

Definition: Two radicals are said to be **similar radicals** if they have the same index and the same radicand.

Rule: To add and subtract radical expressions, put each in simplified form and apply the distributive property, if possible. We can only add similar radicals. We must write each expression in simplified form for radicals before we can tell if the radicals are similar.

Note: Review examples 1 to 5 in the text for guidance.

Note: When adding and subtracting radicals, treat each different radicals as if they are different variables. For example: The statement $4\sqrt{3} + 2\sqrt{5}$ is already in simplified form and we cannot complete the operation because let $x = \sqrt{3}$ and $y = \sqrt{5}$. Then our original statement is same as $4\sqrt{3} + 2\sqrt{5} \Rightarrow 4x + 2y$.

Additional examples

$$2) \quad 6\sqrt{3} - 5\sqrt{3}$$

$$6\sqrt{3} - 5\sqrt{3} = 1\sqrt{3} = \sqrt{3}$$

$$4) \quad 6y\sqrt{a} + 7y\sqrt{a}$$

$$6y\sqrt{a} + 7y\sqrt{a} = 13y\sqrt{a}$$

$$6) \quad 6\sqrt[4]{2} + 9\sqrt[4]{2}$$

$$6\sqrt[4]{2} + 9\sqrt[4]{2} = 15\sqrt[4]{2}$$

$$8) \quad 7\sqrt[6]{7} - \sqrt[6]{7} + 4\sqrt[6]{7}$$

$$7\sqrt[6]{7} - \sqrt[6]{7} + 4\sqrt[6]{7} = 10\sqrt[6]{7}$$

$$10) \quad 5x\sqrt{6} - 3x\sqrt{6} - 2x\sqrt{6}$$

$$5x\sqrt{6} - 3x\sqrt{6} - 2x\sqrt{6} = 0x\sqrt{6} = 0$$

$$12) \quad \sqrt{8} - \sqrt{32} - \sqrt{18}$$

$$\sqrt{8} - \sqrt{32} - \sqrt{18} = \sqrt{(4)(2)} - \sqrt{(16)(2)} - \sqrt{(9)(2)} = \sqrt{4}\sqrt{2} - \sqrt{16}\sqrt{2} - \sqrt{9}\sqrt{2} = 2\sqrt{2} - 4\sqrt{2} - 3\sqrt{2} = -5\sqrt{2}$$

$$14) \quad \sqrt{48} - 3\sqrt{27} + 2\sqrt{75}$$

$$\sqrt{48} - 3\sqrt{27} + 2\sqrt{75} = \sqrt{(16)(3)} - 3\sqrt{(9)(3)} + 2\sqrt{(25)(3)} = \sqrt{16}\sqrt{3} - 3\sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{3}$$

$$= (4)\sqrt{3} - 3(3)\sqrt{3} + 2(5)\sqrt{3} = 4\sqrt{3} - 9\sqrt{3} + 10\sqrt{3} = 5\sqrt{3}$$

