

Note: Since we are working with equality, the idea is not to work with fractions as much as possible. Additionally we need to check our solutions in the original expression to ensure that solutions exist. This criteria is shown in examples 2, 3, 4, 5, and 6 on your text.

Additional examples

$$2) \quad \frac{x}{5} = \frac{x}{2} - 9$$

$$\frac{x}{5} = \frac{x}{2} - \frac{9}{1}$$

$$\left(\frac{10}{1}\right)\left(\frac{x}{5}\right) = \left(\frac{x}{2} - \frac{9}{1}\right)\left(\frac{10}{1}\right)$$

$$LCD = 10 \quad 2x = 5x - 90$$

$$90 = 3x$$

$$30 = x$$

$$4) \quad \frac{a}{4} + \frac{1}{2} = \frac{2}{3}$$

$$\frac{a}{4} + \frac{1}{2} = \frac{2}{3}$$

$$\left(\frac{12}{1}\right)\left(\frac{a}{4} + \frac{1}{2}\right) = \left(\frac{2}{3}\right)\left(\frac{12}{1}\right)$$

$$LCD = 12 \quad 3a + 6 = 8$$

$$3a = 2$$

$$a = \frac{2}{3}$$

$$6) \quad \frac{y}{3} - \frac{y}{6} + \frac{y}{2} = 1$$

$$\frac{y}{3} - \frac{y}{6} + \frac{y}{2} = \frac{1}{1}$$

$$\left(\frac{6}{1}\right)\left(\frac{y}{3} - \frac{y}{6} + \frac{y}{2}\right) = \left(\frac{1}{1}\right)\left(\frac{6}{1}\right)$$

$$LCD = 6 \quad 2y - y + 3y = 6$$

$$4y = 6$$

$$y = \frac{6}{4} = \frac{3}{2}$$

Note: The examples 8 to the end, we need to check our answer because the original expression contains a variable in the denominator.

$$8) \quad \frac{1}{2a} = \frac{2}{a} - \frac{3}{8}$$

$$\frac{1}{2a} = \frac{2}{a} - \frac{3}{8}$$

$$\left(\frac{8a}{1}\right)\left(\frac{1}{2a}\right) = \left(\frac{2}{a} - \frac{3}{8}\right)\left(\frac{8a}{1}\right)$$

$$LCD = 8a$$

$$4 = 16 - 3a$$

$$3a = 12$$

$$a = 4$$

$$10) \quad \frac{5}{2x} = \frac{2}{x} - \frac{1}{12}$$

$$\frac{5}{2x} = \frac{2}{x} - \frac{1}{12}$$

$$LCD = 12x \quad \left(\frac{12x}{1}\right)\left(\frac{5}{2x}\right) = \left(\frac{2}{x} - \frac{1}{12}\right)\left(\frac{12x}{1}\right)$$

$$30 = 24 - x$$

$$x = -6$$

$$12) \quad \frac{2}{x+5} = \frac{2}{5} - \frac{x}{x+5}$$

$$\frac{2}{(x+5)} = \frac{2}{5} - \frac{x}{(x+5)}$$

$$\left(\frac{5(x+5)}{1}\right)\left(\frac{2}{(x+5)}\right) = \left(\frac{2}{5} - \frac{x}{(x+5)}\right)\left(\frac{5(x+5)}{1}\right)$$

$$10 = 2(x+5) - 5x$$

$$LCD = 5(x+5)$$

$$10 = 2x + 10 - 5x$$

$$10 = 10 - 3x$$

$$3x = 0$$

$$x = 0$$

$$14) \quad 2 + \frac{5}{x} = \frac{3}{x^2}$$

$$\frac{2}{1} + \frac{5}{x} = \frac{3}{x^2}$$

$$LCD = x^2 \quad \left(\frac{x^2}{1}\right)\left(\frac{2}{1} + \frac{5}{x}\right) = \left(\frac{3}{x^2}\right)\left(\frac{x^2}{1}\right) \Rightarrow \begin{array}{l} x+3=0 \\ x=-3 \end{array} \quad \begin{array}{l} (x+3)(2x-1)=0 \\ 2x-1=0 \\ 2x=1 \\ x=\frac{1}{2} \end{array}$$

