

Note: In this section, we are doing same operations that we did in section R.4 but with factorable expressions. Therefore, don't forget the phrase "factor before canceling".

Additional examples

$$2) \quad \frac{5}{6} \cdot \frac{7}{8} = \frac{(5)(7)}{(6)(8)} = \frac{35}{48}$$

$$4) \quad \frac{3}{8} \div \frac{5}{4} = \left(\frac{3}{8} \right) \left(\frac{4}{5} \right) = \frac{(3)(4)}{(8)(5)} = \frac{(3)(1)}{(2)(5)} = \frac{3}{10}$$

$$6) \quad \frac{6}{5} \cdot \frac{10}{36} \div \frac{3}{4} = \left(\frac{6}{5} \right) \left(\frac{10}{36} \right) \left(\frac{4}{3} \right) = \frac{(6)(10)(4)}{(5)(36)(3)} = \frac{(1)(2)(4)}{(1)(6)(3)} = \frac{(1)(1)(4)}{(1)(3)(3)} = \frac{4}{9}$$

$$8) \quad \frac{8x^3}{7y^4} \cdot \frac{14y^6}{16x^2} = \left(\frac{8x^3}{7y^4} \right) \left(\frac{14y^6}{16x^2} \right) = \frac{(8x^3)(14y^6)}{(7y^4)(16x^2)} = \frac{(1x)(2y^2)}{(1)(2)} = \frac{(1x)(1y^2)}{(1)(1)} = \frac{y^2}{1} = y^2$$

$$10) \quad \frac{8ab^3}{9a^2b} \div \frac{16a^2b^2}{18ab^3} = \left(\frac{8ab^3}{9a^2b} \right) \left(\frac{18ab^3}{16a^2b^2} \right) = \frac{(8ab^3)(18ab^3)}{(9a^2b)(16a^2b^2)} = \frac{(1b)(2b^2)}{(1a)(2a)} = \frac{(1b)(1b^2)}{(1a)(1a)} = \frac{b^3}{a^2}$$

$$12) \quad \frac{4x^3}{7y^2} \cdot \frac{6z^5}{5x^6} \div \frac{24z^2}{35x^6} = \left(\frac{4x^3}{7y^2} \right) \left(\frac{6z^5}{5x^6} \right) \left(\frac{35x^6}{24z^2} \right) = \frac{(4x^3)(6z^5)(35x^6)}{(7y^2)(5x^6)(24z^2)} = \frac{(4x^3)(1z^3)(7)}{(7y^2)(1)(4)} = \frac{(1x^3)(1z^3)(1)}{(1y^2)(1)(1)} = \frac{x^3z^3}{y^2}$$

$$14) \quad \frac{x^2 - 16}{x^2 - 25} \cdot \frac{x-5}{x-4} = \left(\frac{x^2 - 16}{x^2 - 25} \right) \left(\frac{x-5}{x-4} \right) = \left(\frac{(x+4)(x-4)}{(x+5)(x-5)} \right) \left(\frac{(x-5)}{(x-4)} \right) = \frac{(x+4)(x-4)(x-5)}{(x+5)(x-5)(x-4)} = \frac{(x+4)(1)(1)}{(x+5)(1)(1)}$$

$$= \frac{(x+4)}{(x+5)} = \frac{x+4}{x+5}$$

$$\begin{aligned}
 16) \quad & \frac{y-1}{y^2-y-6} \cdot \frac{y^2+5y+6}{y^2-1} \\
 & \frac{y-1}{y^2-y-6} \cdot \frac{y^2+5y+6}{y^2-1} = \left(\frac{(y-1)}{(y+2)(y-3)} \right) \left(\frac{(y+3)(y+2)}{(y+1)(y-1)} \right) = \frac{(y-1)(y+3)(y+2)}{(y+2)(y-3)(y+1)(y-1)} \\
 & = \frac{(1)(y+3)(1)}{(1)(y-3)(y+1)(1)} = \frac{(y+3)}{(y-3)(y+1)} = \frac{y+3}{(y-3)(y+1)}
 \end{aligned}$$

