Math 392 Quiz 7A

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Name: ANSWERS

Instructions: No calculators! Answer all problems in the space provided! Do your rough work on scrap paper.

1. Define the following:

(a) $\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$

(b) $\int_C \vec{F}\cdot d\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$

(c) $\int_C f(x,y)\,dx = \int_a^b f(x(t),y(t))\,x'(t)\,dt$

(where C is a smooth curve parametrized by $\vec{r}(t) =< x(t), y(t) >$. No shorthand, flesh out full definition.)

2. State the equation in the fundamental theorem for line integrals: $\int_C \nabla f\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$

3. State the equation in Green's Theorem: $\int_C P\,dx + Q\,dy = \iint_D Q_x - P_y\,dA$

4. What does it mean to say "$\vec{F}$ is conservative"? $\vec{F}=\nabla f$ for some scalar function f

5. Let $\vec{F} =< P(x,y), Q(x,y) >$ be defined on an open, simply connected domain D. Suppose P and Q have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

6. Let $\vec{F} =< P(x,y), Q(x,y), R(x,y) >$ be defined on an open, simply connected domain D. Suppose P, Q, and R have continuous first partial derivatives on D. What equation would you use to check if $\vec{F}$ is conservative? $\text{curl}\vec{F} = \vec{0}$

7. Let $\vec{F} =< x\sin y\ ,\ x^2ye^z\ ,\ z\tan(xz) >$, compute:

(a) $curl\vec{F} = \langle -x^2ye^z, -z^2\sec^2(xz), +2xye^z - x\cos y\rangle$

(b) $div\,\vec{F} = \sin y + x^2e^z + \tan(xz) + xz\sec^2(xz)$

8. If $curl\vec{F} = \vec{0}$, then $\vec{F}$ is called Irrotational