Note: You are responsible for word problems of type: Number Problems and Rate Problems: This criteria is shown in examples 1, 2, and 3 on your text.

On a typical rate problem, you will need to set up 2 equations with 2 unknowns of the form:

$\text{distance} = (\text{rate})(\text{time})$ or $\text{distance} = \text{rate} \times \text{time}$

Additional examples

2) Let x be a number. Let $3x$ be another number.

$\frac{1}{x}+\frac{1}{3x}=\frac{4}{9}$

$$\frac{1}{x}+\frac{1}{3x}=\frac{4}{9}$$

$$\left(\frac{9x}{1}\right)\left(\frac{1}{x}+\frac{1}{3x}\right)=\left(\frac{4}{9}\right)\left(\frac{9x}{1}\right)$$

$LCD = 9x$

$$9+3=4x$$
$$12=4x$$
$$3=x$$

The numbers are $x=3$ and $3x=3(3)=9$.

4) Let x be a number.

$x+2\left(\frac{1}{x}\right)=\frac{27}{5} \quad \Rightarrow \quad x+\frac{2}{x}=\frac{27}{5}$

$$x+\frac{2}{x}=\frac{27}{5}$$

$LCD = 5x$

$$\left(\frac{5x}{1}\right)\left(\frac{x}{1}+\frac{2}{x}\right)=\left(\frac{27}{5}\right)\left(\frac{5x}{1}\right) \Rightarrow$$

$$5x^2+10=27x$$
$$5x^2-27x+10=0$$

$$(5x-2)(x-5)=0$$

$5x-2=0$, $5x=2$, $x=\frac{2}{5}$

$x-5=0$, $x=5$

The numbers are $\frac{2}{5}, 5$.

6) Let x be 1st even integer. Let $(x+2)$ be 2nd even integer.

$\frac{1}{x}+\frac{1}{(x+2)}=\frac{3}{4}$

$$\frac{1}{x}+\frac{1}{(x+2)}=\frac{3}{4}$$

$LCD = 4x(x+2)$

$$\left(\frac{4x(x+2)}{1}\right)\left(\frac{1}{x}+\frac{1}{(x+2)}\right)=\left(\frac{3}{4}\right)\left(\frac{4x(x+2)}{1}\right)$$

$$4(x+2)+4x=3x(x+2)$$
$$4x+8+4x=3x^2+6x$$
$$8x+8=3x^2+6x$$
$$0=3x^2-6x-8$$

$\Rightarrow$

$$0=(3x+4)(x-2)$$

$3x+4=0$, $3x=-4$, $x=\frac{-4}{3}$ *discard*

$x-2=0$, $x=2$

The 2 consecutive even integers are 2 and 4.

8) Let x be a number added to both numerator and denominator.

$$\frac{8+x}{11+x}=\frac{6}{7} \quad \Rightarrow \quad \frac{(x+8)}{(x+11)}=\frac{6}{7}$$

$$\frac{(x+8)}{(x+11)}=\frac{6}{7}$$

$$\left(\frac{7(x+11)}{1}\right)\left(\frac{(x+8)}{(x+11)}\right)=\left(\frac{6}{7}\right)\left(\frac{7(x+11)}{1}\right)$$

$LCD=7(x+11)$

$$7(x+8)=6(x+11)$$

$$7x+56=6x+66$$

$$x=10$$

The number is 10.

10) Let x be the speed of the current
Let t be the time the boat traveled (both downstream and upstream)
The speed of the boat in still water is 18 miles per hour.

	Distance	Rate	Time	Equation
Downstream	14 miles	$(18+x)$ MPH	t hours	$14=(18+x)t$
Upstream	10 miles	$(18-x)$ MPH	t hours	$10=(18-x)t$

$$\left.\begin{aligned} 14&=(18+x)t \\ 10=(18-x)t \quad \Rightarrow \quad \frac{10}{(18-x)}&=t \end{aligned}\right\} \Rightarrow \quad \begin{aligned} 14&=(18+x)t \\ 14&=(18+x)\left(\frac{10}{(18-x)}\right) \\ 14&=\frac{10(18+x)}{(18-x)} \end{aligned}$$

$$14=\frac{10(18+x)}{(18-x)}$$

$LCD=(18-x)$

$$\left(\frac{(18-x)}{1}\right)\left(\frac{14}{1}\right)=\left(\frac{10(18+x)}{(18-x)}\right)\left(\frac{(18-x)}{1}\right)$$

$$14(18-x)=10(18+x)$$

$$252-14x=180+10x$$

$$72=24x$$

$$3=x$$

The speed of the current is 3 miles per hour.

12) Let x be the speed of the current
Let t be the time the boat traveled upstream.
Let $(8-t)$ be the time the boat traveled downstream.
The speed of the boat in still water is 4 miles per hour.

	Distance	Rate	Time	Equation
Upstream	12 miles	$(4+x)$ MPH	t hours	$12=(4+x)t$
Downstream	12 miles	$(4-x)$ MPH	$(8-t)$ hours	$12=(4-x)(8-t)$

$$\left.\begin{array}{l} 12=(4+x)t \quad \Rightarrow \quad \dfrac{12}{(4+x)}=t \\ 12=(4-x)(8-t) \quad \Rightarrow \quad \begin{array}{l} 12=32-8x-4t+xt \\ 0=20-8x-4t+xt \end{array} \end{array}\right\} \Rightarrow \begin{array}{l} 0=20-8x-4t+xt \\ 0=20-8x-4\left(\dfrac{12}{(4+x)}\right)+x\left(\dfrac{12}{(4+x)}\right) \\ 0=20-8x-\dfrac{48}{(4+x)}+\dfrac{12x}{(4+x)} \end{array}$$

$$0=20-8x-\frac{48}{(4+x)}+\frac{12x}{(4+x)}$$

$$LCD=(4+x) \qquad \left(\frac{(4+x)}{1}\right)\left(\frac{0}{1}\right)=\left(\frac{20}{1}-\frac{8x}{1}-\frac{48}{(4+x)}+\frac{12x}{(4+x)}\right)\left(\frac{(4+x)}{1}\right)$$

$$0=20(4+x)-8x(4+x)-48+12x$$

$$0=80+20x-32x-8x^2-48+12x$$

$$0=32-8x^2$$

$$8x^2-32=0$$

$$\Rightarrow \quad 8(x^2-4)=0$$

$$8(x+2)(x-2)=0$$

$$x+2=0 \qquad x-2=0$$

$$x=-2 \qquad x=2$$

$$discard$$

The speed of the current is 2 miles per hour.

