

Note: A standard Complex Fraction is a rational expression where at least one numerator or denominator is a fraction. The most reliable method is the method 1 shown in examples 1, 2, and 4 of the text. I modified the writing so that less confusing and I'm calling the *LCD* of complex fraction *GLCD*.

Additional examples

$$2) \quad \frac{5}{\frac{9}{\frac{7}{12}}} \quad GLCD = 36$$

$$\frac{5}{\frac{9}{\frac{7}{12}}} = \left(\begin{array}{c} \frac{5}{9} \\ \frac{7}{12} \end{array} \right) \left(\begin{array}{c} 36 \\ \frac{1}{36} \\ 1 \end{array} \right) = \frac{5(36)}{7(36)} = \frac{5(4)}{7(3)} \frac{20}{21}$$

$$4) \quad \frac{\frac{1}{6} - \frac{1}{3}}{\frac{1}{4} - \frac{1}{8}} \quad GLCD = 24$$

$$\frac{\frac{1}{6} - \frac{1}{3}}{\frac{1}{4} - \frac{1}{8}} = \left(\begin{array}{c} \frac{1}{6} - \frac{1}{3} \\ \frac{1}{4} - \frac{1}{8} \end{array} \right) \left(\begin{array}{c} 24 \\ \frac{1}{24} \\ 1 \end{array} \right) = \frac{\frac{1(24)}{6} - \frac{1(24)}{3}}{\frac{1(24)}{4} - \frac{1(24)}{8}} = \frac{4-8}{6-3} = \frac{-4}{3}$$

$$6) \quad \frac{2 + \frac{5}{6}}{1 - \frac{7}{8}} \quad GLCD = 24$$

$$\frac{2 + \frac{5}{6}}{1 - \frac{7}{8}} = \left(\begin{array}{c} 2 + \frac{5}{6} \\ 1 - \frac{7}{8} \end{array} \right) \left(\begin{array}{c} 24 \\ \frac{1}{24} \\ 1 \end{array} \right) = \frac{\frac{2(24)}{1} + \frac{5(24)}{6}}{\frac{1(24)}{1} - \frac{7(24)}{8}} = \frac{2(24) + 5(4)}{1(24) - 7(3)} = \frac{48 + 20}{24 - 21} = \frac{68}{3}$$

$$8) \quad \frac{1 - \frac{1}{x}}{\frac{1}{1 - \frac{1}{x}}} \quad GLCD = x$$

$$\frac{1 - \frac{1}{x}}{\frac{1}{1 - \frac{1}{x}}} = \left(\frac{1 - \frac{1}{x}}{\frac{1}{1 - \frac{1}{x}}} \right) \left(\frac{x}{\frac{1}{1 - \frac{1}{x}}} \right) = \frac{1(x) - \frac{1(x)}{x}}{\frac{1}{1(x)} - \frac{1}{x}} = \frac{x - 1}{1} = x - 1$$

$$10) \quad \frac{1 - \frac{2}{a}}{1 - \frac{3}{a}} \quad GLCD = a$$

$$\frac{1 - \frac{2}{a}}{1 - \frac{3}{a}} = \left(\frac{1 - \frac{2}{a}}{1 - \frac{3}{a}} \right) \left(\frac{a}{\frac{1}{1 - \frac{3}{a}}} \right) = \frac{a - 2}{a - 3}$$

$$12) \quad \frac{\frac{1}{x} + \frac{2}{y}}{\frac{2}{x} + \frac{1}{y}} \quad GLCD = xy$$

$$\frac{\frac{1}{x} + \frac{2}{y}}{\frac{2}{x} + \frac{1}{y}} = \left(\frac{\frac{1}{x} + \frac{2}{y}}{\frac{2}{x} + \frac{1}{y}} \right) \left(\frac{xy}{\frac{1}{xy}} \right) = \frac{y + 2x}{2y + x} = \frac{2x + y}{x + 2y}$$