$$2x^2 + 5x = 3$$

$$2x^2 + 5x - 3 = 0$$

$$16) \quad \frac{y}{2} - \frac{4}{y} = -\frac{7}{2}$$

$$\frac{y}{2} - \frac{4}{y} = \frac{-7}{2}$$

$$LCD = 2y \quad \left(\frac{2y}{1}\right)\left(\frac{y}{2} - \frac{4}{y}\right) = \left(\frac{-7}{2}\right)\left(\frac{2y}{1}\right) \Rightarrow \begin{array}{l} (y+8)(y-1) = 0 \\ y+8=0 \quad y-1=0 \\ y=-8 \quad y=1 \\ y^2 - 8 = -7y \\ y^2 + 7y - 8 = 0 \end{array}$$

$$18) \quad \frac{x+6}{x+3} = \frac{3}{x+3} + 2$$

$$\frac{(x+6)}{(x+3)} = \frac{3}{(x+3)} + \frac{2}{1}$$

$$LCD = (x+3) \quad \left(\frac{(x+3)}{1}\right)\left(\frac{(x+6)}{(x+3)}\right) = \left(\frac{3}{(x+3)} + \frac{2}{1}\right)\left(\frac{(x+3)}{1}\right) \Rightarrow \begin{array}{l} x+6 = 2x+9 \\ -3 = x \\ \text{discard} \\ (x+6) = 3 + 2(x+3) \\ x+6 = 3 + 2x+6 \end{array}$$

This equation has no solution because if we replace x by -3 we get division by zero.

$$20) \quad \frac{5}{a+1} = \frac{4}{a+2}$$

$$\frac{5}{(a+1)} = \frac{4}{(a+2)}$$

$$\left(\frac{(a+2)(a+1)}{1}\right)\left(\frac{5}{(a+1)}\right) = \left(\frac{4}{(a+2)}\right)\left(\frac{(a+2)(a+1)}{1}\right)$$

$$LCD = (a+2)(a+1) \quad \begin{array}{l} 5(a+2) = 4(a+1) \\ 5a+10 = 4a+4 \\ a = -6 \end{array}$$

$$22) \quad 10 - \frac{3}{x^2} = -\frac{1}{x}$$

$$\frac{10}{1} - \frac{3}{x^2} = \frac{-1}{x}$$

$$LCD = x^2 \quad \left(\frac{x^2}{1}\right)\left(\frac{10}{1} - \frac{3}{x^2}\right) = \left(\frac{-1}{x}\right)\left(\frac{x^2}{1}\right) \Rightarrow \begin{array}{l} (5x+3)(2x-1) = 0 \\ 5x+3=0 \quad 2x-1=0 \\ 5x=-3 \quad 2x=1 \\ x = \frac{-3}{5} \quad x = \frac{1}{2} \\ 10x^2 - 3 = -x \\ 10x^2 + x - 3 = 0 \end{array}$$

$$24) \quad \frac{5}{x-1} + \frac{2}{x-1} = \frac{4}{x+1}$$

$$\frac{5}{(x-1)} + \frac{2}{(x-1)} = \frac{4}{(x+1)}$$

$$\left(\frac{(x+1)(x-1)}{1}\right)\left(\frac{5}{(x-1)} + \frac{2}{(x-1)}\right) = \left(\frac{4}{(x+1)}\right)\left(\frac{(x+1)(x-1)}{1}\right)$$

$$5(x+1) + 2(x+1) = 4(x-1)$$

$$LCD = (x+1)(x-1)$$

$$5x+5+2x+2=4x-4$$

$$7x+7=4x-4$$

$$3x=-11$$

$$x = \frac{-11}{3}$$

$$26) \quad \frac{2}{x+5} + \frac{3}{x+4} = \frac{2x}{x^2+9x+20}$$

$$\frac{2}{(x+5)} + \frac{3}{(x+4)} = \frac{2x}{(x+5)(x+4)}$$

$$\left(\frac{(x+5)(x+4)}{1}\right)\left(\frac{2}{(x+5)} + \frac{3}{(x+4)}\right) = \left(\frac{2x}{(x+5)(x+4)}\right)\left(\frac{(x+5)(x+4)}{1}\right)$$

$$2(x+4) + 3(x+5) = 2x$$

$$LCD = (x+5)(x+4)$$

$$2x+8+3x+15=2x$$

$$5x+23=2x$$

$$3x=-23$$

$$x = \frac{-23}{3}$$

$$28) \quad \frac{2}{x} - \frac{1}{x+1} = \frac{-2}{5x+5}$$

$$\frac{2}{x} - \frac{1}{(x+1)} = \frac{-2}{5(x+1)}$$

$$\left(\frac{5x(x+1)}{1}\right)\left(\frac{2}{x} - \frac{1}{(x+1)}\right) = \left(\frac{-2}{5(x+1)}\right)\left(\frac{5x(x+1)}{1}\right)$$

