2 + 12a + 36 - 100b^2 &= (a^2 + 12a + 36) - 100b^2 = (a+6)^2 - 100b^2 = (a+6)^2 - (10b)^2 \\ &= ((a+6)+10b)((a+6)-10b) = (a+6+10b)(a+6-10b) \end{aligned}$$

30) $x^3 + 4x^2 - 9x - 36$

This one is standard factor by grouping

$$\begin{aligned} x^3 + 4x^2 - 9x - 36 &= (x^3 + 4x^2) - (9x + 36) = x^2(x+4) - 9(x+4) = \{x^2(x+4) - 9(x+4)\} \\ &= (x+4)\{x^2 - 9\} = (x+4)\{x^2 - (3)^2\} = (x+4)\{(x+3)(x-3)\} = (x+4)(x+3)(x-3) \end{aligned}$$

32) $3x^3 + 2x^2 - 27x - 18$

This one is standard factor by grouping

$$\begin{aligned} 3x^3 + 2x^2 - 27x - 18 &= (3x^3 + 2x^2) - (27x + 18) = x^2(3x+2) - 9(3x+2) = \{x^2(3x+2) - 9(3x+2)\} \\ &= (3x+2)\{x^2 - 9\} = (3x+2)\{x^2 - (3)^2\} = (3x+2)\{(x+3)(x-3)\} \\ &= (3x+2)(x+3)(x-3) \end{aligned}$$

36) $x^3 + y^3$

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

38) $a^3 + 8$

$$a^3 + 8 = a^3 + (2)^3 = (a+2)(a^2 - a(2) + (2)^2) = (a+2)(a^2 - 2a + 4)$$

40) $64y^3 - \frac{1}{8}$

$$64y^3 - \frac{1}{8} = (4y)^3 + \left(\frac{1}{2}\right)^3 = \left(4y + \frac{1}{2}\right) \left((4y)^2 - (4y)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 \right) = \left(4y + \frac{1}{2}\right) \left(16y^2 - 2y + \frac{1}{4} \right)$$

42) $10r^3 + 1250$

$$\begin{aligned} 10r^3 + 1250 &= 10\{r^3 + 125\} = 10\{r^3 + (5)^3\} = 10\{(r+5)(r^2 - r(5) + (5)^2)\} = 10\{(r+5)(r^2 - 5r + 25)\} \\ &= 10(r+5)(r^2 - 5r + 25) \end{aligned}$$

44) $27 - 64a^3$

$$27 - 64a^3 = (3)^3 - (4a)^3 = (3 - 4a)((3)^2 + (3)(4a) + (4a)^2) = (3 - 4a)(9^2 + 12a + 16a^2)$$

46) $27t^3 - \frac{1}{27}$

$$27t^3 - \frac{1}{27} = (3t)^3 + \left(\frac{1}{3}\right)^3 = \left(3t + \frac{1}{3}\right) \left((3t)^2 - (3t)\left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2\right) = \left(3t + \frac{1}{3}\right) \left(9t^2 - t + \frac{1}{9}\right)$$

48) $x^2 - 18x + 81$

This one is perfect square

$$\begin{aligned} x^2 - 18x + 81 &= (x-9)(x-9) = (x-9)^2 \\ &\quad \begin{array}{c} \uparrow \quad \uparrow \\ -(9) \quad -(9) \\ = -18 \end{array} \end{aligned}$$

50) $15x^2 + 13x - 6$

$$\begin{aligned} 15x^2 + 13x - 6 &= (5x+6)(3x-1) \\ &\quad \begin{array}{c} \nearrow \searrow \\ +\{3\}(6) - \{5\}(1) \\ = +18 - 5 \\ = +13 \end{array} \end{aligned}$$

52) $21y^2 - 25y - 4$

$$\begin{aligned} 21y^2 - 25y - 4 &= (7y+1)(3y-4) \\ &\quad \begin{array}{c} \nearrow \searrow \\ +\{3\}(1) - \{7\}(4) \\ = +3 - 28 \\ = -25 \end{array} \end{aligned}$$