$$14) \quad \frac{\frac{3x+1}{x^2-49}}{\frac{9x^2-1}{x-7}} \quad GLCD = (x+7)(x-7)$$

$$\begin{aligned} \frac{3x+1}{x^2-49} &= \frac{(3x+1)}{(x+7)(x-7)} = \left(\frac{(3x+1)}{(x+7)(x-7)} \right) \left(\frac{(x+7)(x-7)}{\frac{1}{(x+7)(x-7)}} \right) = \frac{(3x+1)}{(3x+1)(3x-1)(x+7)} \\ &= \frac{(1)}{(1)(3x-1)(x+7)} = \frac{1}{(x+7)(3x-1)} \end{aligned}$$

$$16) \quad \frac{2a}{\frac{3a^2 - 3}{\frac{4a}{6a - 6}}}$$

$$GLCD = 6(a+1)(a-1)$$

$$\begin{aligned} \frac{2a}{\frac{3a^2 - 3}{\frac{4a}{6a - 6}}} &= \frac{2a}{\frac{3(a^2 - 1)}{\frac{4a}{6(a-1)}}} = \frac{2a}{\frac{3(a+1)(a-1)}{\frac{4a}{6(a-1)}}} = \left(\frac{2a}{\frac{3(a+1)(a-1)}{\frac{4a}{6(a-1)}}} \right) \left(\frac{\frac{6(a+1)(a-1)}{1}}{\frac{6(a+1)(a-1)}{1}} \right) = \frac{2a(2)}{4a(a+1)} = \frac{(4a)}{(4a)(a+1)} \\ &= \frac{(1)}{(1)(a+1)} = \frac{1}{(a+1)} = \frac{1}{a+1} \end{aligned}$$

$$18) \quad \frac{4 - \frac{1}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}}$$

$$GLCD = x^2$$

$$\frac{4 - \frac{1}{x^2}}{4 + \frac{4}{x} + \frac{1}{x^2}} = \left(\frac{\frac{4}{1} - \frac{1}{x^2}}{\frac{4}{1} + \frac{4}{x} + \frac{1}{x^2}} \right) \left(\frac{x^2}{\frac{1}{x^2}} \right) = \frac{4x^2 - 1}{4x^2 + 4x + 1} = \frac{(2x+1)(2x-1)}{(2x+1)(2x+1)} = \frac{(1)(2x-1)}{(1)(2x+1)} = \frac{(2x-1)}{(2x+1)} = \frac{2x-1}{2x+1}$$

$$20) \quad \frac{3 + \frac{5}{a} - \frac{2}{a^2}}{3 - \frac{10}{a} + \frac{3}{a^2}}$$

$$GLCD = a^2$$

$$\frac{3 + \frac{5}{a} - \frac{2}{a^2}}{3 - \frac{10}{a} + \frac{3}{a^2}} = \left(\frac{\frac{3}{1} + \frac{5}{a} - \frac{2}{a^2}}{\frac{3}{1} - \frac{10}{a} + \frac{3}{a^2}} \right) \left(\frac{a^2}{\frac{1}{a^2}} \right) = \frac{3a^2 + 5a - 2}{3a^2 - 10a + 3} = \frac{(a+2)(3a-1)}{(3a-1)(a-3)} = \frac{(a+2)(1)}{(1)(a-3)} = \frac{(a+2)}{(a-3)} = \frac{a+2}{a-3}$$

$$22) \quad \frac{3+\frac{5}{x}-\frac{12}{x^2}-\frac{20}{x^3}}{3+\frac{11}{x}+\frac{10}{x^2}}$$

