

Note: If the denominator is a single term, then use the split the fraction bar technique shown in class. If the denominator is more than one term, then Polynomial long division must be used.

Additional examples

$$2) \quad \frac{6x^3 + 12x^2 - 9x}{3x}$$

$$\frac{6x^3 + 12x^2 - 9x}{3x} = \frac{6x^3}{3x} + \frac{12x^2}{3x} - \frac{9x}{3x} = 2x^2 + 4x - 3$$

$$4) \quad \frac{12x^5 - 18x^4 - 6x^3}{6x^3}$$

$$\frac{12x^5 - 18x^4 - 6x^3}{6x^3} = \frac{12x^5}{6x^3} - \frac{18x^4}{6x^3} - \frac{6x^3}{6x^3} = 2x^2 - 3x - 1$$

$$6) \quad \frac{6y^4 - 3y^3 + 18y^2}{9y^2}$$

$$\frac{6y^4 - 3y^3 + 18y^2}{9y^2} = \frac{6y^4}{9y^2} - \frac{3y^3}{9y^2} + \frac{18y^2}{9y^2} = \frac{2y^2}{3} - \frac{y}{3} + 6 = \frac{2}{3}y^2 - \frac{1}{3}y + 6$$

$$8) \quad \frac{-9x^5 + 10x^3 - 12x}{-6x^4}$$

$$\frac{-9x^5 + 10x^3 - 12x}{-6x^4} = \frac{-(-9x^5 + 10x^3 - 12x)}{6x^4} = \frac{9x^5 - 10x^3 + 12x}{6x^4} = \frac{9x^5}{6x^4} - \frac{10x^3}{6x^4} + \frac{12x}{6x^4} = \frac{3x}{2} - \frac{5}{3x} + \frac{2}{x^3}$$

$$10) \quad \frac{a^2b + ab^2}{ab}$$

$$\frac{a^2b + ab^2}{ab} = \frac{a^2b}{ab} + \frac{ab^2}{ab} = a + b$$

$$12) \quad \frac{9x^4y^4 + 18x^3y^4 - 27x^2y^4}{-9xy^3}$$

$$\begin{aligned} \frac{9x^4y^4 + 18x^3y^4 - 27x^2y^4}{-9xy^3} &= \frac{-(9x^4y^4 + 18x^3y^4 - 27x^2y^4)}{9xy^3} = \frac{-9x^4y^4 - 18x^3y^4 + 27x^2y^4}{9xy^3} \\ &= \frac{-9x^4y^4}{9xy^3} - \frac{18x^3y^4}{9xy^3} + \frac{27x^2y^4}{9xy^3} = -x^3y - 2x^2y + 3xy \end{aligned}$$

$$14) \quad \frac{x^2 - x - 6}{x + 2}$$

$$\frac{x^2 - x - 6}{x + 2} = \frac{(x+2)(x-3)}{(x+2)} = \frac{(x-3)}{1} = x-3$$

$$16) \quad \frac{2a^2 + 3a - 9}{2a - 3}$$

$$\frac{2a^2 + 3a - 9}{2a - 3} = \frac{(a+3)(2a-3)}{(2a-3)} = \frac{(a+3)}{1} = a+3$$

$$18) \quad \frac{5x^2 - 26xy - 24y^2}{5x + 4y}$$

$$\frac{5x^2 - 26xy - 24y^2}{5x + 4y} = \frac{(5x+4y)(x-6y)}{(5x+4y)} = \frac{(x-6y)}{1} = x-6y$$

$$20) \quad \frac{x^3 + 8}{x + 2}$$

$$\frac{x^3 + 8}{x + 2} = \frac{(x+2)(x^2 - 2x + 4)}{(x+2)} = \frac{(x^2 - 2x + 4)}{1} = x^2 - 2x + 4$$

$$22) \quad \frac{y^4 - 81}{y - 3}$$

$$\frac{y^4 - 81}{y - 3} = \frac{(y^2 + 9)(y^2 - 9)}{(y-3)} = \frac{(y^2 + 9)(y+3)(y-3)}{(y-3)} = \frac{(y^2 + 9)(y+3)}{1} = (y^2 + 9)(y+3)$$