54) $6a^2 - ab - 15b^2$

$$\begin{aligned} 6a^2 - ab - 15b^2 &= (2a+3b)(3a-5b) \\ &\quad \begin{array}{c} \nearrow \searrow \\ +\{3\}(3) - \{2\}(5) \\ = +9 - 10 \\ = -1 \end{array} \end{aligned}$$

56) $x^2y + 3y + 2x^2 + 6$

This one is standard factor by grouping

$$\begin{aligned} x^2y + 3y + 2x^2 + 6 &= (x^2y + 3y) + (2x^2 + 6) = y(x^2 + 3) + 2(x^2 + 3) = \{y(x^2 + 3) + 2(x^2 + 3)\} \\ &= (x^2 + 3)\{y + 2\} = (x^2 + 3)(y + 2) \end{aligned}$$

58) $18a^2 - 50$

$$18a^2 - 50 = 2\{9a^2 - 25\} = 2\{(3a)^2 - (5)^2\} = 2\{(3a + 5)(3a - 5)\} = 2(3a + 5)(3a - 5)$$

60) $t^2 + 4t + 4 - y^2$

This is perfect square and then difference of two squares

$$\begin{aligned} t^2 + 4t + 4 &= (t+2)(t+2) = (t+2)^2 \\ &\quad \uparrow \quad \uparrow \\ &\quad +(2) \quad +(2) \\ &= +4 \end{aligned}$$

$$t^2 + 4t + 4 - y^2 = (t^2 + 4t + 4) - y^2 = (t+2)^2 - y^2 = ((t+2) + y)((t+2) - y) = (t+2+y)(t+2-y)$$

62) $16x^2 - 49y^2$

$$16x^2 - 49y^2 = (4x)^2 - (7y)^2 = (4x + 7y)(4x - 7y)$$

64) $x^3 + 5x^2 - 9x - 45$

This one is standard factor by grouping

$$\begin{aligned} x^3 + 5x^2 - 9x - 45 &= (x^3 + 5x^2) - (9x + 45) = x^2(x+5) - 9(x+5) = \{x^2(x+5) - 9(x+5)\} \\ &= (x+5)\{x^2 - 9\} = (x+5)\{x^2 - (3)^2\} = (x+5)\{(x+3)(x-3)\} = (x+5)(x+3)(x-3) \end{aligned}$$

66) $16 - x^4$

$$\begin{aligned} 16 - x^4 &= (4)^2 - (x^2)^2 = (4 + x^2)(4 - x^2) = (4 + x^2)((2)^2 - x^2) = (4 + x^2)((2+x)(2-x)) \\ &= (4 + x^2)(2+x)(2-x) \end{aligned}$$

68) $x^2 + 3ax - 2bx - 6ab$

This one is standard factor by grouping

$$\begin{aligned} x^2 + 3ax - 2bx - 6ab &= (x^2 + 3ax) - (2bx + 6ab) = x(x+3a) - 2b(x+3a) = \{x(x+3a) - 2b(x+3a)\} \\ &= (x+3a)\{x-2b\} = (x+3a)(x-2b) \end{aligned}$$

70) $5x^4 + 14x^2 - 3$

$$\begin{aligned}
 5x^4 + 14x^2 - 3 &= 5(x^2)^2 + 14x^2 - 3 = (1x^2 + 3)(5x^2 - 1) = (x^2 + 3)(5x^2 - 1) \\
 &\quad \text{+}\{5\}(3) - \{1\}(1) \\
 &= +15 - 1 \\
 &= +14
 \end{aligned}$$

This expression is not complete if we allow real numbers. If the real number is allowed, then:

$$\begin{aligned}
 5x^4 + 14x^2 - 3 &= 5(x^2)^2 + 14x^2 - 3 = (1x^2 + 3)(5x^2 - 1) = (x^2 + 3)(5x^2 - 1) = (x^2 + 3)\left(\left(\sqrt{5}x\right)^2 - (1)^2\right) \\
 &= (x^2 + 3)(\sqrt{5}x + 1)(\sqrt{5}x - 1)
 \end{aligned}$$

