Math 392 Quiz 8B

March 18, 2019

Name: ANSWERS

Instructions: No calculators! Answer <u>all</u> problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

   (a) $\int_C f(x,y)\,dy = \int_a^b f(x(t),y(t))\,y'(t)\,dt$

   (b) $\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

   (c) $\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

   (where C is a smooth curve parametrized by $\vec{r}(t) =< x(t), y(t) >$. No shorthand, flesh out full definition.)

2. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F} = \nabla f$ for some scalar function f

3. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

4. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

5. What does it mean to say "$\vec{G}$ is a vector potential of $\vec{F}$"? $\vec{F} = \text{curl}\,\vec{G}$

6. Let $\vec{F} =< P(x,y), Q(x,y) >$ be defined on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

7. Let $\vec{F} =< P(x,y), Q(x,y), R(x,y) >$ be defined on an open, simply connected domain D. Suppose P, Q, and R have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\text{curl}\,\vec{F} = \vec{0}$

8. Let $\vec{F} =< yz - x^3\,, y^2 + xz\,,\ xy - z^3 >$:

   (a) Compute $curl\vec{F} = <0,0,0> = \vec{0}$

   (b) Compute $\int_C \vec{F}\cdot d\vec{r}$, where C is the negatively oriented curve in the yz-plane given by the line segment from (-1,0,1) to (1,0,1), followed by the line segment from (1,0,1) to the origin, followed by another line segment from the origin to (-1,0,1). $\int_C \vec{F}\cdot d\vec{r} = 0$

   (c) Justify/show your work for part (b). Begin your answer below, you may use the reverse side of this sheet if necessary.

   Since $\text{curl}\,\vec{F} = \vec{0}$ and $\vec{F}$ has continuous partials, $\vec{F}$ is conservative. Thus, $\int_C \vec{F}\cdot d\vec{r} = 0$, by the Fund. Thm. for line integrals.