$$24) \quad \frac{x^3 + 2x^2 - 25x - 50}{x + 5}$$

$$\frac{x^3 + 2x^2 - 25x - 50}{x + 5} = \frac{x^2(x+2) - 25(x+2)}{(x+5)} = \frac{(x+2)\{x^2 - 25\}}{(x+5)} = \frac{(x+2)(x+5)(x-5)}{(x+5)}$$

$$= \frac{(x+2)(x-5)}{1} = (x+2)(x-5)$$

$$26) \quad \frac{9x^3 + 18x^2 - 4x - 8}{x + 2}$$

$$\frac{9x^3 + 18x^2 - 4x - 8}{x + 2} = \frac{9x^2(x+2) - 4(x+2)}{(x+2)} = \frac{(x+2)\{9x^2 - 4\}}{(x+2)} = \frac{\{9x^2 - 4\}}{1} = 9x^2 - 4$$

$$28) \quad \frac{x^2 + 4x - 8}{x - 3}$$

$$\begin{array}{r} x+7 \\ x-3 \overline{) x^2 + 4x - 8} \\ -(x^2 - 3x) \\ \hline +7x - 8 \\ -(7x - 21) \\ \hline +13 \end{array}$$

$$\frac{x^2 + 4x - 8}{x - 3} = x + 7 + \frac{(+13)}{x-3} = x + 7 + \frac{13}{x-3}$$

$$30) \quad \frac{8x^2 - 26x - 9}{2x - 7}$$

$$\begin{array}{r} 4x + 1 \\ 2x - 7 \overline{)8x^2 - 26x - 9} \\ -(8x^2 - 28x) \\ \hline +2x - 9 \\ -(2x - 7) \\ \hline -2 \end{array}$$

$$\frac{8x^2 - 26x - 9}{2x - 7} = 4x + 1 + \frac{(-2)}{2x - 7} = 4x + 1 - \frac{2}{2x - 7}$$

$$32) \quad \frac{3x^3 - 5x^2 + 2x - 1}{x - 2}$$

$$\begin{array}{r} 3x^2 + x + 4 \\ x - 2 \overline{)3x^3 - 5x^2 + 2x - 1} \\ -(3x^3 - 6x^2) \\ \hline +x^2 + 2x \\ -(x^2 - 2x) \\ \hline +4x - 1 \\ -(4x - 8) \\ \hline +7 \end{array}$$

$$\frac{3x^3 - 5x^2 + 2x - 1}{x - 2} = 3x^2 + x + 4 + \frac{(+7)}{x - 2} = 3x^2 + x + 4 + \frac{7}{x - 2}$$

$$34) \quad \frac{3y^3 - 19y^2 + 17y + 4}{3y - 4}$$

$$\begin{array}{r} y^2 - 5y - 1 \\ 3y - 4 \overline{)3y^3 - 19y^2 + 17y + 4} \\ -(3y^3 - 4y^2) \\ \hline -15y^2 + 17y \\ -(-15y^2 + 20y) \\ \hline -3y + 4 \\ -(-3y + 4) \\ \hline 0 \end{array}$$

$$\frac{3y^3 - 19y^2 + 17y + 4}{3y - 4} = y^2 - 5y - 1$$

$$36) \quad \begin{array}{r} 6x^3 + 7x^2 - x + 3 \\ \hline 3x^2 - x + 1 \end{array} \quad \begin{array}{r} 2x + 3 \\ \hline 6x^3 + 7x^2 - x + 3 \\ -(6x^3 - 2x^2 + 2x) \\ \hline 9x^2 - 3x + 3 \\ -(9x^2 - 3x + 3) \\ \hline 0 \end{array}$$

$$\frac{6x^3 + 7x^2 - x + 3}{3x^2 - x + 1} = 2x + 3$$

$$38) \quad \begin{array}{r} 9y^3 - 6y^2 + 8 \\ \hline 3y - 3 \end{array} \quad \begin{array}{r} 3y^2 + y + 1 \\ \hline 9y^3 - 6y^2 + 0y + 8 \\ -(9y^3 - 9y^2) \\ \hline +3y^2 + 0y \\ -(3y^2 - 3y) \\ \hline +3y + 8 \\ -(3y - 3) \\ \hline +11 \end{array}$$