72) $27 - r^3$

$$27 - r^3 = (3)^3 - r^3 = (3 - r)((3)^2 + (3)r + r^2) = (3 - r)(9 + 3r + r^2)$$

74) $12x^4 - 62x^3 + 70x^2$

$$\begin{aligned}
 12x^4 - 62x^3 + 70x^2 &= 2x^2\{6x^2 - 31x + 35\} = 2x^2\{(3x - 5)(2x - 7)\} = 2x^2(3x - 5)(2x - 7) \\
 &\quad \text{-}\{2\}(5) - \{3\}(7) \\
 &= -10 - 21 \\
 &= -31
 \end{aligned}$$

76) $100x^2 - 100x - 1200$

$$\begin{aligned}
 100x^2 - 100x - 1200 &= 100\{x^2 - x - 12\} = 100\{(x + 3)(x - 4)\} = 100(x + 3)(x - 4) \\
 &\quad \text{+}\{3\} \quad \text{-}\{4\} \\
 &= -1
 \end{aligned}$$

78) $8 - 2x - 15x^2$

$$\begin{aligned}
 8 - 2x - 15x^2 &= (4 + 5x)(2 - 3x) \\
 &\quad \text{+}\{2\}(5) - \{4\}(3) \\
 &= +10 - 12 \\
 &= -2
 \end{aligned}$$

80) $18a^4b^2 - 24a^3b^3 + 8a^2b^4$

$$\begin{aligned}
 18a^4b^2 - 24a^3b^3 + 8a^2b^4 &= 2a^2b^2\{9a^2 - 12ab + 4b^2\} = 2a^2b^2\{(3a - 2b)(3a - 2b)\} = 2a^2b^2(3a - 2b)^2 \\
 &\quad \text{-}\{3\}(2) - \{3\}(2) \\
 &= -6 - 6 \\
 &= -12
 \end{aligned}$$

82) $r^2 - \frac{1}{9}$

$$r^2 - \frac{1}{9} = r^2 - \left(\frac{1}{3}\right)^2 = \left(r + \frac{1}{3}\right)\left(r - \frac{1}{3}\right)$$

84) $16x^3 + 16x^2 + 3x$

$$16x^3 + 16x^2 + 3x = x\{16x^2 + 16x + 3\} = x\{(4x+1)(4x+3)\} = x(4x+1)(4x+3)$$

$$\begin{aligned} &+ \{4\}(1) + \{4\}(3) \\ &= +4 + 12 \\ &= +16 \end{aligned}$$

86) $16x^2 + 16x - 1$

This trinomial is not factorable using only integer and rational (fraction) numbers/coefficients. All patterns does not work for this trinomial.

88) $25y^7 - 16y^5$

$$25y^7 - 16y^5 = y^5\{25y^2 - 16\} = y^5\{(5y)^2 - (4)^2\} = y^5\{(5y+4)(5y-4)\} = y^5(5y+4)(5y-4)$$

90) $4a^2 + 2a + \frac{1}{4}$

$$LCD = 4$$

$$\begin{aligned} 4a^2 + 2a + \frac{1}{4} &= \frac{4}{4} \cdot 4a^2 + \frac{4}{4} \cdot 2a + \frac{1}{4} = \frac{16a^2 + 8a + 1}{4} = \frac{1}{4}\{16a^2 + 8a + 1\} = \frac{1}{4}\{(4a+1)(4a+1)\} \\ &= \frac{(4a+1)(4a+1)}{4} = \frac{(4a+1)^2}{(2)^2} = \left(\frac{4a+1}{2}\right)^2 \\ &= \left(\frac{4a}{2} + \frac{1}{2}\right)^2 = \left(2a + \frac{1}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} &+ \{4\}(1) + \{4\}(1) \\ &= +4 + 4 \\ &= +8 \end{aligned}$$

92) $x^2 - 12x + 36 - b^2$

This is prefect square and then difference of two squares

$$x^2 - 12x + 36 = (x-6)(x-6) = (x-6)^2$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ -(6) \quad -(6) \\ = -12 \end{array}$$

$$x^2 - 12x + 36 - b^2 = (x^2 - 12x + 36) - b^2 = (x-6)^2 - b^2 = ((x-6)+b)((x-6)-b) = (x-6+b)(x-6-b)$$