- 16) $2\sqrt{50x^2} - 8x\sqrt{18} - 3\sqrt{72x^2}$
- $$\begin{aligned} 2\sqrt{50x^2} - 8x\sqrt{18} - 3\sqrt{72x^2} &= 2\sqrt{(25x^2)(2)} - 8x\sqrt{(9)(2)} - 3\sqrt{(36x^2)(2)} \\ &= 2\sqrt{25x^2}\sqrt{2} - 8x\sqrt{9}\sqrt{2} - 3\sqrt{36x^2}\sqrt{2} = 2(5x)\sqrt{2} - 8x(3)\sqrt{2} - 3(6x)\sqrt{2} \\ &= 10x\sqrt{2} - 24x\sqrt{2} - 18x\sqrt{2} = -32x\sqrt{2} \end{aligned}$$
- 18) $\sqrt[3]{81} + 3\sqrt[3]{24}$
- $$\sqrt[3]{81} + 3\sqrt[3]{24} = \sqrt[3]{(27)(3)} + 3\sqrt[3]{(8)(3)} = \sqrt[3]{27}\sqrt[3]{3} + 3\sqrt[3]{8}\sqrt[3]{3} = (3)\sqrt[3]{3} + 3(2)\sqrt[3]{3} = 3\sqrt[3]{3} + 6\sqrt[3]{3} = 9\sqrt[3]{3}$$
- 20) $2\sqrt[3]{x^8y^6} - 3y^2\sqrt[3]{8x^8}$
- $$\begin{aligned} 2\sqrt[3]{x^8y^6} - 3y^2\sqrt[3]{8x^8} &= 2\sqrt[3]{(x^6y^6)(x^2)} - 3y^2\sqrt[3]{(8x^6)(x^2)} = 2\sqrt[3]{x^6y^6}\sqrt[3]{x^2} - 3y^2\sqrt[3]{8x^6}\sqrt[3]{x^2} \\ &= 2(x^2y^2)\sqrt[3]{x^2} - 3y^2(2x^2)\sqrt[3]{x^2} = 2x^2y^2\sqrt[3]{x^2} - 6x^2y^2\sqrt[3]{x^2} = -4x^2y^2\sqrt[3]{x^2} \end{aligned}$$
- 22) $9a\sqrt{20a^3b^2} + 7b\sqrt{45a^5}$
- $$\begin{aligned} 9a\sqrt{20a^3b^2} + 7b\sqrt{45a^5} &= 9a\sqrt{(4a^2b^2)(5a)} + 7b\sqrt{(9a^4)(5a)} = 9a\sqrt{4a^2b^2}\sqrt{5a} + 7b\sqrt{9a^4}\sqrt{5a} \\ &= 9a(2ab)\sqrt{5a} + 7b(3a^2)\sqrt{5a} = 18a^2b\sqrt{5a} + 21a^2b\sqrt{5a} = 39a^2b\sqrt{5a} \end{aligned}$$
- 24) $7\sqrt[3]{a^4b^3c^2} - 6ab\sqrt[3]{ac^2}$
- $$\begin{aligned} 7\sqrt[3]{a^4b^3c^2} - 6ab\sqrt[3]{ac^2} &= 7\sqrt[3]{(a^3b^3)(ac^2)} - 6ab\sqrt[3]{ac^2} = 7\sqrt[3]{a^3b^3}\sqrt[3]{ac^2} - 6ab\sqrt[3]{ac^2} \\ &= 7(ab)\sqrt[3]{ac^2} - 6ab\sqrt[3]{ac^2} = 7ab\sqrt[3]{ac^2} - 6ab\sqrt[3]{ac^2} = 1ab\sqrt[3]{ac^2} = ab\sqrt[3]{ac^2} \end{aligned}$$
- 26) $x\sqrt[4]{5xy^8} + y\sqrt[4]{405x^5y^4} + y^2\sqrt[4]{80x^5}$
- $$\begin{aligned} x\sqrt[4]{5xy^8} + y\sqrt[4]{405x^5y^4} + y^2\sqrt[4]{80x^5} &= x\sqrt[4]{(y^8)(5x)} + y\sqrt[4]{(81x^4y^4)(5x)} + y^2\sqrt[4]{(16x^4)(5x)} \\ &= x\sqrt[4]{y^8}\sqrt[4]{5x} + y\sqrt[4]{81x^4y^4}\sqrt[4]{5x} + y^2\sqrt[4]{16x^4}\sqrt[4]{5x} \\ &= x(y^2)\sqrt[4]{5x} + y(3xy)\sqrt[4]{5x} + y^2(2x)\sqrt[4]{5x} \\ &= xy^2\sqrt[4]{5x} + 3xy^2\sqrt[4]{5x} + 2xy^2\sqrt[4]{5x} = 6xy^2\sqrt[4]{5x} \end{aligned}$$
- 28) $\frac{\sqrt{3}}{3} + \frac{1}{\sqrt{3}}$
- $$\frac{\sqrt{3}}{3} + \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} + \frac{1}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right) = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$$

