

Additional examples:

2) $12x^5 + 18x^7$
 $12x^5 + 18x^7 = 6x^5(2 + 3x^2)$

4) $24y^4 - 8y^2$
 $24y^4 - 8y^2 = 8y^2(3y^2 - 1)$

6) $30a^3b^4 + 20a^4b^3$
 $30a^3b^4 + 20a^4b^3 = 10a^3b^3(3b + 2a)$

8) $14x^6y^3 - 6x^2y^4$
 $14x^6y^3 - 6x^2y^4 = 2x^2y^3(7x^4 - 3y)$

10) $3a^2 - 3a - 6$
 $3a^2 - 3a - 6 = 3(a^2 - a - 2) = 3(a+1)(a-2)$

$\begin{matrix} & \uparrow & \uparrow \\ a^2 & -a & \\ \uparrow & \uparrow \\ +1 & -2 \\ = -1 \end{matrix}$

12) $2x^3 - 14x^2 + 20x$
 $2x^3 - 14x^2 + 20x = 2x(x^2 - 7x + 10) = 2x(x-2)(x-5)$

$\begin{matrix} & \uparrow & \uparrow \\ x^2 & -7x & \\ \uparrow & \uparrow \\ -2 & -5 \\ = -7 \end{matrix}$

14) $3a(x-y) - 7b(x-y)$
 $3a(x-y) - 7b(x-y) = \{3a(x-y) - 7b(x-y)\} = (x-y)\{3a - 7b\} = (x-y)(3a - 7b)$

16) $10x^3(2x-3y) - 15x^2(2x-3y)$
 $10x^3(2x-3y) - 15x^2(2x-3y) = \{10x^3(2x-3y) - 15x^2(2x-3y)\}$
 $= 5x^2(2x-3y)\{2x-3\} = 5x^2(2x-3y)(2x-3)$

18) $2x^2(x+2) + 13x(x+2) + 15(x+2)$
 $2x^2(x+2) + 13x(x+2) + 15(x+2) = \{2x^2(x+2) + 13x(x+2) + 15(x+2)\} = (x+2)\{2x^2 + 13x + 15\}$
 $= (x+2)\{(2x+3)(1x+5)\} = (x+2)(2x+3)(x+5)$

$\begin{matrix} & \nearrow & \searrow & \uparrow \\ 2x & 1x & 5 \\ \uparrow & \uparrow \\ 3 & 5 \\ +\{1\}(3) + \{2\}(5) \\ = +3 + 10 \\ = +13 \end{matrix}$

20) $5xy^2 + 5y^2 + 3ax + 3a$

$$\begin{aligned} 5xy^2 + 5y^2 + 3ax + 3a &= (5xy^2 + 5y^2) + (3ax + 3a) = 5y^2(x+1) + 3a(x+1) = \{5y^2(x+1) + 3a(x+1)\} \\ &= (x+1)\{5y^2 + 3a\} = (x+1)(5y^2 + 3a) \end{aligned}$$

22) $x^3y^3 + 2x^3 + 5x^2y^3 + 10x^2$

$$\begin{aligned} x^3y^3 + 2x^3 + 5x^2y^3 + 10x^2 &= (x^3y^3 + 2x^3) + (5x^2y^3 + 10x^2) = x^3(y^3 + 2) + 5x^2(y^3 + 2) \\ &= \{x^3(y^3 + 2) + 5x^2(y^3 + 2)\} = (y^3 + 2)\{x^3 + 5x^2\} \\ &= (x^3 + 5x^2)(y^3 + 2) = x^2(x+5)(y^3 + 2) \end{aligned}$$

24) $ax - x^2 - bx + ab$

$$\begin{aligned} ax - x^2 - bx + ab &= ax - x^2 + ab - bx = (ax - x^2) + (ab - bx) = x(a - x) + b(a - x) = \{x(a - x) + b(a - x)\} \\ &= (a - x)\{x + b\} = (a - x)(x + b) \end{aligned}$$

26) $x^2 - xy - ax + ay$

$$\begin{aligned} x^2 - xy - ax + ay &= (x^2 - xy) - (ax - ay) = x(x - y) - a(x - y) = \{x(x - y) - a(x - y)\} \\ &= (x - y)\{x - a\} = (x - y)(x - a) \end{aligned}$$

28) $x^2 - 7x + 12$

$$\begin{array}{r} x^2 - 7x + 12 = (x - 3)(x - 4) \\ \uparrow \quad \uparrow \\ -(3) \quad -(4) \\ = -7 \end{array}$$

30) $x^2 + x - 12$

$$\begin{array}{r} x^2 + x - 12 = (x + 4)(x - 3) \\ \uparrow \quad \uparrow \\ +(4) \quad -(3) \\ = +1 \end{array}$$

32) $y^2 - y - 6$

$$\begin{array}{r} y^2 - y - 6 = (y + 2)(x - 3) \\ \uparrow \quad \uparrow \\ +(2) \quad -(3) \\ = -1 \end{array}$$