$$\begin{aligned}
 18) \quad & \frac{x^2+5x+1}{4x-4} \cdot \frac{x-1}{x^2+5x+1} \\
 & \frac{x^2+5x+1}{4x-4} \cdot \frac{x-1}{x^2+5x+1} = \left(\frac{x^2+5x+1}{4(x-1)} \right) \left(\frac{(x-1)}{x^2+5x+1} \right) = \frac{(x^2+5x+1)(x-1)}{4(x-1)(x^2+5x+1)} = \frac{(1)(1)}{4(1)(1)} = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 20) \quad & \frac{y}{x} \div \frac{xy}{xy-1} \\
 & \frac{y}{x} \div \frac{xy}{xy-1} = \left(\frac{y}{x} \right) \left(\frac{(xy-1)}{xy} \right) = \frac{(1)(xy-1)}{(x)(x1)} = \frac{(xy-1)}{x^2} = \frac{xy-1}{x^2}
 \end{aligned}$$

$$\begin{aligned}
 22) \quad & \frac{1}{x^2-9} \div \frac{1}{(x-3)^2} \\
 & \frac{1}{x^2-9} \div \frac{1}{(x-3)^2} = \left(\frac{1}{x^2-9} \right) \left(\frac{(x-3)^2}{1} \right) = \left(\frac{1}{(x+3)(x-3)} \right) \left(\frac{(x-3)(x-3)}{1} \right) = \frac{(x-3)(x-3)}{(x+3)(x-3)} = \frac{(x-3)(1)}{(x+3)(1)} \\
 & = \frac{(x-3)}{(x+3)} = \frac{x-3}{x+3}
 \end{aligned}$$

$$\begin{aligned}
 24) \quad & \frac{y-3}{y^2-6y+9} \div \frac{y-3}{4} \\
 & \frac{y-3}{y^2-6y+9} \div \frac{y-3}{4} = \left(\frac{y-3}{y^2-6y+9} \right) \left(\frac{4}{y-3} \right) = \left(\frac{(y-3)}{(y-3)(y-3)} \right) \left(\frac{4}{(y-3)} \right) = \frac{(y-3)(4)}{(y-3)(y-3)(y-3)} \\
 & = \frac{(1)(4)}{(1)(y-3)(y-3)} = \frac{4}{(y-3)(y-3)} = \frac{4}{(y-3)^2}
 \end{aligned}$$

$$\begin{aligned}
 26) \quad & \frac{7x+3y}{42x^2-17xy-15y^2} \cdot \frac{12x^2-4xy-5y^2}{2x+y} \\
 & \frac{7x+3y}{42x^2-17xy-15y^2} \cdot \frac{12x^2-4xy-5y^2}{2x+y} = \left(\frac{(7x+3y)}{(7x+3y)(6x-5y)} \right) \left(\frac{(2x+y)(6x-5y)}{(2x+y)} \right) \\
 & = \frac{(7x+3y)(2x+y)(6x-5y)}{(7x+3y)(6x-5y)(2x+y)} = \frac{(1)(1)(1)}{(1)(1)(1)} = \frac{1}{1} = 1
 \end{aligned}$$

28) $\frac{a^2 + 7a + 12}{a - 5} \div \frac{a^2 + 9a + 18}{a^2 - 7a + 10}$

$$\begin{aligned}\frac{a^2 + 7a + 12}{a - 5} \div \frac{a^2 + 9a + 18}{a^2 - 7a + 10} &= \left(\frac{a^2 + 7a + 12}{a - 5} \right) \left(\frac{a^2 - 7a + 10}{a^2 + 9a + 18} \right) = \left(\frac{(a+4)(a+3)}{(a-5)} \right) \left(\frac{(a-2)(a-5)}{(a+6)(a+3)} \right) \\ &= \frac{(a+4)(a+3)(a-2)(a-5)}{(a-5)(a+6)(a+3)} = \frac{(a+4)(1)(a-2)(1)}{(a-5)(1)(1)} \\ &= \frac{(a+4)(a-2)}{(a-5)} = \frac{(a+4)(a-2)}{a-5}\end{aligned}$$