$GLCD = x^3$

$$\begin{aligned} 3+\frac{5}{x}-\frac{12}{x^2}-\frac{20}{x^3} &= \left(\frac{3}{1} + \frac{5}{x} - \frac{12}{x^2} - \frac{20}{x^3} \right) \left(\frac{x^3}{1} \right) \\ 3+\frac{11}{x}+\frac{10}{x^2} &= \left(\frac{3}{1} + \frac{11}{x} + \frac{10}{x^2} \right) \left(\frac{1}{1} \right) \\ &= \frac{3x^3 + 5x^2 - 12x - 20}{3x^3 + 11x^2 + 10x} = \frac{x^2(3x+5) - 4(3x+5)}{x\{3x^2 + 11x + 10\}} \\ &= \frac{(3x+5)\{x^2 - 4\}}{x(x+2)(3x+5)} = \frac{(3x+5)(x+2)(x-2)}{x(x+2)(3x+5)} = \frac{(1)(1)(x-2)}{x(1)(1)} = \frac{(x-2)}{x} = \frac{x-2}{x} \end{aligned}$$

$$24) \quad \frac{1+\frac{1}{x-2}}{1-\frac{1}{x-2}}$$

$GLCD = (x-2)$

$$\begin{aligned} 1+\frac{1}{x-2} &= \left(\frac{1}{1} + \frac{1}{(x-2)} \right) \left(\frac{(x-2)}{1} \right) \\ 1-\frac{1}{x-2} &= \left(\frac{1}{1} - \frac{1}{(x-2)} \right) \left(\frac{1}{(x-2)} \right) \\ &= \frac{(x-2)+1}{(x-2)-1} = \frac{x-2+1}{x-2-1} = \frac{x-1}{x-3} \end{aligned}$$

$$26) \quad \frac{1+\frac{1}{x-2}}{1-\frac{3}{x+2}}$$

$GLCD = (x+2)(x-2)$

$$\begin{aligned} 1+\frac{1}{x-2} &= \left(\frac{1}{1} + \frac{1}{(x-2)} \right) \left(\frac{(x+2)(x-2)}{1} \right) \\ 1-\frac{3}{x+2} &= \left(\frac{1}{1} - \frac{3}{(x+2)} \right) \left(\frac{1}{(x+2)(x-2)} \right) \\ &= \frac{(x+2)(x-2) + (x+2)}{(x+2)(x-2) - 3(x-2)} = \frac{(x^2 - 4) + (x+2)}{(x^2 - 4) - 3(x-2)} = \frac{x^2 - 4 + x + 2}{x^2 - 4 - 3x + 6} \\ &= \frac{x^2 + x - 2}{x^2 - 3x + 2} = \frac{(x+2)(x-1)}{(x-1)(x-2)} = \frac{(x+2)(1)}{(1)(x-2)} = \frac{(x+2)}{(x-2)} = \frac{x+2}{x-2} \end{aligned}$$

$$28) \quad \frac{\frac{1}{a-1}+1}{\frac{1}{a+1}-1} \quad GLCD = (a+1)(a-1)$$

$$\begin{aligned} \frac{\frac{1}{a-1}+1}{\frac{1}{a+1}-1} &= \left(\frac{\frac{1}{(a-1)} + \frac{1}{1}}{\frac{1}{(a+1)} - \frac{1}{1}} \right) \left(\frac{(a+1)(a-1)}{\frac{1}{(a+1)(a-1)}} \right) = \frac{(a+1)+(a+1)(a-1)}{(a-1)-(a+1)(a-1)} = \frac{(a+1)+(a^2-1)}{(a-1)-(a^2-1)} = \frac{a+1+a^2-1}{a-1-a^2+1} \\ &= \frac{a+a^2}{a-a^2} = \frac{a(1+a)}{a(1-a)} = \frac{(a)(1+a)}{(a)(1-a)} = \frac{(1)(1+a)}{(1)(1-a)} = \frac{(1+a)}{(1-a)} = \frac{1+a}{1-a} \end{aligned}$$

$$30) \quad \frac{\frac{1}{x+a} + \frac{1}{x-a}}{\frac{1}{x+a} - \frac{1}{x-a}} \quad GLCD = (x+a)(x-a)$$