$$2(5(x+1)) - 1(5x) = -2(x)$$

$$LCD = 5x(x+1)$$

$$10x+10-5x=-2x$$

$$5x+10=-2x$$

$$7x=-10$$

$$x = \frac{-10}{7}$$

$$30) \quad \frac{t+3}{t^2-2t} = \frac{10}{t^2-4}$$

$$\frac{(t+3)}{t(t-2)} = \frac{10}{(t+2)(t-2)}$$

$$\left(\frac{t(t+2)(t-2)}{1}\right)\left(\frac{(t+3)}{t(t-2)}\right) = \left(\frac{10}{(t+2)(t-2)}\right)\left(\frac{t(t+2)(t-2)}{1}\right)$$

$$(t+2)(t+3) = 10(t)$$

$$LCD = t(t+2)(t-2)$$

$$t^2 + 5t + 6 = 10t$$

$$t^2 - 5t + 6 = 0$$

$$(t-2)(t-3) = 0$$

$$t-2=0 \quad t-3=0$$

$$t=2 \quad t=3$$

discard

The solution of $t=2$ does not satisfy the original statement.

$$32) \quad \frac{1}{y+2} - \frac{2}{y-3} = \frac{-2y}{y^2-y-6}$$

$$\frac{1}{(y+2)} - \frac{2}{(y-3)} = \frac{-2y}{(y+2)(y-3)}$$

$$LCD = (y+2)(y-3) \quad \left(\frac{(y+2)(y-3)}{1}\right)\left(\frac{1}{(y+2)} - \frac{2}{(y-3)}\right) = \left(\frac{-2y}{(y+2)(y-3)}\right)\left(\frac{(y+2)(y-3)}{1}\right)$$

$$1(y-3) - 2(y+2) = -2y$$

$$y-3-2y-4 = -2y$$

$$-y-7 = -2y$$

$$y=7$$

$$34) \quad \frac{1}{a+3} - \frac{a}{a^2-9} = \frac{2}{3-a}$$

$$\frac{1}{(a+3)} - \frac{a}{(a+3)(a-3)} = \frac{2}{-(a-3)}$$

$$\left(\frac{(a+3)(a-3)}{1}\right)\left(\frac{1}{(a+3)} - \frac{a}{(a+3)(a-3)}\right) = \left(\frac{-2}{(a-3)}\right)\left(\frac{(a+3)(a-3)}{1}\right)$$

$$(a-3) - a = -2(a+3)$$

$$LCD = (a+3)(a-3)$$

$$a-3-a = -2a-6$$

$$-3 = -2a-6$$

$$2a = -3$$

$$a = \frac{-3}{2}$$

$$36) \quad \frac{2x-3}{5x+10} + \frac{3x-2}{4x+8} = 1$$

$$\frac{(2x-3)}{5(x+2)} + \frac{(3x-2)}{4(x+2)} = \frac{1}{1}$$

$$\left(\frac{20(x+2)}{1}\right)\left(\frac{(2x-3)}{5(x+2)} + \frac{(3x-2)}{4(x+2)}\right) = \left(\frac{1}{1}\right)\left(\frac{20(x+2)}{1}\right)$$

$$4(2x-3) + 5(3x-2) = 20(x+2)$$

$$LCD = 20(x+2)$$

$$8x - 12 + 15x - 10 = 20x + 40$$

$$23x - 22 = 20x + 40$$

$$3x = 62$$

$$x = \frac{62}{3}$$

$$38) \quad \frac{y+3}{y^2-y} - \frac{8}{y^2-1} = 0$$

$$\frac{(y+3)}{y(y-1)} - \frac{8}{(y+1)(y-1)} = 0$$

$$\left(\frac{y(y+1)(y-1)}{1}\right)\left(\frac{(y+3)}{y(y-1)} - \frac{8}{(y+1)(y-1)}\right) = \left(\frac{0}{1}\right)\left(\frac{y(y+1)(y-1)}{1}\right)$$

$$(y+1)(y+3) - 8(y) = 0$$

$$LCD = y(y+1)(y-1)$$

$$(y^2 + 4y + 3) - 8y = 0$$

$$y^2 + 4y + 3 - 8y = 0$$

$$y^2 - 4y + 3 = 0$$

$$(y-1)(y-3) = 0$$

$$y-1=0 \quad y-3=0$$

$$y=1 \quad y=3$$

discard

The solution of $y=1$ does not satisfy the original statement.