30) $\frac{9t^2 - 1}{6t^2 + 7t - 3} \div \frac{27t^3 + 1}{8t^3 + 27}$

$$\begin{aligned}\frac{9t^2 - 1}{6t^2 + 7t - 3} \div \frac{27t^3 + 1}{8t^3 + 27} &= \left(\frac{9t^2 - 1}{6t^2 + 7t - 3} \right) \left(\frac{8t^3 + 27}{27t^3 + 1} \right) = \left(\frac{(3t+1)(3t-1)}{(2t+3)(3t-1)} \right) \left(\frac{(2t+3)(4t^2 - 6t + 9)}{(3t+1)(9t^2 - 3t + 1)} \right) \\ &= \frac{(3t+1)(3t-1)(2t+3)(4t^2 - 6t + 9)}{(2t+3)(3t-1)(3t+1)(9t^2 - 3t + 1)} = \frac{(1)(1)(1)(4t^2 - 6t + 9)}{(1)(1)(1)(9t^2 - 3t + 1)} \\ &= \frac{(4t^2 - 6t + 9)}{(9t^2 - 3t + 1)} = \frac{4t^2 - 6t + 9}{9t^2 - 3t + 1}\end{aligned}$$

32) $\frac{x^2 - 2x + 1}{3x^2 + 7x - 20} \div \frac{x^2 + 3x - 4}{3x^2 - 2x - 5}$

$$\begin{aligned}\frac{x^2 - 2x + 1}{3x^2 + 7x - 20} \div \frac{x^2 + 3x - 4}{3x^2 - 2x - 5} &= \left(\frac{x^2 - 2x + 1}{3x^2 + 7x - 20} \right) \left(\frac{3x^2 - 2x - 5}{x^2 + 3x - 4} \right) = \left(\frac{(x-1)(x-1)}{(x+4)(3x-5)} \right) \left(\frac{(x+1)(3x-5)}{(x+4)(x-1)} \right) \\ &= \frac{(x-1)(x-1)(x+1)(3x-5)}{(x+4)(3x-5)(x+4)(x-1)} = \frac{(x-1)(1)(x+1)(1)}{(x+4)(1)(x+4)(1)} \\ &= \frac{(x-1)(x+1)}{(x+4)(x+4)} = \frac{(x-1)(x+1)}{(x+4)^2}\end{aligned}$$

34) $\frac{3a^2 + 7ab - 20b^2}{a^2 + 5ab + 4b^2} \div \frac{3a^2 - 17ab + 20b^2}{3a - 12b}$

$$\begin{aligned}\frac{3a^2 + 7ab - 20b^2}{a^2 + 5ab + 4b^2} \div \frac{3a^2 - 17ab + 20b^2}{3a - 12b} &= \left(\frac{3a^2 + 7ab - 20b^2}{a^2 + 5ab + 4b^2} \right) \left(\frac{3a - 12b}{3a^2 - 17ab + 20b^2} \right) \\ &= \left(\frac{(a+4b)(3a-5b)}{(a+b)(a+4b)} \right) \left(\frac{(3)(a-4b)}{(3a-5b)(a-4b)} \right) \\ &= \frac{(a+4b)(3a-5b)(3)(a-4b)}{(a+b)(a+4b)(3a-5b)(a-4b)} \\ &= \frac{(1)(1)(3)(1)}{(a+b)(1)(1)(1)} = \frac{3}{(a+b)} = \frac{3}{a+b}\end{aligned}$$

$$\begin{aligned}
 36) \quad & \frac{m^2 + 4m - 21}{m^2 - 12m + 27} \cdot \frac{m^2 - 7m + 12}{m^2 + 3m - 28} \\
 & \frac{m^2 + 4m - 21}{m^2 - 12m + 27} \cdot \frac{m^2 - 7m + 12}{m^2 + 3m - 28} = \left(\frac{(m+7)(m-3)}{(m-3)(m-9)} \right) \left(\frac{(m-3)(m-4)}{(m+7)(m-4)} \right) \\
 & = \frac{(m+7)(m-3)(m-3)(m-4)}{(m-3)(m-9)(m+7)(m-4)} \\
 & = \frac{(1)(1)(m-3)(1)}{(1)(m-9)(1)(1)} = \frac{(m-3)}{(m-9)} = \frac{m-3}{m-9}
 \end{aligned}$$