$$\begin{aligned} \frac{\frac{1}{x+a} + \frac{1}{x-a}}{\frac{1}{x+a} - \frac{1}{x-a}} &= \left(\frac{\frac{1}{(x+a)} + \frac{1}{(x-a)}}{\frac{1}{(x+a)} - \frac{1}{(x-a)}} \right) \left(\frac{(x+a)(x-a)}{\frac{1}{(x+a)(x-a)}} \right) = \frac{(x-a)+(x+a)}{(x-a)-(x+a)} = \frac{x-a+x+a}{x-a-x-a} = \frac{2x}{-2a} = \frac{-x}{a} \end{aligned}$$

$$32) \quad \frac{\frac{y-1}{y+1} - \frac{y+1}{y-1}}{\frac{y-1}{y+1} + \frac{y+1}{y-1}} \quad GLCD = (y+1)(y-1)$$

$$\begin{aligned} \frac{\frac{y-1}{y+1} - \frac{y+1}{y-1}}{\frac{y-1}{y+1} + \frac{y+1}{y-1}} &= \left(\frac{\frac{(y-1)}{(y+1)} - \frac{(y+1)}{(y-1)}}{\frac{(y-1)}{(y+1)} + \frac{(y+1)}{(y-1)}} \right) \left(\frac{(y+1)(y-1)}{\frac{1}{(y+1)(y-1)}} \right) = \frac{(y-1)(y-1) - (y+1)(y+1)}{(y-1)(y-1) + (y+1)(y+1)} \\ &= \frac{(y^2 - 2y + 1) - (y^2 + 2y + 1)}{(y^2 - 2y + 1) + (y^2 + 2y + 1)} = \frac{y^2 - 2y + 1 - y^2 - 2y - 1}{y^2 - 2y + 1 + y^2 + 2y + 1} = \frac{-4y}{2y^2 + 2} \\ &= \frac{-4y}{2(y^2 + 1)} = \frac{-2y}{(y^2 + 1)} = \frac{-2y}{y^2 + 1} \end{aligned}$$

$$34) \quad x - \frac{1}{x - \frac{1}{2}}$$

Since the expression is more than one term, we should do our operations in stages, 1st the complex fraction:

$$GLCD = 2$$

$$\frac{1}{x - \frac{1}{2}} = \left(\begin{array}{c} \frac{1}{1} \\ \frac{1}{x - \frac{1}{2}} \\ \hline 1 \end{array} \right) \left(\begin{array}{c} 2 \\ \frac{1}{2} \\ \hline 1 \end{array} \right) = \frac{2}{2x-1}$$

Now we can use this result in the original expression:

$$LCD = (2x-1)$$

$$x - \frac{1}{x - \frac{1}{2}} = x - \frac{2}{2x-1} = \frac{x}{1} - \frac{2}{(2x-1)} = \frac{x(2x-1)-2}{(2x-1)} = \frac{2x^2-x-2}{(2x-1)} = \frac{2x^2-x-2}{2x-1}$$

$$36) \quad 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{2}}}$$

Just like exercise 34, this one has multiple terms and complex fractions. We should start with the complex fraction of the denominator:

$$GLCD1 = 2$$

$$\frac{1}{1 - \frac{1}{2}} = \left(\begin{array}{c} \frac{1}{1} \\ \frac{1}{1 - \frac{1}{2}} \\ \hline 1 \end{array} \right) \left(\begin{array}{c} 2 \\ \frac{1}{2} \\ \hline 1 \end{array} \right) = \frac{2}{2-1} = \frac{2}{1} = 2$$

Now we replace this answer to the complex fraction of the denominator and continue:

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{2}}} = 1 - \frac{1}{1-2} = 1 - \frac{1}{(-1)} = 1 - (-1) = 1 + 1 = 2$$

$$38) \quad \frac{2 + \frac{1}{x - \frac{3}{2 - \frac{1}{x - \frac{1}{3}}}}}{2 - \frac{1}{x - \frac{3}{2 + \frac{1}{x - \frac{1}{3}}}}}$$