$$40) \quad \frac{1}{x+2} + \frac{1}{x-2} = \frac{4}{x^2-4}$$

$$\frac{1}{(x+2)} + \frac{1}{(x-2)} = \frac{4}{(x+2)(x-2)}$$

$$\left(\frac{(x+2)(x-2)}{1}\right)\left(\frac{1}{(x+2)} + \frac{1}{(x-2)}\right) = \left(\frac{4}{(x+2)(x-2)}\right)\left(\frac{(x+2)(x-2)}{1}\right)$$

$$LCD = (x+2)(x-2)$$

$$(x-2) + (x+2) = 4$$

$$x-2+x+2=4$$

$$2x=4$$

$$x=2$$

discard

The solution of $x=2$ does not satisfy the original statement. Therefore, this equation has no solution.

$$42) \quad \frac{1}{y^2+5y+4} + \frac{3}{y^2-1} = \frac{-1}{y^2+3y-4}$$

$$LCD = (y+4)(y+1)(y-1)$$

$$\frac{1}{(y+4)(y+1)} + \frac{3}{(y+1)(y-1)} = \frac{-1}{(y+4)(y-1)}$$

$$\left(\frac{(y+4)(y+1)(y-1)}{1}\right)\left(\frac{1}{(y+4)(y+1)} + \frac{3}{(y+1)(y-1)}\right) = \left(\frac{-1}{(y+4)(y-1)}\right)\left(\frac{(y+4)(y+1)(y-1)}{1}\right)$$

$$(y-1) + 3(y+4) = -1(y+1)$$

$$y-1+3y+12 = -y-1$$

$$4y+11 = -y-1$$

$$5y = -12$$

$$y = \frac{-12}{5}$$

$$44) \quad 3x^{-1} - 5 = 2x^{-1} - 3$$

$$3x^{-1} - 5 = 2x^{-1} - 3$$

$$\frac{3}{x} - \frac{5}{1} = \frac{2}{x} - \frac{3}{1}$$

$$LCD = x \quad \left(\frac{x}{1}\right)\left(\frac{3}{x} - \frac{5}{1}\right) = \left(\frac{2}{x} - \frac{3}{1}\right)\left(\frac{x}{1}\right)$$

$$3-5x=2-3x$$

$$1=2x$$

$$\frac{1}{2} = x$$

46) $1 + 3x^{-2} = 4x^{-1}$

$$1 + 3x^{-2} = 4x^{-1}$$

$$\frac{1}{1} + \frac{3}{x^2} = \frac{4}{x}$$

$$LCD = x^2 \quad \left(\frac{x^2}{1}\right)\left(\frac{1}{1} + \frac{3}{x^2}\right) = \left(\frac{4}{x}\right)\left(\frac{x^2}{1}\right) \Rightarrow \begin{array}{l} (x-1)(x-3) = 0 \\ x-1 = 0 \quad x-3 = 0 \\ x = 1 \quad x = 3 \end{array}$$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

48) Solve $\frac{1}{x} = \frac{1}{a} - \frac{1}{b}$ for x .

$$\frac{1}{x} = \frac{1}{a} - \frac{1}{b}$$

$$\left(\frac{abx}{1}\right)\left(\frac{1}{x}\right) = \left(\frac{1}{a} - \frac{1}{b}\right)\left(\frac{abx}{1}\right)$$

$$LCD = abx$$

$$ab = bx - ax$$

$$ab = x(b - a)$$

$$\frac{ab}{(b-a)} = x$$

$$x = \frac{ab}{b-a}$$

50) $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\left(\frac{RR_1R_2R_3}{1}\right)\left(\frac{1}{R}\right) = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{RR_1R_2R_3}{1}\right)$$

$$LCD = RR_1R_2R_3$$

$$R_1R_2R_3 = RR_2R_3 + RR_1R_3 + RR_1R_2$$

$$R_1R_2R_3 = R(R_2R_3 + R_1R_3 + R_1R_2)$$

$$\frac{R_1R_2R_3}{(R_2R_3 + R_1R_3 + R_1R_2)} = R$$

$$R = \frac{R_1R_2R_3}{R_2R_3 + R_1R_3 + R_1R_2}$$