$$\begin{aligned}
 38) \quad & \frac{12a^2b - 3ab^2 - 42b^3}{9a^2 - 36b^2} \cdot \frac{6a^2 - 15ab + 6b^2}{8a^3b - b^4} \\
 & \frac{12a^2b - 3ab^2 - 42b^3}{9a^2 - 36b^2} \cdot \frac{6a^2 - 15ab + 6b^2}{8a^3b - b^4} = \left(\frac{3b(4a^2 - ab - 14b^2)}{9(a^2 - 4b^2)} \right) \left(\frac{3(2a^2 - 5ab + 2b^2)}{b(8a^3 - b^3)} \right) \\
 & = \left(\frac{(3b)(4a + 7b)(a - 2b)}{(9)(a + 2b)(a - 2b)} \right) \left(\frac{(3)(2a - b)(a - 2b)}{(b)(2a - b)(4a^2 + 2ab + b^2)} \right) \\
 & = \frac{(1)(4a + 7b)(1)(3)(1)(a - 2b)}{(3)(a + 2b)(1)(1)(1)(4a^2 + 2ab + b^2)} \\
 & = \frac{(1)(4a + 7b)(1)(1)(1)(a - 2b)}{(1)(a + 2b)(1)(1)(1)(4a^2 + 2ab + b^2)} \\
 & = \frac{(4a + 7b)(a - 2b)}{(a + 2b)(4a^2 + 2ab + b^2)}
 \end{aligned}$$

$$\begin{aligned}
 40) \quad & \frac{490x^2 - 640}{49x^2 - 112x + 64} \cdot \frac{28x^2 - 95x + 72}{56x^3 - 62x^2 - 144x} \\
 & \frac{490x^2 - 640}{49x^2 - 112x + 64} \cdot \frac{28x^2 - 95x + 72}{56x^3 - 62x^2 - 144x} = \left(\frac{10(49x^2 - 64)}{(7x - 8)(7x - 8)} \right) \left(\frac{(4x - 9)(7x - 8)}{2x(28x^2 - 31x - 72)} \right) \\
 & = \left(\frac{(10)(7x + 8)(7x - 8)}{(7x - 8)(7x - 8)} \right) \left(\frac{(4x - 9)(7x - 8)}{(2x)(7x + 8)(4x - 9)} \right) \\
 & = \frac{(10)(7x + 8)(7x - 8)(4x - 9)(7x - 8)}{(7x - 8)(7x - 8)(2x)(7x + 8)(4x - 9))} \\
 & = \frac{(5)(1)(1)(1)(1)}{(1)(1)(1x)(1)(1)} \\
 & = \frac{5}{x}
 \end{aligned}$$

$$\begin{aligned}
 42) \quad & \frac{2x^4 - 16x}{3x^6 - 48x^2} \cdot \frac{6x^5 + 24x^3}{4x^4 + 8x^3 + 16x^2} \\
 & \frac{2x^4 - 16x}{3x^6 - 48x^2} \cdot \frac{6x^5 + 24x^3}{4x^4 + 8x^3 + 16x^2} = \left(\frac{2x(x^3 - 8)}{3x^2(x^4 - 16)} \right) \left(\frac{6x^3(x^2 + 4)}{4x^2(x^2 + x + 4)} \right) \\
 & = \left(\frac{(2x)(x-2)(x^2 + 2x + 4)}{(3x^2)(x^2 + 4)(x^2 - 4)} \right) \left(\frac{(6x^3)(x^2 + 4)}{(4x^2)(x^2 + x + 4)} \right) \\
 & = \left(\frac{(2x)(x-2)(x^2 + 2x + 4)}{(3x^2)(x^2 + 4)(x+2)(x-2)} \right) \left(\frac{(6x^3)(x^2 + 4)}{(4x^2)(x^2 + x + 4)} \right) \\
 & = \frac{(2x)(x-2)(x^2 + 2x + 4)(6x^3)(x^2 + 4)}{(3x^2)(x^2 + 4)(x+2)(x-2)(4x^2)(x^2 + x + 1)} \\
 & = \frac{(1)(1)(1)(2x)(1)}{(1)(1)(x+2)(1)(2x)(1)} = \frac{(1)(1)(1)(1)(1)}{(1)(1)(x+2)(1)(1)(1)} = \frac{1}{(x-2)} = \frac{1}{x-2}
 \end{aligned}$$