$$30) \quad \frac{\sqrt{6}}{2} + \frac{1}{\sqrt{6}}$$

LCD = 6

$$\frac{\sqrt{6}}{2} + \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{2} + \frac{1}{\sqrt{6}} \left(\frac{\sqrt{6}}{\sqrt{6}} \right) = \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{6} = \frac{3\sqrt{6} + \sqrt{6}}{6} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

$$32) \quad \sqrt{x} + \frac{1}{\sqrt{x}}$$

LCD = x

$$\sqrt{x} + \frac{1}{\sqrt{x}} = \frac{\sqrt{x}}{1} + \frac{1}{\sqrt{x}} \left(\frac{\sqrt{x}}{\sqrt{x}} \right) = \frac{\sqrt{x}}{1} + \frac{\sqrt{x}}{x} = \frac{x\sqrt{x} + \sqrt{x}}{x} = \frac{\sqrt{x}(x+1)}{x} = \frac{(x+1)\sqrt{x}}{x}$$

$$34) \quad \frac{\sqrt{12}}{6} + \sqrt{\frac{1}{3}} + \frac{\sqrt{3}}{3}$$

$$\begin{aligned} \frac{\sqrt{12}}{6} + \sqrt{\frac{1}{3}} + \frac{\sqrt{3}}{3} &= \frac{\sqrt{(4)(3)}}{6} + \frac{\sqrt{1}}{\sqrt{3}} + \frac{\sqrt{3}}{3} = \frac{\sqrt{4}\sqrt{3}}{6} + \frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{6} + \frac{1}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) + \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} + \frac{\sqrt{3}}{3} \\ &= \frac{3\sqrt{3}}{3} = \frac{\sqrt{3}}{1} = \sqrt{3} \end{aligned}$$

$$36) \quad \sqrt{15} - \sqrt{\frac{3}{5}} + \sqrt{\frac{5}{3}}$$

LCD = 15

$$\begin{aligned} \sqrt{15} - \sqrt{\frac{3}{5}} + \sqrt{\frac{5}{3}} &= \frac{\sqrt{15}}{1} - \frac{\sqrt{3}}{\sqrt{5}} + \frac{\sqrt{5}}{\sqrt{3}} = \frac{\sqrt{15}}{1} - \frac{\sqrt{3}}{\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right) + \frac{\sqrt{5}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{15}}{1} - \frac{\sqrt{3}\sqrt{5}}{5} + \frac{\sqrt{5}\sqrt{3}}{3} \\ &= \frac{\sqrt{15}}{1} - \frac{\sqrt{15}}{5} + \frac{\sqrt{15}}{3} = \frac{15\sqrt{15} - 3\sqrt{15} + 5\sqrt{15}}{15} = \frac{17\sqrt{15}}{15} \end{aligned}$$

$$38) \quad \sqrt[4]{8} + \frac{1}{\sqrt[4]{2}}$$

Method 1:

LCD = 2

$$\sqrt[4]{8} + \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{1} + \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{8}}{1} + \frac{1}{\sqrt[4]{(2)^1}} \left(\frac{\sqrt[4]{(2)^3}}{\sqrt[4]{(2)^3}} \right) = \frac{\sqrt[4]{8}}{1} + \frac{\sqrt[4]{(2)^3}}{2} = \frac{\sqrt[4]{8}}{1} + \frac{\sqrt[4]{8}}{2} = \frac{2\sqrt[4]{8} + \sqrt[4]{8}}{2} = \frac{3\sqrt[4]{8}}{2}$$

Method 2 (keeping radicals in the denominator)

$$\sqrt[4]{8} + \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{(2)^3}}{1} + \frac{1}{\sqrt[4]{2}} = \frac{\sqrt[4]{(2)^3}}{1} \left(\frac{\sqrt[4]{(2)^1}}{\sqrt[4]{(2)^1}} \right) + \frac{1}{\sqrt[4]{2}} = \frac{2}{\sqrt[4]{(2)^1}} + \frac{1}{\sqrt[4]{2}} = \frac{2}{\sqrt[4]{2}} + \frac{1}{\sqrt[4]{2}} = \frac{3}{\sqrt[4]{2}}$$