This exercise has complex fraction within the numerator and denominator. Since the small complex fractions are identical, we just have to solve one to replace on both:

$$GLCD1 = 3$$

$$\frac{1}{x - \frac{1}{3}} = \left(\begin{array}{c} \frac{1}{\frac{1}{x-1}} \\ \hline \frac{1}{1} \end{array} \right) \left(\begin{array}{c} \frac{3}{\frac{1}{3}} \\ \hline \frac{1}{1} \end{array} \right) = \frac{3}{3x-1}$$

Now we can replace the solution above into both numerator and denominator.

$$GLCD2 = (3x-1)$$

$$\frac{2 + \frac{1}{x - \frac{1}{3}}}{2 - \frac{1}{x - \frac{1}{3}}} = \frac{2 + \frac{3}{3x-1}}{2 - \frac{3}{3x-1}} = \left(\begin{array}{c} \frac{2}{\frac{2}{1}} + \frac{\frac{3}{3x-1}}{\frac{3}{(3x-1)}} \\ \hline \frac{2}{1} - \frac{\frac{3}{3x-1}}{\frac{3}{(3x-1)}} \end{array} \right) \left(\begin{array}{c} \frac{(3x-1)}{\frac{1}{(3x-1)}} \\ \hline \frac{1}{1} \end{array} \right) = \frac{2(3x-1)+3}{2(3x-1)-3} = \frac{6x-2+3}{6x-2-3} = \frac{6x+1}{6x-5}$$

$$40) \quad \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$GLCD = x^2(x+h)^2$$

$$\begin{aligned} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} &= \left(\begin{array}{c} \frac{1}{(x+h)^2} - \frac{1}{x^2} \\ \hline \frac{h}{1} \end{array} \right) \left(\begin{array}{c} \frac{x^2(x+h)^2}{1} \\ \hline \frac{x^2(x+h)^2}{1} \end{array} \right) = \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} = \frac{x^2 - (x^2 + 2xh + h^2)}{hx^2(x+h)^2} = \frac{x^2 - x^2 - 2xh - h^2}{hx^2(x+h)^2} \\ &= \frac{-2xh - h^2}{hx^2(x+h)^2} = \frac{h(-2x-h)}{hx^2(x+h)^2} = \frac{(-2x-h)}{x^2(x+h)^2} = \frac{-2x-h}{x^2(x+h)^2} \end{aligned}$$

$$42) \quad \frac{\frac{x}{yz} - \frac{y}{xz} + \frac{z}{xy}}{\frac{1}{x^2y^2} - \frac{1}{x^2z^2} + \frac{1}{y^2z^2}}$$

$GLCD = x^2y^2z^2$

$$\begin{aligned} \frac{\frac{x}{yz} - \frac{y}{xz} + \frac{z}{xy}}{\frac{1}{x^2y^2} - \frac{1}{x^2z^2} + \frac{1}{y^2z^2}} &= \left(\frac{\frac{x}{yz} - \frac{y}{xz} + \frac{z}{xy}}{\frac{1}{x^2y^2} - \frac{1}{x^2z^2} + \frac{1}{y^2z^2}} \right) \left(\frac{x^2y^2z^2}{x^2y^2z^2} \right) = \frac{x(x^2yz) - y(xy^2z) + z(xyz^2)}{z^2 - y^2 + x^2} \\ &= \frac{x^3yz - xy^3z + xyz^3}{z^2 - y^2 + x^2} = \frac{xyz(x^2 - y^2 + z^2)}{x^2 - y^2 + z^2} = \frac{xyz(x^2 - y^2 + z^2)}{(x^2 - y^2 + z^2)} \\ &= \frac{xyz(1)}{(1)} = \frac{xyz}{1} = xyz \end{aligned}$$

$$44) \quad \frac{\frac{y^2 - 5x - 14}{y^2 + 3y - 10}}{\frac{y^2 - 8x + 7}{y^2 + 6y + 5}}$$