$$\begin{aligned}
 44) \quad & \frac{a^2 - 6ab + 9b^2}{a^2 - 4b^2} \cdot \frac{a^2 - 5ab + 6b^2}{(a-3b)^2} \div \frac{a^2 - 9b^2}{a^2 - ab - 6b^2} \\
 & \frac{a^2 - 6ab + 9b^2}{a^2 - 4b^2} \cdot \frac{a^2 - 5ab + 6b^2}{(a-3b)^2} \div \frac{a^2 - 9b^2}{a^2 - ab - 6b^2} = \left(\frac{a^2 - 6ab + 9b^2}{a^2 - 4b^2} \right) \left(\frac{a^2 - 5ab + 6b^2}{(a-3b)^2} \right) \left(\frac{a^2 - ab - 6b^2}{a^2 - 9b^2} \right) \\
 & = \left(\frac{(a-3b)(a-3b)}{(a+2b)(a-2b)} \right) \left(\frac{(a-2b)(a-3b)}{(a-3b)(a-3b)} \right) \left(\frac{(a+2b)(a-3b)}{(a+3b)(a-3b)} \right) \\
 & = \frac{(a-3b)(a-3b)(a-2b)(a-3b)(a+2b)(a-3b)}{(a+2b)(a-2b)(a-3b)(a-3b)(a+3b)(a-3b)} \\
 & = \frac{(1)(1)(1)(a-3b)(1)(1)}{(1)(1)(1)(1)(a+3b)(1)} = \frac{(a-3b)}{(a+3b)} = \frac{a-3b}{a+3b}
 \end{aligned}$$

$$\begin{aligned}
 46) \quad & \frac{8y^2 + 18y - 5}{21y^2 - 16y + 3} \cdot \frac{35y^2 - 22y + 3}{6y^2 + 17y + 5} \div \frac{16y^2 - 1}{9y^2 - 1} \\
 & \frac{8y^2 + 18y - 5}{21y^2 - 16y + 3} \cdot \frac{35y^2 - 22y + 3}{6y^2 + 17y + 5} \div \frac{16y^2 - 1}{9y^2 - 1} = \left(\frac{8y^2 + 18y - 5}{21y^2 - 16y + 3} \right) \left(\frac{35y^2 - 22y + 3}{6y^2 + 17y + 5} \right) \left(\frac{9y^2 - 1}{16y^2 - 1} \right) \\
 & = \left(\frac{(2y+5)(4y-1)}{(7y-3)(3y-1)} \right) \left(\frac{(7y-3)(5y-1)}{(2y+5)(3y+1)} \right) \left(\frac{(3y+1)(3y-1)}{(4y+1)(4y-1)} \right) \\
 & = \frac{(2y+5)(4y-1)(7y-3)(5y-1)(3y+1)(3y-1)}{(7y-3)(3y-1)(2y+5)(3y+1)(4y+1)(4y-1)} \\
 & = \frac{(1)(1)(1)(5y-1)(1)(1)}{(1)(1)(1)(1)(4y+1)(1)} = \frac{(5y-1)}{(4y+1)} = \frac{5y-1}{4y+1}
 \end{aligned}$$