34) $3 + 2x - x^2$

$$\begin{array}{r} 3 + 2x - x^2 = -x^2 + 2x + 3 = -(x^2 - 2x - 3) = -(x + 1)(x - 3) = -(-3 + x)(1 + x) = (3 - x)(1 + x) \\ \uparrow \quad \uparrow \\ +(1) \quad -(3) \\ = -2 \end{array}$$

36) $15 - 2x - x^2$

$$\begin{aligned}
 15 - 2x - x^2 &= -x^2 - 2x + 15 = -(x^2 + 2x - 15) = -(x + 5)(x - 3) = -(-3 + x)(5 + x) = (3 - x)(1 + x) \\
 &\quad \uparrow \quad \uparrow \\
 &\quad +(5) \quad -(3) \\
 &\quad = +2
 \end{aligned}$$

38) $x^2 - 5xy - 24y^2$

$$\begin{aligned}
 x^2 - 5xy - 24y^2 &= (x + 3y)(x - 8y) \\
 &\quad \uparrow \quad \uparrow \\
 &\quad +(3) \quad -(8) \\
 &\quad = -5
 \end{aligned}$$

40) $a^2 - 8ab - 9b^2$

$$\begin{aligned}
 a^2 - 8ab - 9b^2 &= (a + 1b)(a - 9b) = (a + b)(a - 9b) \\
 &\quad \uparrow \quad \uparrow \\
 &\quad +(1) \quad -(9) \\
 &\quad = -8
 \end{aligned}$$

42) $x^2 + 14xa + 48a^2$

$$\begin{aligned}
 x^2 + 14xa + 48a^2 &= (x + 8a)(x + 6a) \\
 &\quad \uparrow \quad \uparrow \\
 &\quad +(8) \quad +(6) \\
 &\quad = +14
 \end{aligned}$$

44) $x^2 + 10xb + 25b^2$

$$\begin{aligned}
 x^2 + 10xb + 25b^2 &= (x + 5b)(x + 5b) = (x + 5b)^2 \\
 &\quad \uparrow \quad \uparrow \\
 &\quad +(5) \quad +(5) \\
 &\quad = +10
 \end{aligned}$$

46) $2x^2 - 7x - 15$

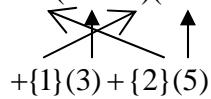
$$\begin{aligned}
 2x^2 - 7x - 15 &= (2x + 3)(1x - 5) = (2x + 3)(x - 5) \\
 &\quad \cancel{\uparrow} \quad \cancel{\uparrow} \quad \uparrow \\
 &\quad +\{1\}(3) - \{2\}(5) \\
 &\quad = +3 - 10 \\
 &\quad = -7
 \end{aligned}$$

48) $2x^2 - x - 15$

$$\begin{aligned}
 2x^2 - x - 15 &= (2x + 5)(1x - 3) = (2x + 5)(x - 3) \\
 &\quad \cancel{\uparrow} \quad \cancel{\uparrow} \quad \uparrow \\
 &\quad +\{1\}(5) - \{2\}(3) \\
 &\quad = +5 - 6 \\
 &\quad = -1
 \end{aligned}$$

50) $2x^2 + 13x + 15$

$2x^2 + 13x + 15 = (2x+3)(1x+5) = (2x+3)(x+5)$



$$\begin{aligned} &+ \{1\}(3) + \{2\}(5) \\ &= +3 + 10 \\ &= +13 \end{aligned}$$

52) $2x^2 + 11x + 15$

$2x^2 + 11x + 15 = (2x+5)(1x+3) = (2x+5)(x+3)$

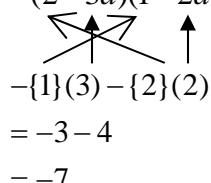


$$\begin{aligned} &+ \{1\}(5) + \{2\}(3) \\ &= +5 + 6 \\ &= +11 \end{aligned}$$

54) $2x^2 - 7x - 15$ [This is the same as exercise 46]

56) $2 - 7a + 6a^2$

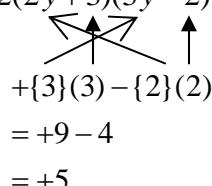
$2 - 7a + 6a^2 = (2 - 3a)(1 - 2a)$



$$\begin{aligned} &- \{1\}(3) - \{2\}(2) \\ &= -3 - 4 \\ &= -7 \end{aligned}$$

58) $72y^2 + 60y - 72$

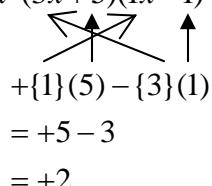
$72y^2 + 60y - 72 = 12(6y^2 + 5y - 6) = 12(2y+3)(3y-2)$