14) Let x be the speed of the car
Let $(x+30)$ be the speed of the train
Let t be the time traveled (both car and train) and train traveled 120 miles and car 80 miles.

	Distance	Rate	Time	Equation
Train	120 miles	$(x+30)$ MPH	t hours	$120=(x+30)t$
Car	80 miles	x MPH	t hours	$80=xt$

$$\left.\begin{array}{l} 120=(x+30)t \\ 80=xt \quad \Rightarrow \quad \dfrac{80}{x}=t \end{array}\right\} \Rightarrow \begin{array}{l} 120=(x+30)t \\ 120=(x+30)\left(\dfrac{80}{x}\right) \\ 120=\dfrac{80(x+30)}{x} \end{array}$$

$$120=\frac{80(x+30)}{x}$$

$$LCD=x \qquad \left(\frac{x}{1}\right)\left(\frac{120}{1}\right)=\left(\frac{80(x+30)}{x}\right)\left(\frac{x}{1}\right)$$

$$120x=80(x+30)$$

$$120x=80x+2400$$

$$40x=2400$$

$$x=60$$

$$(x+30)=(60)+30=90$$

The speed of the cat is 60 miles per hour and the speed of the train is 90 miles per hour.

16) The speed of the bicycle is 20 miles per hour. Both bicycle and car is traveling 30 miles. Since we are measuring time in hours $15 \text{ minutes} = \frac{1}{4} \text{ hour}$.

Let x be the speed of the car
Let t be the time traveled by car
The bicycle left 15 minutes before the car, so let $\left(t+\frac{1}{4}\right)$ be the time traveled by bicycle

	Distance	Rate	Time	Equation
Bicycle	30 miles	20 MPH	$\left(t+\frac{1}{4}\right)$ hours	$30 = 20\left(t+\frac{1}{4}\right)$
Car	30 miles	x MPH	t hours	$30 = xt$

$$\left.\begin{aligned} 30 = 20\left(t+\frac{1}{4}\right) &\Rightarrow 30 = 20t+5 \\ 30 = xt &\Rightarrow \frac{30}{x} = t \end{aligned}\right\} \Rightarrow \begin{aligned} 30 &= 20t+5 \\ 30 &= 20\left(\frac{30}{x}\right)+5 \\ 25 &= \frac{600}{x} \end{aligned}$$

$$25 = \frac{600}{x}$$

$$LCD = x \quad \left(\frac{x}{1}\right)\left(\frac{25}{1}\right) = \left(\frac{600}{x}\right)\left(\frac{x}{1}\right)$$

$$25x = 600$$

$$x = 24$$

The speed of the car is 24 miles per hour.

18) The bakery route is 140 miles either on-time or late. Since we are measuring time in hours $30 \text{ minutes} = \frac{1}{2} \text{ hour}$.

Let x be the speed of the truck when the driver leaves on-time
Let t be the time traveled truck when the driver leaves on-time
Let $(x+5)$ be the speed of the truck when the driver is late 30 minutes
The driver is late 30 minutes, so let $\left(t-\frac{1}{2}\right)$ be the time when the driver is late 30 minutes

	Distance	Rate	Time	Equation
Late 30 minutes	140 miles	$(x+5)$ MPH	$\left(t-\frac{1}{2}\right)$ hours	$140 = (x+5)\left(t-\frac{1}{2}\right)$
On-time	140 miles	x MPH	t hours	$140 = xt$

$$\left.\begin{array}{l} 140=(x+5)\left(t-\dfrac{1}{2}\right) \quad \Rightarrow \quad 140=xt+5t-\dfrac{x}{2}-\dfrac{5}{2} \\ 140=xt \quad \Rightarrow \quad \dfrac{140}{x}=t \end{array}\right\} \Rightarrow \begin{array}{l} 140=xt+5t-\dfrac{x}{2}-\dfrac{5}{2} \\ 140=x\left(\dfrac{140}{x}\right)+5\left(\dfrac{140}{x}\right)-\dfrac{x}{2}-\dfrac{5}{2} \\ 140=140+\dfrac{5(140)}{x}-\dfrac{x}{2}-\dfrac{5}{2} \\ 0=\dfrac{700}{x}-\dfrac{x}{2}-\dfrac{5}{2} \end{array}$$

$$0=\frac{700}{x}-\frac{x}{2}-\frac{5}{2}$$

$$\left(\frac{2x}{1}\right)\left(\frac{0}{1}\right)=\left(\frac{700}{x}-\frac{x}{2}-\frac{5}{2}\right)\left(\frac{2x}{1}\right)$$

$LCD=2x$

$$0=2(700)-x(x)-5(x)$$

$$0=1400-x^2-5x$$

$$x^2+5x-1400=0$$

$$(x+40)(x-35)=0$$

$$x+40=0 \qquad x-35=0$$

$$x=-40 \qquad x=35$$

discard

The speed of the driver when he is on-time is 35 miles per hour.