48)
$$\frac{ax+bx+2a+2b}{ax-3a+bx-3b} \cdot \frac{ax-bx-3a+3b}{ax-bx-2a+2b}$$

$$\frac{ax+bx+2a+2b}{ax-3a+bx-3b} \cdot \frac{ax-bx-3a+3b}{ax-bx-2a+2b} = \left(\frac{x(a+b)+2(a+b)}{a(x-3)+b(x-3)} \right) \left(\frac{x(a-b)-3(a-b)}{x(a-b)-2(a-b)} \right)$$

$$= \left(\frac{(a+b)(x+2)}{(x-3)(a+b)} \right) \left(\frac{(a-b)(x-3)}{(a-b)(x-2)} \right) = \frac{(a+b)(x+2)(a-b)(x-3)}{(x-3)(a+4)(a-b)(x-2)}$$

$$= \frac{(1)(x+2)(1)(1)}{(1)(1)(1)(x-2)} = \frac{(x+2)}{(x-2)} = \frac{x+2}{x-2}$$

50)
$$\frac{4xb-8b+12x-24}{xb^2+3b^2+3xb+9b} \div \frac{4xb-8b-8x+16}{xb^2+3b^2-2xb-6b}$$

$$\frac{4xb-8b+12x-24}{xb^2+3b^2+3xb+9b} \div \frac{4xb-8b-8x+16}{xb^2+3b^2-2xb-6b} = \left(\frac{4xb-8b+12x-24}{xb^2+3b^2+3xb+9b} \right) \left(\frac{xb^2+3b^2-2xb-6b}{4xb-8b-8x+16} \right)$$

$$= \left(\frac{4\{xb-2b+3x-6\}}{b\{xb+3b+3x+9\}} \right) \left(\frac{b\{xb+3b-2x-6\}}{4\{xb-2b-2x+4\}} \right)$$

$$= \left(\frac{4\{b(x-2)+3(x-2)\}}{b\{b(x+3)+3(x+3)\}} \right) \left(\frac{b\{b(x+3)-2(x+3)\}}{4\{b(x-2)-2(x-2)\}} \right)$$

$$= \left(\frac{(4)(x-2)(b+3)}{(b)(x+3)(b+3)} \right) \left(\frac{(b)(x+3)(b-2)}{(4)(x-2)(b-2)} \right)$$

$$= \frac{(4)(x-2)(b+3)(b)(x+3)(b-2)}{(b)(x+3)(b+3)(4)(x-2)(b-2)}$$

$$= \frac{(1)(1)(1)(1)(1)}{(1)(1)(1)(1)(1)} = \frac{1}{1} = 1$$

52)
$$\frac{x^3+2x^2-9x-18}{x^4+3x^3-4x^2-12x} \cdot \frac{x^3+5x^2+6x}{x^2-x-6}$$

$$\frac{x^3+2x^2-9x-18}{x^4+3x^3-4x^2-12x} \cdot \frac{x^3+5x^2+6x}{x^2-x-6} = \left(\frac{x^2(x+2)-9(x+2)}{x\{x^3+3x^2-4x-12\}} \right) \left(\frac{x\{x^2+5x+6\}}{(x+2)(x-3)} \right)$$

$$= \left(\frac{(x+2)\{x^2-9\}}{x\{x^2(x+3)-4(x+3)\}} \right) \left(\frac{x(x+2)(x+3)}{(x+2)(x-3)} \right)$$

$$= \left(\frac{(x+2)(x+3)(x-3)}{x(x+3)\{x^2-4\}} \right) \left(\frac{x(x+2)(x+3)}{(x+2)(x-3)} \right)$$

$$= \left(\frac{(x+2)(x+3)(x-3)}{(x)(x+3)(x+2)(x-2)} \right) \left(\frac{(x)(x+2)(x+3)}{(x+2)(x-3)} \right)$$

$$= \frac{(x+2)(x+3)(x-3)(x)(x+2)(x+3)}{(x)(x+3)(x+2)(x-2)(x+2)(x-3)}$$

$$= \frac{(1)(1)(1)(1)(1)(x+3)}{(1)(1)(1)(x-2)(1)(1)} = \frac{(x+3)}{(x-2)} = \frac{x+3}{x-2}$$