$$\begin{aligned} &+ \{3\}(3) - \{2\}(2) \\ &= +9 - 4 \\ &= +5 \end{aligned}$$

60) $3x^4 + 2x^3 - 5x^2$

$3x^4 + 2x^3 - 5x^2 = x^2(3x^2 + 2x - 5) = x^2(3x+5)(1x-1) = x^2(3x+5)(x-1)$



$$\begin{aligned} &+ \{1\}(5) - \{3\}(1) \\ &= +5 - 3 \\ &= +2 \end{aligned}$$

$$62) \quad 40r^3 + 200r^2 + 250r$$

$$40r^3 + 200r^2 + 250r = 10r(4r^2 + 20r + 25) = 10r(2r+5)(2r+5) = 10r(2r+5)^2$$

$$\begin{array}{c}
 \text{Diagram showing two crossed lines with arrows pointing away from each other.} \\
 +\{2\}(5) + \{2\}(5) \\
 = +10 + 10 \\
 = +20
 \end{array}$$

$$64) \quad 3x^2 + 19xy - 14y^2$$

$$3x^2 + 19xy - 14y^2 = (1x + 7y)(3x - 2y) = (x + 7y)(3x - 2y)$$

$$\begin{array}{c}
 \text{Diagram showing two crossed lines with arrows pointing away from each other.} \\
 +\{3\}(7) - \{1\}(2) \\
 = +21 - 2 \\
 = +19
 \end{array}$$

$$66) \quad 9x^2 + 9xa - 10a^2$$

$$9x^2 + 9xa - 10a^2 = (3x + 5y)(3x - 2y)$$

$$\begin{aligned}
 & +\{3\}(5) - \{3\}(2) \\
 & = +15 - 6 \\
 & = +9
 \end{aligned}$$

$$68) \quad 6a^2 - 7ab - 5b^2$$

$$6a^2 - 7ab - 5b^2 = (2a + 1b)(3a - 5b) = (2a + b)(3a - 5b)$$

$$\begin{array}{r}
 \begin{array}{c} \swarrow \uparrow \searrow \\ +\{3\}(1) - \{2\}(5) \end{array} \\
 = +3 - 10 \\
 = -7
 \end{array}$$

$$70) \quad 200 - 600t - 350t^2$$

$$200 - 600t - 350t^2 = 50(4 - 12t - 7t^2) = 50(2 + 1t)(2 - 7t) = 50(2 + t)(2 - 7t)$$

$$\begin{array}{c}
 \nearrow \uparrow \searrow \\
 +\{2\}(1) - \{2\}(7) \\
 = +2 - 14 \\
 = -12
 \end{array}$$

$$72) \quad 4y^5 + 7y^4 - 2y^3$$

$$4y^5 + 7y^4 - 2y^3 = y^3(4y^2 + 7y - 2) = y^3(1y + 2)(4y - 1) = y^3(y + 2)(4y - 1)$$

$$+ \{4\}(2) - \{1\}(1)$$

$$= +8 - 1$$

$$= +7$$

74) $60a^2 + 65a^3 - 20a^4$

$$60a^2 + 65a^3 - 20a^4 = 5a^2(12 + 13a - 4a^2) = 5a^2(3 + 4a)(4 - 1a) = 5a^2(3 + 4a)(4 - a)$$

$$+\{4\}(4) - \{3\}(1)$$

$$= +16 - 3$$

$$= +13$$

76) $8x^4y^2 - 47x^3y^3 - 6x^2y^4$

$$8x^4y^2 - 47x^3y^3 - 6x^2y^4 = x^2y^2(8x^2 - 47xy - 6y^2) = x^2y^2(8x + 1y)(1x - 6y) = x^2y^2(8x + y)(x - 6y)$$

$$+\{1\}(1) - \{8\}(6)$$

$$= +1 - 48$$

$$= -47$$

78) $600x^4 - 100x^2 - 200$

$$600x^4 - 100x^2 - 200 = 100(6x^4 - x^2 - 2) = 100(2x^2 + 1)(3x^2 - 2) = 100(2x^2 + 1)(3x^2 - 2)$$

$$+\{3\}(1) - \{2\}(2)$$

$$= +3 - 4$$

$$= -1$$

80) $2x^2(x+2) + 13x(x+2) + 15(x+2)$

$$2x^2(x+2) + 13x(x+2) + 15(x+2) = \{2x^2(x+2) + 13x(x+2) + 15(x+2)\}$$

$$= (x+2)\{2x^2 + 13x + 15\}$$

$$= (x+2)\{(2x+3)(1x+5)\} = (x+2)(2x+3)(x+5)$$

$$+\{1\}(3) + \{2\}(5)$$

$$= +3 + 10$$

$$= +13$$

82) $2x^2(x+1) + 7x(x+1) + 6(x+1)$

$$2x^2(x+1) + 7x(x+1) + 6(x+1) = \{2x^2(x+1) + 7x(x+1) + 6(x+1)\}$$

$$= (x+1)\{2x^2 + 7x + 6\}$$

$$= (x+1)\{(1x+2)(2x+3)\} = (x+1)(x+2)(2x+3)$$

$$+\{2\}(2) + \{1\}(3)$$

$$= +4 + 3$$

$$= +7$$