$GLCD = (y+5)(y+1)(y-2)$

$$\begin{aligned} \frac{\frac{y^2 - 5x - 14}{y^2 + 3y - 10}}{\frac{y^2 - 8x + 7}{y^2 + 6y + 5}} &= \frac{\frac{(y+2)(y-7)}{(y+5)(y-2)}}{\frac{(y-1)(y-7)}{(y+5)(y+1)}} = \left(\frac{\frac{(y+2)(y-7)}{(y+5)(y-2)}}{\frac{(y-1)(y-7)}{(y+5)(y+1)}} \right) \left(\frac{\frac{(y+5)(y+1)(y-2)}{1}}{\frac{(y+5)(y+1)(y-2)}{1}} \right) = \frac{(y+2)(y-7)(y+1)}{(y-1)(y-7)(y-2)} \\ &= \frac{(y+2)(1)(y+1)}{(y-1)(1)(y-2)} = \frac{(y+2)(y+1)}{(y-1)(y-2)} \end{aligned}$$

$$46) \quad \frac{\frac{6}{x+5} - 7}{\frac{8}{x+5} - \frac{9}{x+3}}$$

$GLCD = (x+5)(x+3)$

$$\begin{aligned} \frac{\frac{6}{x+5} - 7}{\frac{8}{x+5} - \frac{9}{x+3}} &= \left(\frac{\frac{6}{x+5} - 7}{\frac{8}{x+5} - \frac{9}{x+3}} \right) \left(\frac{\frac{(x+5)(x+3)}{1}}{\frac{(x+5)(x+3)}{1}} \right) = \frac{6(x+3) - 7(x+5)(x+3)}{8(x+3) - 9(x+5)} = \frac{6x + 18 - 7(x^2 + 8x + 15)}{8x + 24 - 9x - 45} \\ &= \frac{6x + 18 - 7x^2 - 56x - 105}{8x + 24 - 9x - 45} = \frac{-7x^2 - 50x - 87}{-x - 21} = \frac{-(7x^2 + 50x + 87)}{-(x + 21)} \\ &= \frac{7x^2 + 50x + 87}{x + 21} = \frac{(7x + 29)(x + 3)}{x + 21} \end{aligned}$$

$$48) \quad \frac{\frac{9}{a-7} + \frac{8}{2a+3}}{\frac{10}{2a^2 - 11a - 21}}$$

$$GLCD = (2a+3)(a-7)$$

$$\begin{aligned} \frac{\frac{9}{a-7} + \frac{8}{2a+3}}{\frac{10}{2a^2 - 11a - 21}} &= \frac{\frac{9}{(a-7)} + \frac{8}{(2a+3)}}{\frac{10}{(2a+3)(a-7)}} = \left(\frac{\frac{9}{(a-7)} + \frac{8}{(2a+3)}}{\frac{10}{(2a+3)(a-7)}} \right) \left(\frac{(2a+3)(a-7)}{\frac{1}{(2a+3)(a-7)}} \right) = \frac{9(2a+3) + 8(a-7)}{10} \\ &= \frac{18a+27+8a-56}{10} = \frac{26a-29}{10} \end{aligned}$$

$$50) \quad \frac{\frac{1}{k^2 - 7k + 12}}{\frac{1}{k-3} + \frac{1}{k-4}}$$

$$GLCD = (k-3)(k-4)$$

$$\begin{aligned} \frac{\frac{1}{k^2 - 7k + 12}}{\frac{1}{k-3} + \frac{1}{k-4}} &= \frac{\frac{1}{(k-3)(k-4)}}{\frac{1}{(k-3)} + \frac{1}{(k-4)}} = \left(\frac{\frac{1}{(k-3)(k-4)}}{\frac{1}{(k-3)} + \frac{1}{(k-4)}} \right) \left(\frac{(k-3)(k-4)}{\frac{1}{(k-3)(k-4)}} \right) = \frac{1}{(k-4)+(k-3)} = \frac{1}{k-4+k-3} \\ &= \frac{1}{2k-7} \end{aligned}$$