54)
$$\frac{a^3 - a^2 b}{ac - a} \div \left(\frac{a-b}{c-1} \right)^2$$

$$\frac{a^3 - a^2 b}{ac - a} \div \left(\frac{a-b}{c-1} \right)^2 = \frac{a^3 - a^2 b}{ac - a} \div \frac{(a-b)^2}{(c-1)^2} = \left(\frac{a^3 - a^2 b}{ac - a} \right) \left(\frac{(c-1)^2}{(a-b)^2} \right) = \left(\frac{(a^2)(a-b)}{(a)(c-1)} \right) \left(\frac{(c-1)(c-1)}{(a-b)(a-b)} \right)$$

$$= \frac{(a^2)(a-b)(c-1)(c-1)}{(a)(c-1)(a-b)(a-b)} = \frac{(a)(1)(1)(c-1)}{(1)(1)(1)(a-b)} = \frac{(a)(c-1)}{(a-b)} = \frac{a(c-1)}{a-b}$$

56)
$$\frac{ax - 2a + 2xy - 4y}{ax + 2a - 2xy - 4y} \div \frac{ax + 2a + 2xy + 4y}{ax - 2a - 2xy + 4y}$$

$$\frac{ax - 2a + 2xy - 4y}{ax + 2a - 2xy - 4y} \div \frac{ax + 2a + 2xy + 4y}{ax - 2a - 2xy + 4y} = \left(\frac{ax - 2a + 2xy - 4y}{ax + 2a - 2xy - 4y} \right) \left(\frac{ax - 2a - 2xy + 4y}{ax + 2a + 2xy + 4y} \right)$$

$$= \left(\frac{a(x-2) + 2y(x-2)}{a(x+2) - 2y(x+2)} \right) \left(\frac{a(x-2) - 2y(x-2)}{a(x+2) + 2y(x+2)} \right)$$

$$= \left(\frac{(x-2)(a+2y)}{(x+2)(a-2y)} \right) \left(\frac{(x-2)(a-2y)}{(x+2)(a+2y)} \right)$$

$$= \frac{(x-2)(a+2y)(x-2)(a-2y)}{(x+2)(a-2y)(x+2)(a+2y)}$$

$$= \frac{(x-2)(1)(x-2)(1)}{(x+2)(1)(x+2)(1)} = \frac{(x-2)(x-2)}{(x+2)(x+2)} = \frac{(x-2)^2}{(x+2)^2}$$

58)
$$\frac{k^2 - 1}{k^3 + 1} \div \frac{k^3 - 1}{k - 1} \cdot (k^2 + k + 1)$$

$$\frac{k^2 - 1}{k^3 + 1} \div \frac{k^3 - 1}{k - 1} \cdot (k^2 + k + 1) = \left(\frac{k^2 - 1}{k^3 + 1} \right) \left(\frac{k - 1}{k^3 - 1} \right) \left(\frac{(k^2 + k + 1)}{1} \right)$$

$$= \left(\frac{(k+1)(k-1)}{(k+1)(k^2 - k + 1)} \right) \left(\frac{(k-1)}{(k-1)(k^2 + k + 1)} \right) \left(\frac{(k^2 + k + 1)}{1} \right)$$

$$= \frac{(k+1)(k-1)(k-1)(k^2 + k + 1)}{(k+1)(k^2 - k + 1)(k-1)(k^2 + k + 1)} = \frac{(1)(k-1)(1)(1)}{(1)(k^2 - k + 1)(1)(1)} = \frac{(k-1)}{(k^2 - k + 1)} = \frac{k-1}{k^2 - k + 1}$$