$$\frac{9y^3 - 6y^2 + 8}{3y - 3} = 3y^2 + y + 1 + \frac{(+11)}{3y - 3} = 3y^2 + y + 1 + \frac{11}{3y - 3}$$

$$40) \quad \begin{array}{r} a^4 + a^3 - 1 \\ \hline a + 2 \end{array} \quad \begin{array}{r} a^3 - a^2 + 2a - 4 \\ \hline a^4 + a^3 + 0a^2 + 0a - 1 \\ -(a^4 + 2a^3) \\ \hline -a^3 + 0a^2 \\ -(-a^3 - 2a^2) \\ \hline +2a^2 + 0a \\ -(2a^2 + 4a) \\ \hline -4a - 1 \\ -(-4a - 8) \\ \hline +7 \end{array}$$

$$\begin{aligned} \frac{a^4 + a^3 - 1}{a + 2} &= a^3 - a^2 + 2a - 4 + \frac{(+7)}{a + 2} \\ &= a^3 - a^2 + 2a - 4 + \frac{7}{a + 2} \end{aligned}$$

$$\begin{array}{r}
 42) \quad \frac{y^4 - 81}{y - 3} \\
 \overline{y - 3) \quad y^4 + 0y^3 + 0y^2 + 0y - 81} \\
 \underline{-(y^4 - 3y^3)} \\
 \qquad 3y^3 + 0y^2 \\
 \underline{-(3y^3 - 9y^2)} \\
 \qquad + 9y^2 + 0y \\
 \underline{-(9y^2 - 27y)} \\
 \qquad + 27y - 81 \\
 \underline{-(27y - 81)} \\
 \qquad 0
 \end{array}
 \qquad \frac{y^4 - 81}{y - 3} = y^3 + 3y^2 + 9y + 27$$

$$\begin{array}{r}
 44) \quad \frac{2x^4 + x^3 + 4x - 3}{2x^2 - x + 3} \\
 \overline{2x^2 - x + 3) \quad 2x^4 + x^3 + 0x^2 + 4x - 3} \\
 \underline{-(2x^4 - x^3 + 3x^2)} \\
 \qquad + 2x^3 - 3x^2 + 4x \\
 \underline{-(2x^3 - x^2 + 3x)} \\
 \qquad - 2x^2 + x - 3 \\
 \underline{(-2x^2 + x - 3)} \\
 \qquad 0
 \end{array}
 \qquad \frac{2x^4 + x^3 + 4x - 3}{2x^2 - x + 3} = x^2 + x - 1$$

58) Factor $x^3 + 10x^2 + 29x + 20$ completely if one of its factors is $x + 4$.

$$\begin{array}{r}
 x + 4 \overline{) x^3 + 10x^2 + 29x + 20} \\
 \underline{-(x^3 + 4x^2)} \\
 \qquad + 6x^2 + 29x \\
 \underline{-(6x^2 + 24x)} \\
 \qquad + 5x + 20 \\
 \underline{-(5x - 20)} \\
 \qquad 0
 \end{array}
 \qquad \begin{aligned}
 x^3 + 10x^2 + 29x + 20 &= (x + 4)(x^2 + 6x + 5) \\
 &= (x + 4)(x + 1)(x + 5)
 \end{aligned}$$

- 60) Factor $x^3 + 3x^2 - 10x - 24$ completely if one of its factors is $x+2$.

$$\begin{array}{r} x^2 + x - 12 \\ \hline x+2 \overline{) x^3 + 3x^2 - 10x - 24} \\ -(x^3 + 2x^2) \\ \hline +x^2 - 10x \\ -(x^2 + 2x) \\ \hline -12x - 24 \\ -(-12x - 24) \\ \hline 0 \end{array}$$
$$\begin{aligned} x^3 + 3x^2 - 10x - 24 &= (x+2)(x^2 + x - 12) \\ &= (x+2)(x+4)(x-3) \end{aligned}$$