60)
$$\frac{r^3 - s^3}{s - r} \cdot \frac{(r+s)^2}{r^2 + s^2}$$

$$\frac{r^3 - s^3}{s - r} \cdot \frac{(r+s)^2}{r^2 + s^2} = \left(\frac{(r-s)(r^2 + rs + s^2)}{(s-r)} \right) \left(\frac{(r+s)(r+s)}{(r^2 + s^2)} \right) = \frac{(r-s)(r^2 + rs + s^2)(r+s)(r+s)}{(s-r)(r^2 + s^2)}$$

$$= \frac{\{-s+r\}(r^2 + rs + s^2)(r+s)(r+s)}{(s-r)(r^2 + s^2)} = \frac{\{-1(s-r)\}(r^2 + rs + s^2)(r+s)(r+s)}{(s-r)(r^2 + s^2)}$$

$$= \frac{\{-1(1)\}(r^2 + rs + s^2)(r+s)(r+s)}{(1)(r^2 + s^2)} = \frac{-(r^2 + rs + s^2)(r+s)^2}{r^2 + s^2}$$

$$62) \quad (4x+8) \cdot \frac{x}{x+2}$$

$$(4x+8) \cdot \frac{x}{x+2} = \left(\frac{(4x+8)}{1} \right) \left(\frac{x}{x+2} \right) = \left(\frac{(4)(x+2)}{1} \right) \left(\frac{(x)}{(x+2)} \right) = \frac{(4)(1)(x)}{(1)} = \frac{4x}{1} = 4x$$

$$64) \quad (x^2 - 49) \cdot \frac{5}{x+7}$$

$$\begin{aligned} (x^2 - 49) \cdot \frac{5}{x+7} &= \left(\frac{(x^2 - 49)}{1} \right) \left(\frac{5}{x+7} \right) = \left(\frac{(x+7)(x-7)}{1} \right) \left(\frac{(5)}{(x+7)} \right) = \frac{(x+7)(x-7)(5)}{(x+7)} = \frac{(1)(x-7)(5)}{(1)} \\ &= \frac{5(x-7)}{1} = 5(x-7) = 5x - 35 \end{aligned}$$

$$66) \quad (x^2 - 3x + 2) \cdot \frac{-1}{x-2}$$

$$\begin{aligned} (x^2 - 3x + 2) \cdot \frac{-1}{x-2} &= \left(\frac{(x^2 - 3x + 2)}{1} \right) \left(\frac{-1}{x-2} \right) = \left(\frac{(x-1)(x-2)}{1} \right) \left(\frac{(-1)}{(x-2)} \right) = \frac{(x-1)(x-2)(-1)}{(x-2)} \\ &= \frac{(x-1)(1)(-1)}{(1)} = \frac{-(x-1)}{1} = -(x-1) = (-x+1) = 1-x \end{aligned}$$

$$68) \quad (y+1)(y+4)(y-1) \cdot \frac{3}{y^2 - 1}$$

$$\begin{aligned} (y+1)(y+4)(y-1) \cdot \frac{3}{y^2 - 1} &= \left(\frac{(y+1)(y+4)(y-1)}{1} \right) \left(\frac{3}{y^2 - 1} \right) = \left(\frac{(y+1)(y+4)(y-1)}{1} \right) \left(\frac{(3)}{(y+1)(y-1)} \right) \\ &= \frac{(y+1)(y+4)(y-1)(3)}{(y+1)(y-1)} = \frac{(1)(y+4)(1)(3)}{(1)(1)} = \frac{3(y+4)}{1} = 3(y+1) \end{aligned}$$

$$70) \quad a(a+3)(a-3) \cdot \frac{a-1}{a^2 - 3a}$$

$$\begin{aligned} a(a+3)(a-3) \cdot \frac{a-1}{a^2 - 3a} &= \left(\frac{a(a+3)(a-3)}{1} \right) \left(\frac{a-1}{a^2 - 3a} \right) = \left(\frac{(a)(a+3)(a-3)}{1} \right) \left(\frac{(a-1)}{(a)(a-3)} \right) \\ &= \frac{(a)(a+3)(a-3)(a-1)}{(a)(a-3)} = \frac{(1)(a+3)(1)(a-1)}{(1)(1)} = \frac{(a+3)(a-1)}{1} = (a+3)(a-1) \end{aligned}$$